

STUDY OF OPTICAL REFRACTION ON NEAR-GROUND PATHS IN ARID ZONE

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Regular and random optical refraction on near-ground paths under conditions of hot climate in arid zone is studied. Diurnal variations of the vertical refraction angle, its dependence on the altitude of a target under observation, and relation of the regular component of the refraction angle variations to the variance of its random fluctuations are the objects of this study. The formulas derived on the basis of the Kolmogorov—Obukhov theory of the surface atmospheric layer and the theory of optical refraction enable us to increase the accuracy of the refraction studies by several times as compared to the well-known technique of metrological measurements of the refraction angle.

In analysis of the light beam propagation in the surface atmospheric layer one should take into consideration the corrections associated with the optical refraction phenomenon.¹ The regular refraction occurs mainly owing to the presence of the constant vertical temperature gradient along the ray path, while the random refraction is engendered by the fast spatiotemporal temperature fluctuations.^{2,3} Under conditions of hot climate of arid zone the underlying surface is strongly heated resulting in the developed thermal processes and large vertical temperature gradients in the surface atmospheric layer. In particular, in the indicated regions the difference between the temperature of underlying surface and air temperature at an altitude of two meters reaches twenty and even more degrees on separate summer days. Such well-developed temperature processes in their turn intensify the refraction effects.

In this paper we present the results of investigations of the regular and random optical refraction on the near-ground paths under conditions of hot climate of arid zone.

To perform the refraction measurements with high accuracy, we used the formulas derived on the basis of the theory of optical refraction and the Kolmogorov—Obukhov theory of the surface layer. Since the final representation of these equations differs from that given in Ref. 2, below we will briefly describe them.

1. CALCULATION OF THE ANGLE OF VERTICAL REFRACTION FOR LARGE TEMPERATURE GRADIENTS

We consider the well-known exact formula used for calculation of the angle of regular refraction on the horizontal near-ground paths²

$$r = \frac{lP}{\alpha_0 T^2} \{\beta_0 + \gamma\}, \quad \gamma = \frac{2}{l^2} \int_0^l \frac{\partial T}{\partial z} x dx, \quad (1)$$

where r is the refraction angle (seconds of arc), l is the path length (m), P is the air pressure (hPa), T is the air temperature (K), $\alpha_0 = 0.123$, $\beta_0 = 0.0342$, and z is the altitude (m).

Since, as a rule, it is difficult to determine directly such a term of Eq. (1) as the temperature lapse rate γ averaged over the path, its various approximations based on

the results of measuring the parameters of the atmosphere at the observation point are commonly used. The most exact approximation is obtained on the basis of the Kolmogorov—Obukhov theory⁴ of the surface atmospheric layer, in which the temperature lapse rate assumes the form⁵

$$\frac{\partial T}{\partial z} = \frac{T^*}{z} \varphi(\zeta), \quad (2)$$

where

$$T^* = \frac{\Delta T(z)}{\Delta f(z)}, \quad \varphi(\zeta) = \zeta \frac{df}{d\zeta}, \quad \zeta = \frac{z}{L^*},$$

$$f = \begin{cases} \ln \zeta + \mu \zeta, & 0 < \zeta, \\ \ln |\zeta|, & \Gamma \leq \zeta \leq 0, \\ v_0 + v\zeta^{-1/3}, & \zeta < \Gamma, \end{cases} \quad (3)$$

T^* and L^* are the characteristic scales of temperature and length, $\mu \approx 10$, $v_0 \approx 0.25$, $v \approx 1.2$, $\Gamma \approx -0.07$ (see Ref. 6), and Δ is the increment to the altitude.

Assuming that the variations of the scales T^* and L^* along the light ray path are insignificant, we substitute Eqs. (2) and (3) into Eq. (1) and finally obtain

$$\gamma = \Delta T \gamma_0, \quad (4)$$

where

$$\gamma_0 = \begin{cases} c^2 H^{-1} / \ln(z_2/z_1) + (1 - c^2) / \Delta z, & 0 < \zeta, \\ H^{-1} / \ln(z_2/z_1), & \Gamma \leq \zeta \leq 0, \\ -\frac{1}{3} H^{-4/3} / (z_2^{-1/3} - z_1^{-1/3}), & \zeta < \Gamma, \end{cases} \quad (5)$$

$$H^{-n} = \frac{2}{l^2} \int_0^l \frac{x dx}{z^n(x)}, \quad c^2 = 1 / \left[1 + \frac{\mu}{L^*} \frac{\Delta z}{\ln(z_2/z_1)} \right],$$

$z(x)$ is the ray path profile, $\Delta z = z_2 - z_1$, z_2 and z_1 are the chosen altitudes of measurement of the atmospheric

meteorological parameters above the observation point ($z_2 > z_1$).

