Investigation of time correlation of orthogonal components of wind velocity in the atmospheric boundary layer based on acoustic sensing data

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The results of the statistical analysis of time correlation of instantaneous and averaged (over 0.5, 2, 10, 20, and 30 minute intervals) values of wind velocity horizontal components measured in different regions using the Volna-3 three-channel Doppler sodar are under discussion. Along with the consideration of peculiarities of time correlation functions of the wind velocity horizontal components their approximation by means of found analytical expressions is given.

Introduction

The introduction of efficient means of remote sensing in practice of environmental monitoring, particularly, laser and acoustic sensing, has opened up fresh opportunities for a detailed study of the structure and dynamics of the atmospheric boundary layer (ABL), including the investigation of time correlation of instantaneous and averaged (over different time intervals) values of wind velocity at various altitude levels. This data are necessary in investigating the peculiarities of behavior of wind velocity pulsations and averages in ABL for small time shifts (from several minutes to some hours), as well as in other practical problems, for example, very short-range forecasting (up to 1 h and from 2 h up to 12 h)[1] of wind field. Such a forecasting is performed based on the dynamic-stochastic approach in the framework of meteorological provision of monitoring of limited territories (for example, within the limits of a large city).

It should be noted that up to now the time correlation of instantaneous and averaged values of wind velocity orthogonal components in ABL at small time shifts (from one to several tens of minutes) is poorly known. The papers on the problem of parametrization of time correlation functions for wind velocity orthogonal components are unknown for us.

Therefore, we present in this paper our results on analysis and parametrization of time correlation functions of wind velocity orthogonal components calculated for its instantaneous and average values, which were obtained via acoustic sensing of ABL in the vicinity of Tomsk and in the Lake Baikal coastal area.

1. Some methodical questions and characteristics of initial material

It is well known that any random time function is generally estimated whether it is stationary or not.[2] In case of atmospheric random processes, the assumption on such a function stability holds only for comparatively small time intervals and fails for somewhat increased intervals. In our case, the time normalized correlation functions of wind velocity orthogonal components (zonal U and meridian V) were calculated at time intervals of the order of 3 h, for which we assumed ergodicity of the process and its stationarity in a broad sense (constancy of averages and variances).

Therefore, to calculate the normalized correlation (later on simply correlation) functions $\mu_\xi(\tau_k)$, we can use the formula written for a stationary process in the form[2,3]:

$$\hat{\mu}_\xi(\tau_k) = \frac{1}{N-k}\frac{\sum_{i=1}^{N-k}[\xi(t_i)-\hat{m}_\xi][\xi(t_i+\tau_k)-\hat{m}_\xi]}{\hat{\sigma}_\xi^2}, \quad (1)$$

where $\hat{m}_\xi$ and $\hat{\sigma}_\xi^2$ are the estimates of the average and variance of the meteorological parameter ξ; $\tau_k = k\Delta t$ ($k = 0, 1, 2, ..., m$) are the time shifts; $\xi(t_i)$ and $\xi(t_i+\tau_k)$ are the terms in the sequence of observations of ξ (in our case, U and V); $t_i = i\Delta t$ and $t_i+\tau_k = (i+k)\Delta t$ (here Δt is the interval between members of the time series); N is the number of terms in the time series.

The normalized correlation functions $\mu_U(\tau_k)$ and $\mu_V(\tau_k)$ were calculated[2] based on the independent ten-hour realizations of wind velocity orthogonal components $U(i\Delta t)$ and $V(i\Delta t)$, which at these time intervals can be considered as stationary.[3] Further, assuming some uniformity of $\mu_U(\tau_k)$ and $\mu_V(\tau_k)$ for a particular thermodynamic state of the atmosphere, the obtained selected values of $\hat{\mu}_U(\tau_k)$ and $\hat{\mu}_V(\tau_k)$ were averaged. To minimize the effect of a possible daily behavior of $U(i\Delta t)$ and $V(i\Delta t)$ on $\hat{\mu}_U(\tau_k)$ and $\hat{\mu}_V(\tau_k)$, their calculation was conducted up to the maximum time shift τ_k not exceeding 3 hours.

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One more important condition, noted in Ref. 2, is noteworthy. The white noise $n(t)$ in the data under processing has a pronounced effect on calculation of $\mu_\xi(\tau_k)$, which can result in significant systematic errors in estimating $\mu_\xi(\tau_k)$, namely, a considerable underestimating of calculated $\hat{\mu}_\xi(\tau_k)$ at $\tau_k > 0$ relative to their true values. In our case, random errors $\varepsilon_U(i\Delta t)$, $\varepsilon_V(i\Delta t)$ of sodar measurements of wind components $U(i\Delta t)$ and $V(i\Delta t)$ mainly contribute in $n(t)$. Their variances $D(\varepsilon_U)$ and $D(\varepsilon_V)$ are determined mainly by levels and fluctuations of acoustic signal and surrounding noise received from the atmosphere during measurements, i.e., the signal-to-noise ratio. The distorting effect of ε_U and ε_V can be eliminated when analyzing the jumps of $\hat{\mu}_\xi(k\Delta t)$ at $k = 0$ (Ref. 2). For this purpose it is necessary to approximate $\hat{\mu}_\xi(k\Delta t)$ in the vicinity of zero time shifts with subsequent extrapolation to the point $\tau_k = 0$.

To compensate systematic errors in determination of $\mu_\xi(\tau_k)$, we use in this paper another approach, which is more simple and better adapted to fast wind prediction. It consists in preliminary averaging of actual values of $\xi(i\Delta t)$ during the time T_m. For example, for even M

$$\xi_m(iT_m) = \frac{1}{M} \sum_{m=-M/2}^{M/2-1} \xi[(i+m)\Delta t], \tag{2}$$

where $M = T_m/\Delta t$ is the number of averaged $\xi(i\Delta t)$ readings. Application of Eq. (2) allows the variance of white noise $n(t)$ to be decreased by the factor M, which results in a decrease of estimate shift (1) (when changing ξ by ξ_m and Δt by T_m) relative to the true $\mu_\xi(\tau_k)$ values. The contribution of small-scale turbulence to the estimation of $\mu_\xi(\tau_k)$ is also suppressed. This high-frequency turbulent range has no information content for predicting the wind velocity field, therefore it is not the subject of our investigation (the wind values corresponding to this range are practically uncorrelated beyond the time interval from 20 to 70 s).[4] However, very large values of T_m can result in suppression of the analyzed mesoscale components. As a result, the obtained values of $\hat{\mu}_\xi(\tau_k)$ can be also distorted. In particular, at some relations between T_m, the correlation radius of the studied processes, and the level of white noise in them, $\hat{\mu}_\xi(\tau_k)$ values can be overestimated relative to actual values of $\mu_\xi(\tau_k)$. Consequently, there exists a problem of choosing an optimal averaging interval T_m. It is recommended to select T_m of the order of 10–20 min for the wind velocity modulus.[5]

Since in our case the orthogonal components of the wind velocity are used, the question of selecting the averaging interval T_m for these components remains open. Therefore in this paper, as applied to orthogonal components of wind velocity, we investigated a choice of the optimal averaging interval T_m, which enables one to obtain for U- and V-components the most stable correlation functions for $\mu_\xi(\tau_k)$ (from the point of view of their use in the problem of very short-range forecasting of the wind field).

We have performed the analytical approximation of corresponding correlation functions as well. Note that there is a series of analytical expressions for approximation of the time correlation functions (see, e.g., Ref. 2) but no one is suitable for describing the obtained by us empirical dependences, because they relate to macroscale processes. Therefore, we had to find approximating expressions suitable for adequate description of $\hat{\mu}_U(\tau_k)$ and $\hat{\mu}_V(\tau_k)$ behavior in the mesoscale region (i.e., in the region with a characteristic time of processes from tens of minutes to several hours).[6]

As the initial material for calculating empirical time correlation functions of wind velocity orthogonal components (U and V) we used the data on wind velocity pulsations at the time interval between the successive measurements of the order of 15–17 s, obtained in the Tomsk region and in the coastal area of the Lake Baikal. The data were obtained by means of the Volna-3 three-channel Doppler sodar[7,8] designed at the IAO SB RAS. Underline that to estimate the time correlation for the wind velocity orthogonal components, we used data obtained under relatively stationary conditions, i.e., in the absence of sharp variations of thermodynamic state of the atmospheric boundary layer. Therefore, we considered the cases of stable (temperature inversions) and unstable (developed convection) atmospheric stratifications only for Tomsk region. In the coastal area of Lake Baikal with its complex local circulation and corresponding very low correlation relations such a separation makes no sense.

As initial samples for calculating time correlation functions we used for stable and unstable stratifications (Tomsk region) 5 000 measurements each, and of the order of 10 000 measurements for the Lake Baikal coastal area.

The time correlation functions were estimated for four altitude levels: 100, 150, 200, and 250 m in the Tomsk region and three altitude levels: 100, 150, and 200 m in the coastal area of Lake Baikal. Besides, along with instantaneous values we used the averaged values of wind velocity orthogonal components calculated for different averaging intervals T_m = 0.5, 1, 2, 10, 20, 30 min. This allowed us to estimate the effect of the initial data averaging on the time correlation relations and to find the optimal interval of such averaging, which is of importance from the practical point of view.

2. Peculiarities of time correlation of instantaneous and averaged values of wind velocity depending on the type of temperature stratification

In this section we analyze statistically the effect of the temperature stratification type on time correlation of instantaneous (pulsations) and averaged (over 0.5-, 1-, 2-, 10-, 20-, 30-min intervals) values of wind velocity orthogonal components in ABL (up to 250 m height).