We consider the determination of the parameters ζ and c^2 by means of measuring the altitude increments to the wind velocity Δv . To this end, we use the dependence⁵

$$\zeta/\varphi(\zeta) = \text{Ri}' . \quad (6)$$

Here $\text{Ri}' = \alpha^2 \text{Ri}$ is the Richardson number, $\alpha^2 = \alpha^2(\text{Ri})$ is a certain function, and $\alpha^2 \approx 1$ for $|\zeta| \ll 1$ (see Refs. 4 and 6).

Using formula (6) with allowance for the relations⁴

$$\text{Ri} = \frac{g}{T} \frac{\partial T / \partial z}{(\partial v / \partial z)^2} , \quad (7)$$

$$\frac{\partial T}{\partial z} = \frac{\Delta T}{z} \frac{\varphi}{\Delta f} , \quad \frac{\partial v}{\partial z} = \frac{\Delta v}{z} \frac{\varphi}{\Delta f}$$

one can obtain

$$\zeta = \text{Ri} = \tilde{\text{Ri}} , \quad \Gamma \leq \zeta \leq 0 ,$$

$$c^2 = 1 - \mu \tilde{\text{Ri}} \alpha^2 / \frac{z}{\Delta z} \ln \frac{z_2}{z_1} , \quad (8)$$

where g is the acceleration of gravity,

$$\tilde{\text{Ri}} = \frac{g}{T} \frac{\Delta T}{(\Delta v)^2} z \ln \frac{z_2}{z_1} .$$

Note that the parameter c^2 and the interval of variation of ζ , which enter into Eq. (5), can also be determined by means of the measurements of the air temperature at three or more altitudes.

Equations (1), (4), (5), and (8) allow us to calculate the angle of vertical refraction by means of measuring the increments ΔT and Δv above the observation point.

2. THE VERTICAL PROFILE OF THE REFRACTION ANGLE

Formula (4) for calculating the temperature lapse rate was derived under assumption of constant scales T_* and L_* along the entire light ray path. However, in practice these conditions may be violated (especially in the case of strongly inhomogeneous paths) that calls into question the use of the above-presented relations in such cases. These violations can be controlled indirectly by means of examining the vertical profile of the optical refraction angle. Let us consider it consecutively for the unstable ($\zeta < \Gamma$), neutral ($\Gamma \leq \zeta \leq 0$), and stable ($\zeta > 0$) stratifications of the atmosphere. In what follows the parameters of these stratifications will be indicated by the superscripts U , N , and S , respectively.

The vertical profile of the refraction angle assumes its simplest form for $\zeta \leq 0$ ($\Delta T / \Delta z \leq 0$). In this case from Eqs. (1), (4), and (5) it can be easily obtained that

$$r(t) = \kappa \Delta r(t) + \kappa_0(t) . \quad (9)$$

Here $\kappa = \kappa^U = H^{-4/3} / \Delta H^{-4/3}$ for $\zeta > 0$ and $\kappa = \kappa^N = H^{-1} / \Delta H^{-1}$ for $\Gamma \leq \zeta \leq 0$, $\kappa_0 = \frac{\beta_0 P}{\alpha_0 T^2} [l - \kappa \Delta l]$, and t is the time.

Using Eq. (9) for two targets located at different altitudes along the same viewing direction (with the conventional numbers 1 and 2) and assuming $r = r_1$, $\Delta r = r_2 - r_1$, $\kappa^U = H_1^{-4/3} / (H_2^{-4/3} - H_1^{-4/3})$, and $\kappa^N = H_1^{-1} / (H_2^{-1} - H_1^{-1})$, we obtain

$$r_2(t) = r_1(t) (1 + 1/\kappa) - \kappa_0(t) / \kappa . \quad (10)$$

For $\zeta > 0$ ($\Delta T / \Delta z > 0$) the time dependence of the vertical profile is much more complicated. Now three targets are required (because the additional need arises to eliminate the indefinite parameter c^2 depending on the meteorological parameters of the atmosphere). For simplicity we consider only the case in which the targets (with the conventional numbers 1, 2, and 3) are located at the distances $l_1 = l_2 \neq l_3$ and altitudes $z_1 < z_2 < z_3$ (such an arrangement of targets was used in our experiments). From Eqs. (1), (4), and (5) we then obtain