We consider first the behavior of empirical time correlation functions, calculated using instantaneous and averaged (over 0.5- and 1-min intervals) values of U- and V-components, obtained for the Tomsk region under conditions of stable atmospheric stratification. Figure 1 shows the time correlation functions constructed on the basis of wind pulsations measured by the sodar at a 150 m height with a 15 s step, as well as based on averaged data (over 30 s and 1 min).

Figure 1 demonstrates a significant effect of random errors in U and V measurements on estimates of $\mu_U(\tau_k)$ and $\mu_V(\tau_k)$. This is seen from a sharp decrease of values of time correlation functions near $\tau_k = 0$ that is characteristic of estimates distorted by the white noise.[2] Note that at unstable atmospheric stratification, i.e., in the presence of convection, the character of time correlation relation of instantaneous and averaged ($T_m = 0.5$ and 1 min) values of wind velocity orthogonal components does not change. It is evident that at $T_m = 1$ min ($M = 4$) systematic errors in estimation of the sought correlation coefficients do not disappear.

Figures 2 and 3 exemplify the empirical time correlation functions at stable and unstable stratification at increasing averaging interval up to 2, 10, and 30 min. The above-mentioned sharp decreases of $\hat{\mu}_U(\tau_k)$ and $\hat{\mu}_V(\tau_k)$ at $\tau_k = 0$, although less pronounced, are observed at a 2 min averaging ($M = 8$), that also testifies about insufficient magnitude of T_m. The behavior of $\hat{\mu}_U(\tau_k)$ and $\hat{\mu}_V(\tau_k)$ changes as T_m approaches 10 min. In this case the type of time correlation relations is practically unchangeable for each of the heights at increasing T_m up to 30 min. At the same time, Figs. 2 and 3 show that it depends greatly on the type of temperature stratification. Under conditions of the stable atmospheric stratification, $\mu_U(\tau_k)$ and $\mu_V(\tau_k)$ reach zero, on average, for 2.5–3 hours, and in case of unstable one – for 1.5–2 hours.

At $T_m > 30$ min, the distortions of $\mu_U(\tau_k)$ and $\mu_V(\tau_k)$ occur due to the factors considered briefly earlier. This statement is based on the results of comparison with $\mu_U(\tau_k)$ and $\mu_V(\tau_k)$ obtained when realizing the above-mentioned technique,[2] i.e., when approximating selected $\hat{\mu}_U(\tau_k)$ and $\hat{\mu}_V(\tau_k)$ in the vicinity of small $\tau > 0$ and their extrapolation to $\tau = 0$.

This method enables one to neutralize the displacing effect of measurement noises on estimates of the sought time correlation functions without distorting their frequency components. Fluctuations of $\hat{\mu}_U(\tau_k)$ and $\hat{\mu}_V(\tau_k)$ decrease with further averaging over the ensemble. In this case we have used selectively both initial realizations of $U(i\Delta t)$, $V(i\Delta t)$ and averaged ones over 30 s and 1 min intervals, where the analyzed mesoscale wind components are not yet suppressed. The obtained $\mu_U(\tau_k)$, $\mu_V(\tau_k)$ were further compared with those calculated by the basic method by Eq. (2) at $T_m = 2$, 10, 20, 30, 40 min. Mainly, the U- and V-components corresponding to stable atmospheric stratification (Fig. 4), were subjected to such a procedure.

It is found that most often (at commonly reachable signal-to-noise ratios) the value $T = 20$ min corresponds to the greatest proximity of $\mu_U(\tau_k)$ and $\mu_V(\tau_k)$ calculated by two methods. With increasing the signal-to-noise ratio (at decreasing omissions of wind data[5]) the best agreement was at $T_m = 10$ min, and with decrease of the ratio – at $T_m = 30$ min. At $T_m = 2$ min, the neutralization of white noise remains should take place[2] after the use of formula (2).

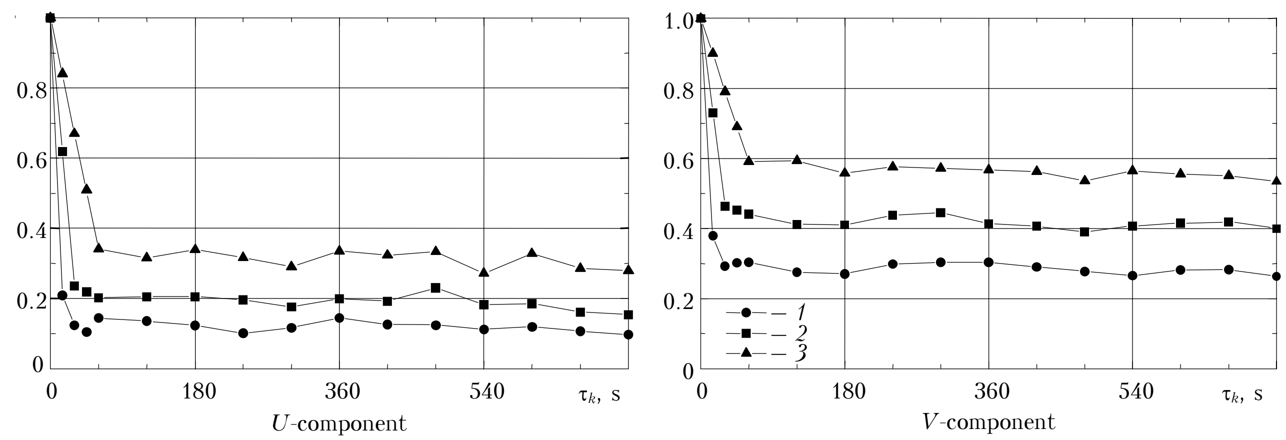


Fig. 1. Time correlation functions of orthogonal components of wind velocity constructed for 150 m altitude according to instantaneous (*1*) and averaged over a period of 30 s (*2*) and 1 min (*3*) data of acoustic sensing in the Tomsk region.