$$r(t) = \kappa_{21}^S \Delta r_{21}(t) + \kappa_{31}^S \Delta r_{31}(t) , \quad (11)$$

where

$$\kappa_{21}^S = \frac{H^{-1}}{\Delta H_{21}^{-1}} - \frac{l}{\Delta l_{31}} \frac{\Delta H_{31}^{-1}}{\Delta H_{21}^{-1}} , \quad \kappa_{31}^S = \frac{l}{\Delta l_{31}} .$$

For $\kappa_{31}^S \gg 1$ from Eq. (11) we derive the formula

$$\Delta r_{31}(t) \approx \tilde{\kappa} \Delta r_{21}(t) , \quad (12)$$

where

$$\tilde{\kappa} = \Delta H_{31}^{-1} / \Delta H_{21}^{-1} .$$

As can be seen from Eq. (9), dependence (12) will be observed for the other atmospheric stratifications ($\tilde{\kappa} = \Delta H_{31}^{-4/3} / \Delta H_{21}^{-4/3}$ should be taken for $\zeta \leq \Gamma$).

When using the above relations for monitoring of the homogeneity of the light ray path, it is convenient to replace the refraction angle r by the difference between the angle φ of radiation arrival measured with a theodolite and the true angle of sight of the object $\varphi_0 = \text{const}$: $r = \varphi - \varphi_0$. Then, as can be seen from Eqs. (9) and (11), the scales T_* and L_* will vary slightly along the examined path if the following conditions are satisfied:

$$\begin{aligned} \varphi(t) - \kappa^U, N \Delta \varphi(t) - \kappa_0^U, N(t) &\approx \text{const} , \\ \varphi(t) - \kappa_{21}^S \Delta \varphi_{21}(t) - \kappa_{31}^S \Delta \varphi_{31}(t) &\approx \text{const} \quad (13) \\ (l_1 = l_2 \neq l_3) . \end{aligned}$$

Deviations from the time dependences given by Eq. (13) will serve as an indication of the inhomogeneity in the meteorological parameters of the atmosphere along the path of light ray propagation.

3. INTERRELATIONSHIP OF REGULAR AND RANDOM OPTICAL REFRACTIONS IN THE SURFACE ATMOSPHERIC LAYER

One of the most important characteristics of the random refraction is the variance of the random fluctuations of angles of arrival of the light field averaged over the

receiving aperture. In practice its measurement involves the use of highly sophisticated experimental apparatus.^{3,7} The performance of such an apparatus can be controlled by exploiting the interrelationship of the characteristics of regular and random optical refractions in the surface atmospheric layer. This interrelationship can be used in calculation of the angle of regular vertical refraction by means of measuring the variances of the random refraction.⁸ Realizing the importance of the above-indicated interrelationship, we will consider it in more detail.^{3,8}

For horizontal paths the variance of the random fluctuations of the angles of arrival of a light beam is defined by the relation⁹

$$\sigma_r^2 = \kappa_1 D^{-1/3} \int_0^l c_n^2(x) dx, \quad (14)$$

where D is the diameter of the receiving aperture and κ_1 is the coefficient of proportionality.

The structure constant of the refractive index c_n involved in this equation is directly proportional to the temperature structure constant c_T (see Ref. 3)

$$c_n = \kappa_2 c_T P/T^2, \quad (15)$$

where κ_2 is the coefficient of proportionality.

It is well known from the theory of the surface atmospheric layer that the structure constant c_T depends on the temperature lapse rate⁴

$$c_T^2 = \kappa_3 [\kappa z]^{4/3} a^2(\zeta) [\partial T/\partial z]^2, \quad (16)$$

where κ_3 is the coefficient of proportionality and $\kappa \approx 0.4$ is the von Karman constant. The theoretical function $a^2(\zeta)$ is often taken to be in the form³

$$a^2(\zeta) = \varphi^{-1}(\zeta) [a\varphi(\zeta) - \zeta]^{-1/3}, \quad (17)$$

while its experimental representations can be found in Refs. 9 and 10.