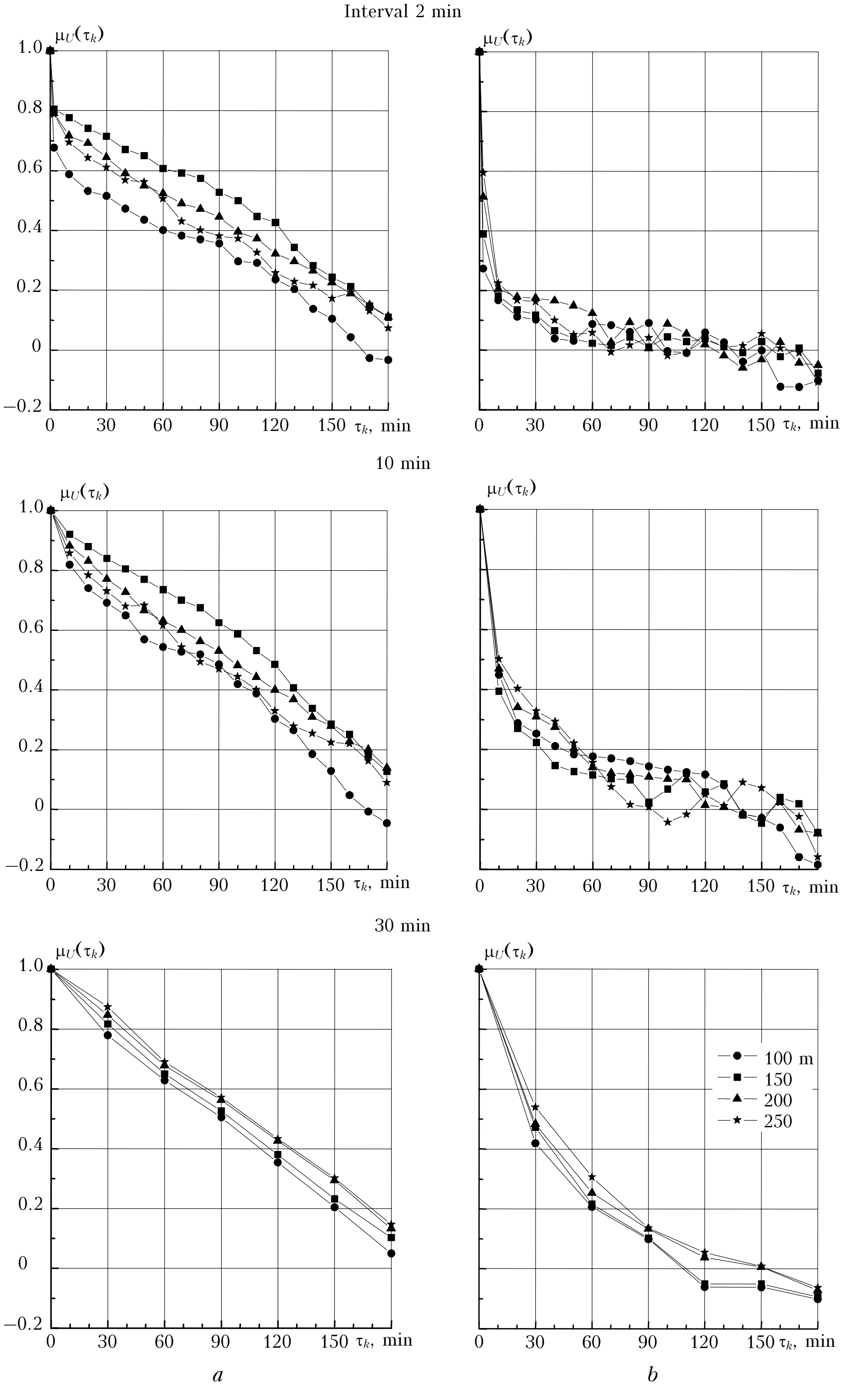
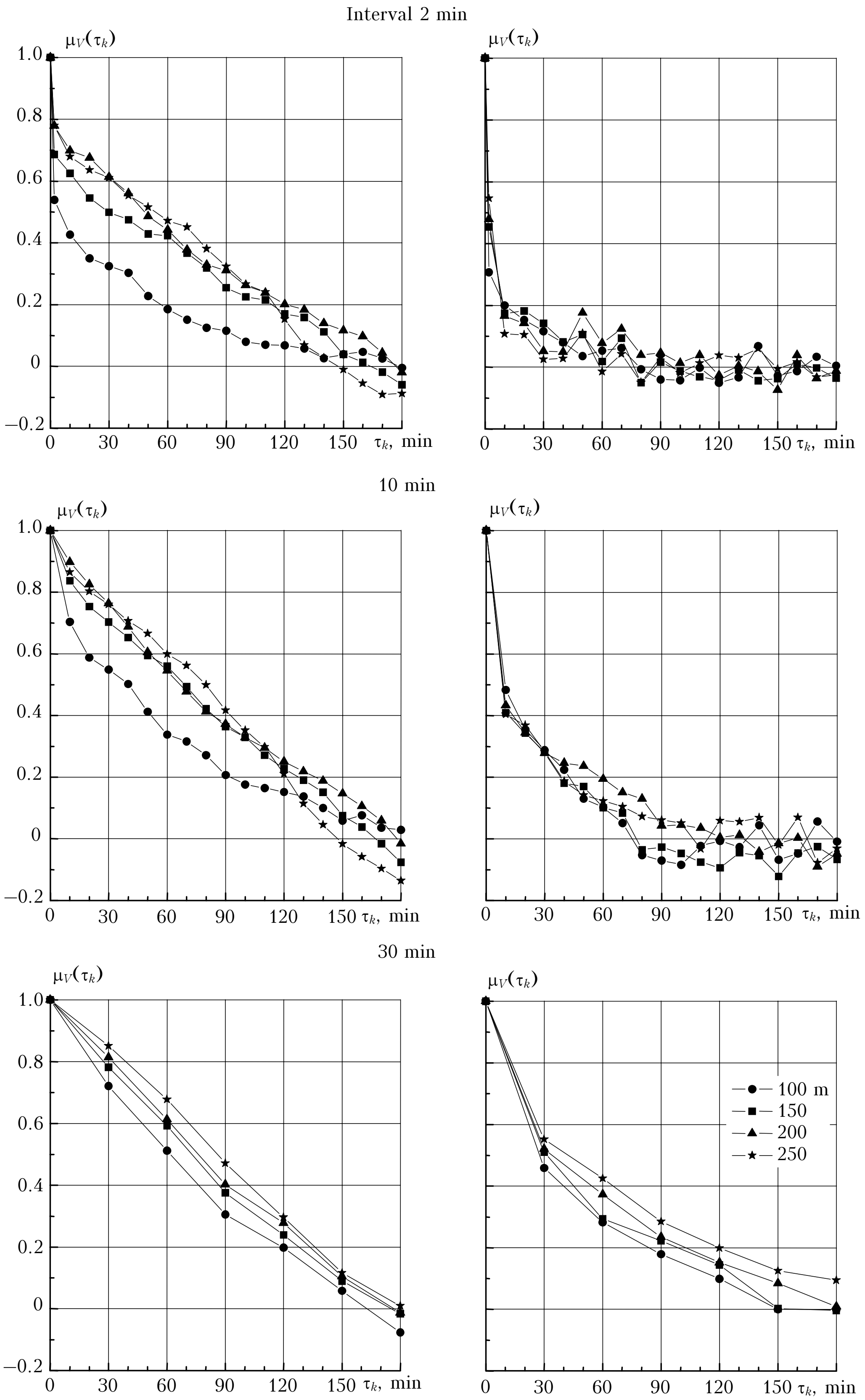


Fig. 2. Time correlation functions of wind velocity U-component constructed for stable (*a*) and unstable (*b*) stratifications according to sodar measurements averaged over different time intervals. Tomsk region.



a *b*

Fig. 3. Time correlation functions of wind velocity V-component constructed for stable (*a*) and unstable (*b*) stratifications according to sodar measurements averaged over different periods of time. Tomsk region.

This is partly illustrated in Fig. 4, where the selected $\hat{\mu}_U(\tau_k)$ are exemplified. They were obtained at 150 m height and different T_m for actual $U(i\Delta t)$ readings under the stable atmospheric stratification observed on January 4, 2000 in the Tomsk region. The dashed line shows the result of the reconstruction (using methods from Ref. 2) of $\tilde{\mu}_U(\tau_k)$ from a previously calculated $\hat{\mu}_U(\tau_k)$ at $T_m = 0.5$ min. The behavior of $\tilde{\mu}_U(\tau_k)$ is characterized by strong fluctuations caused by a significant level of white noise in the data under processing. In this particular case the signal-to-noise ratio in power was only 0.29. The best correspondence of $\tilde{\mu}_U(\tau_k)$ to rather complex mean behavior of $\hat{\mu}_U(\tau_k)$ is observed at $T_m = 20$ min. A sufficient agreement is at $T_m = 10$ and 30 min. The use of $T_m = 40$ min is characterized by significant distortions of $\hat{\mu}_U(\tau_k)$ relative to $\tilde{\mu}_U(\tau_k)$ because of excessive suppression of frequency components in the initial $U(i\Delta t)$ sampling.

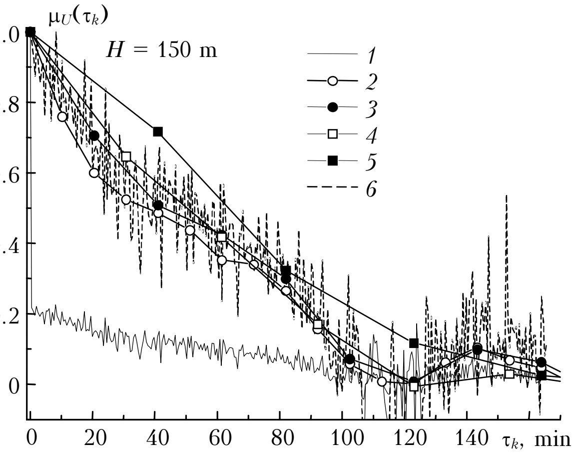


Fig. 4. Comparison of empirical correlation functions of wind velocity U-components obtained at different intervals of averaging T_m: 0.5 (*1*), 10 (*2*), 20 (*3*), 30 (*4*), and 40 min (*5*), by methods from Ref. 2 (*6*).

In contrast to stable geophysical conditions of the Tomsk region, the wind conditions in the Lake Baikal coastal zone are characterized by a significant instability both in time and height that is connected with local peculiarities of the atmospheric circulation.[9] Therefore, the time correlations of averaged (over 2, 10, 20, and 30 min) values of wind velocity orthogonal components there differ considerably (they are close to the correlations under conditions of unstable stratification). This is illustrated in Fig. 5, where the time correlation functions of U- and V-components are shown, which are constructed for the Lake Baikal coastal zone using averaged (over 2, 10 and 30 min) values of wind velocity at various heights (100, 150, and 200 m).

Analysis of Fig. 5 shows:

- time correlations of U- and V-components obtained from averaged (over 2 min) data, even at the shift $\tau_k = 30$ min are much weaker than those determined from acoustic measurements in the Tomsk region at a stable stratification (see Figs. 2*a* and 3*a*). Really, regardless of the wind velocity components and the altitude level, the values of correlation coefficients at $\tau_k = 30$ min and $T_m = 2$ min are 0.18–0.24 in the coastal zone of Lake Baikal; in the Tomsk region at a stable stratification these values are 0.50–0.71;
- at increasing averaging interval up to 10, 20, and 30 min the values of time correlation coefficients of U and V-components, calculated based on data of sodar measurements in the Lake Baikal coastal area, grow markedly. For example, for the same $\tau_k = 30$ min and the $T_m = 30$ min the values of $\mu_U(\tau_k)$ and $\mu_V(\tau_k)$ regardless of altitude grow to 0.39–0.51;
- as τ_k exceeds 30 min, values of the correlation coefficients of U- and V-components rapidly decrease (this is characteristic of all taken averaging intervals) and already at $\tau_k = 60$ min become zero or even fall lower.

Note in conclusion that analysis of all data has shown the interval $T_m \approx 20$ min is optimal in solving the above problems irrespective of the region, type of temperature stratification, and altitude level. The intervals $T_m = 10$ min and $T_m = 30$ min are permissible and can be considered as lower and upper limits in the time averaging.

3. Some results of parametrization of time correlation functions of wind velocity *U*- and *V*-components

In practice, not $\mu_\xi(\tau_k)$ are commonly used, but some analytical expressions approximating them. Therefore, we have made an attempt to find such analytical expressions.

Underline that we subjected to analytical approximation in full measure only those correlation functions $\mu_U(\tau_k)$ and $\mu_V(\tau_k)$, which were obtained using the averaged data (over 10, 20, and 30 min), i.e., from the range of recommended values of T_m. For two cases, as a comparison, the above approximations are given for $T_m = 2$ min.