Using Eqs. (3) and (6), we represent the function $a^2(\zeta)$ in Eq. (17) in the form

$$a^2(\text{Ri}) = (1 - \alpha \text{Ri})^{-1/3} \begin{cases} \alpha^{-1/3} (1 - \mu \alpha^2 \text{Ri})^{4/3}, & \text{Ri} > 0, \\ \alpha^{-1/3}, & \Gamma \leq \text{Ri} \leq 0, \\ -3(\alpha \text{Ri})^{1/3}/\nu, & \text{Ri} < \Gamma. \end{cases} \quad (18)$$

It can be seen from here that for the unstable atmospheric stratification the function $a^2(\text{Ri})$ saturates rapidly and for $\text{Ri} \ll -1$ tends toward its limiting value $a_m^2 = -3/\nu$.

By substituting Eqs. (15) and (16) into Eq. (14) and assuming constancy of the scales T^* and L^* along the ray path, from formulas (1)–(3) we finally obtain

$$r = A + B\sigma_r, \quad (19)$$

where

$$A = \frac{\beta_0}{\alpha_0} \frac{Pl}{T^2}, \quad B = \frac{B_0}{a} D^{1/6} l^{1/2} \text{sign} \left(\frac{\partial T}{\partial z} \right).$$

As the expression for B_0 is quite cumbersome, we will write it out only in the particular case in which the condition $z(x) \approx h = \text{const}$ is satisfied along the path

$$B_0^2 = 1/\alpha_0^2 \kappa^{4/3} \kappa_1 \kappa_2^2 \kappa_3 h^{4/3} = \text{const } h^{-4/3}.$$

Formula (19) can be used for the unstable atmospheric stratification in calculation of the angle of vertical refraction from the measured variances of random refraction⁸ (since in this case $a \rightarrow a_m$, $B \rightarrow \text{const}$, and relation (19) is practically no longer the function of the atmospheric meteorological parameters). In addition, it can be used to check the accuracy of determining the variance of the random refraction through fulfilment of the condition

$$\varphi(t) - A(t) - B(t) \sigma_r(t) = \varphi_0 = \text{const}. \quad (20)$$

Such a check is important when the formulas^{3,11}

$$c_n^2 \approx \sigma_r^2 D^{1/3} / 2.84 l, \quad r_0 \approx 1.68 [(2\pi/\lambda)^2 c_n^2 l]^{-3/5}, \quad (21)$$

are used for determining the structure constant of the refractive index c_n and the Fried coherence radius r_0 . Here λ is the wavelength.

4. EXPERIMENTAL STUDY OF THE REFRACTION IN THE SURFACE ATMOSPHERIC LAYER

The experimental study of the optical refraction was conducted in summer in the arid zone (when the air temperature rose up to 40°C and even higher). A 2T2 standard theodolite and a laser scanning theodolite^{7,12} were used for measurements. The results of measurements were processed with the help of the above-presented formulas. The investigated paths of light ray propagation were collinear but somewhat differ in altitude and length. The paths were located in the foothills above the irregular underlying surface. The paths passed over the desert regions and dwelling houses and crossed a small ravine and highway. Their altitudes varied from 2 to 30 m (above the ravine). These inhomogeneous paths were selected specially to estimate the accuracy of calculation of the refraction above the irregular underlying surface, when the traditional methods of calculation were not necessary successful. The atmospheric meteorological parameters such as P , $T(z_2)$, $T(z_0)$, and $\nu(z_2)$ were measured above the observation point at the altitudes $z_2 = 2$ m and $z_0 = 0.03$ m. The angle of arrival of radiation $\varphi(\varphi = r + \varphi_0)$ was measured at the altitude $z_2 = 2.5$ m above the ground. The parameter z_1 , for which we have assumed the roughness parameter z_0 (by definition $\nu(z_0) = 0$), was chosen to estimate the accuracy of calculation under the worst conditions (for maximum Δz) and to reduce the number of the parameters to be measured. The individual day with typical weather conditions was chosen as an illustration of the results of many observations.