As the result of our studies, we have found that the following unified expression can be used for their approximation (regardless of the wind velocity component, region, and averaging interval) in all above cases:

$$\mu_\xi(\tau_k) = [\exp(-\alpha\tau_k)](1 - \beta\tau_k). \tag{3}$$

Here α and β are the empirical coefficients; τ_k is the time shift, where $\tau_k = \Delta t,\ 2\Delta t,\ 3\Delta t,\ \ldots,\ k\Delta t$, and $\Delta t = t_{i+1} - t_i = \text{const}$ is the time step between neighboring terms of the time series.

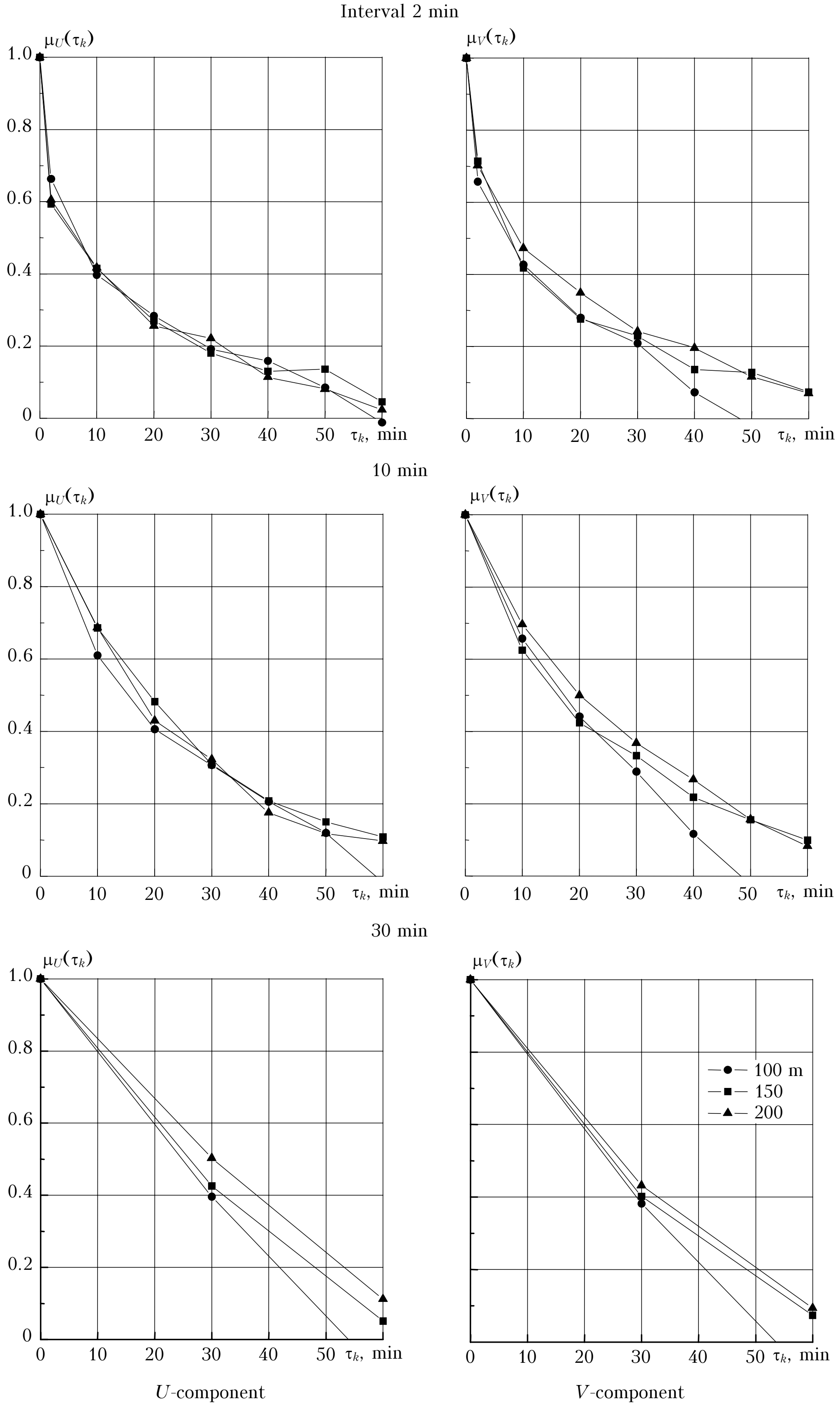


Fig. 5. Time correlation functions of wind velocity orthogonal components constructed according to sodar measurements averaged over different periods of time. Coastal area of Lake Baikal.