Shown in Fig. 1a is the diurnal variation of the temperature difference $\Delta T = T(z_2) - T(z_0)$ on one of these days. It can be seen from it that in the daytime the temperature gradient is negative while at night – positive. Therefore, all three stratifications of the surface atmospheric layer must be observed every twenty–four hours. In order to avoid the laborious measurement of the ray path profile $z(x)$ in calculations of the refraction angles according to formulas (1) and (4), we determined the invariable parameters $\gamma_0^U(\zeta < \Gamma)$ and $\gamma_0^N(\Gamma \leq \zeta \leq 0)$ from the long–term observations of the angle of arrival $\varphi(t)$ of radiation reflected from the chosen target. To this end, we used the formula derived from Eqs. (1) and (4) with allowance for the dependence $\varphi(t_0) = r(t) + \varphi_0$

$$\gamma_0 \approx \frac{\beta_0}{\sum_{i,j} 1} \sum_{i,j} \frac{\frac{\alpha_0}{l\beta_0} (\varphi_i - \varphi_j) - \frac{P_i}{T_i^2} + \frac{P_j}{T_j^2}}{\Delta T_i \frac{P_i}{T_i^2} - \Delta T_j \frac{P_j}{T_j^2}}, \quad (22)$$

where the subscripts *i* and *j* (*i* ≠ *j*) denote the values of the corresponding parameters at the moments of measurements *t_i* and *t_j*.

This formula was used separately for unstable and neutral atmospheric stratifications. In so doing some measurements, whose contribution was twice as much as the value of γ_0 averaged over the measurements or even more, were eliminated from summation (since it did not always happen that the weather conditions on the path corresponded to the assumptions of Eq. (4), i.e., T^* and $L^* = \text{const}$).

As a result, for 2303–m path under investigation (the lengths of all paths were measured with a Topaz SP2 laser range finder) we obtained $\gamma_0^U \approx \gamma_0^N \approx 0.009$. We note that if we used the rough approximation $\gamma \approx \Delta T / \Delta z$, the value of γ_0 would be equal to 0.5 resulting in the discrepancy in the calculated and measured (Fig. 1b) values of *r* by nearly an order of magnitude.

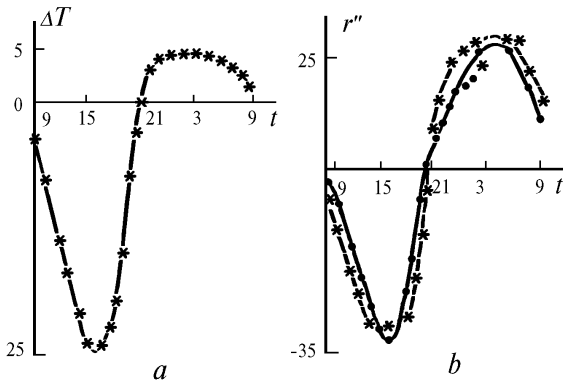


FIG. 1. Diurnal variations of $\Delta T(t)$ (a) and $r''(t)$ (b) (time *t* is in hours): calculated (dots) and measured (asterisks) values.

To calculate the refraction angles is more laborious for the stable atmospheric stratification ($\zeta > 0$). As can be seen from Eq. (5), in this case the knowledge of the function

$c^2(\tilde{R}i)$ is necessary. For this reason this function was extended according to the formulas

$$c^2 = \frac{\gamma_0^N - 1/\Delta z}{\gamma_0^S - 1/\Delta z},$$

$$\gamma_0^S = \frac{1}{\Delta T} \left[\frac{\alpha_0 T^2}{lP} (\varphi - \varphi_0) - \beta_0 \right], \quad (23)$$

where the true constant angle of sight of the target φ_0 was determined from numerous refraction measurements on different summer days when $\Delta T \approx 0$ (in addition, it can also be calculated with the help of formulas (1) and (4) for the other atmospheric stratifications given that $\gamma_0^{U,N}$ are known).

The experimental values of $c^2(\tilde{R}i)$ obtained in such a way were fitted by the linear dependence and, as a result,

the function $c^2(\tilde{R}i) \approx 1 - \tilde{R}i/30$ was reconstructed. With the help of this function we calculated the refraction angles according to formulas (1), (4), and (5) for the stable atmospheric stratification.

The resultant diurnal variation of the angle of optical refraction, calculated according to Eqs. (1) and (4), is shown in Fig. 1b. The figure shows that for the examined inhomogeneous path over the irregularly shaped underlying surface the deviation of the calculated refraction angles from the measured ones ($r = \varphi - \varphi_0$) does not exceed 6'' (the rated accuracy of the 2T2 theodolite is 2'') that is indicative of the high accuracy of the above formulas. As to the diurnal variation of $r(t)$, it should be noted that it qualitatively reproduces that of $\Delta T(t)$ and, in addition, at night for $\Delta T > 0$ the refractive angle is affected by the Richardson number.

We examined the vertical profiles of the optical refraction by observing three targets lying on one line of sight at the distances $l_1 = l_2 = 2303$ m and $l_3 = 2415$ m whose altitudes differ as follows: $z_2 - z_1 = 2$ m and $z_3 - z_1 = 13$ m. The observation with the 2T2 optical theodolite revealed some factors, associated with the inhomogeneity of the light ray propagation path, which were superimposed on the slow diurnal variations of the refraction angles. In addition the spread Δr_{21} was approximately twice as large as the analogous value Δr_{32} . For this reason the second and third targets were chosen for examining the altitude profile given by Eq. (9). For these targets the spreads Δr_{32} were about 15% of the refraction angles. Final results of the experiments are shown in Fig. 2. Figure 2a shows that for $\Delta T / \Delta z \leq 0$ the experimental results are fitted fairly well by linear function (9): $r_2(t) \approx -3\Delta r_{32}(t)$ ($|\kappa_0| \ll |r_2|$). Because of the inhomogeneity of the path and the chosen target locations with $L_1 \gg \Delta L_{31}$, the examination of analogous condition (11) for $\Delta T / \Delta z > 0$ did not yield any satisfactory result (we obtained in this case that $\kappa_{21}^S \gg 1$ and $\kappa_{31}^S \gg 1$ and the effect of the spreads of the results became ten times stronger). At the same time, the dependence Δr_{31} on Δr_{21} shown in Fig. 2b was in a good agreement with the theoretical one given by Eq. (12), i.e., $\Delta r_{31}(t) \approx 3\Delta r_{21}(t)$ for any atmospheric stratification.

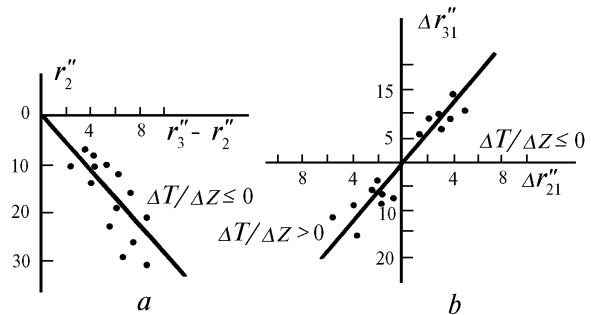


FIG. 2. Final results of the experiment: dependence of the refraction angle on its altitude increment (a) and dependence of the increments to the refraction angles at different altitudes (b).

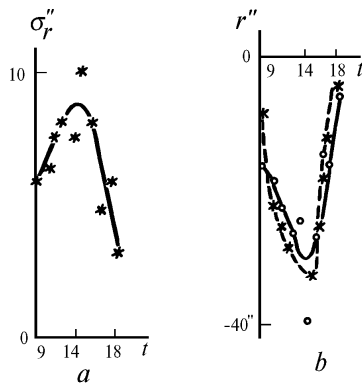


FIG. 3. Daytime dependences of $\sigma_r(t)$ (a) and $r(t)$ (b) calculated according to formula (19) (asterisks) and measured (dots).

The random refraction was investigated by the laser scanning theodolite⁷ in the daytime in September ($\Delta T/\Delta z < 0$) on 2415-m path. The measured dependence of the variance of random fluctuations of the vertical angle of arrival of the laser ray reflected from a corner-cube reflector (we used a He-Ne laser with $\lambda = 0.63 \mu\text{m}$) is shown in Fig. 3a. Shown in Fig. 3b are the diurnal variations of the angle of vertical refraction calculated according to formula (19) and measured based on the formula $r = \varphi - \varphi_0$. It can be seen from Fig. 3 that the curves $r(t)$ agree well and therefore, for unstable stratification formula (19) can be used for estimating the vertical refraction angles including the inhomogeneous paths.⁸ It should be noted additionally that from the known values of σ_r one can easily reconstruct the diurnal variation of the parameters c_n and r_0 with the help of formulas (21), because $c_n \approx 8.6 \cdot 10^{-9} \sigma_r'' \text{ (cm}^{-1/3}\text{)}$ and

$r_0 \approx 4.75 [\sigma_r'']^{-6/5} \text{ (cm)}$ (in our case the diameter D of the receiving aperture was 10 cm).

In conclusion we note that the present study of optical refraction on the near-surface paths may be useful for improving the accuracy of the goniometric and reference optical systems operating under conditions of arid zones.

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