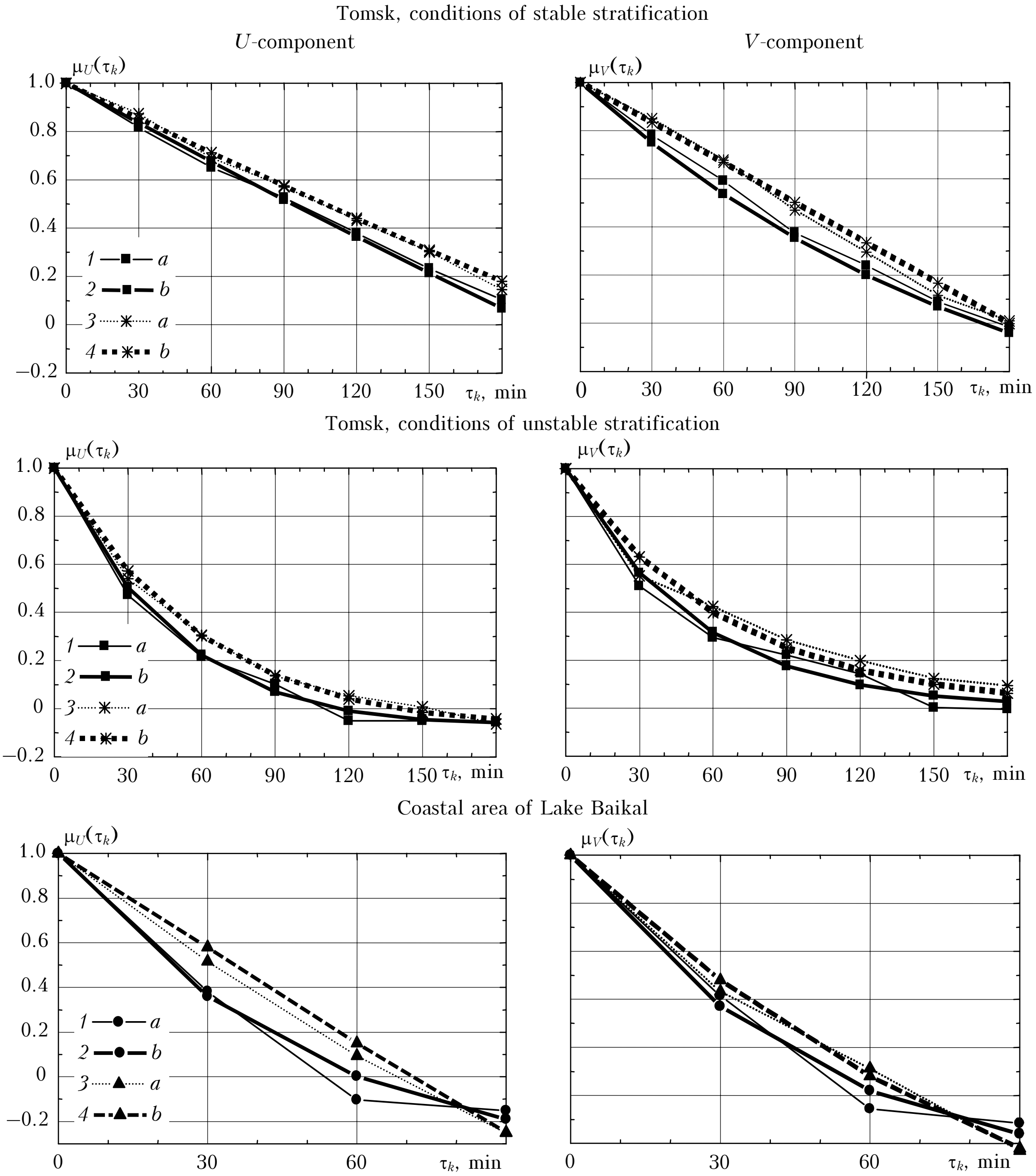


Fig. 6. Empirical (*a*) and approximated (*b*) time correlation functions of wind velocity *U*- and *V*-components constructed according to sodar measurements averaged over 30 min for altitudes of 150 (*1*, *2*) and 250 (*3*, *4*) (Tomsk); 100 (*1*, *2*) and 200 m (*3*, *4*) (Lake Baikal).

Averaging interval, min	α	β
Tomsk region. Stable stratification		
2	0.0147	0.0036
10, 20, 30	0.0068	0.0047
Tomsk region. Unstable stratification		
10, 20, 30	0.0260	0.0100
Lake Baikal coastal zone		
2	0.0650	0.0003
10, 20, 30	0.0153	0.0157

The values of coefficients α and β are given in Table.

The use of analytical expression (3) and the corresponding empirical coefficients, given in Table, enables one to describe empirical time correlation functions of *U*- and *V*-components with an absolute error not exceeding 0.05–0.10. This conclusion is supported by Fig. 6.

Thus, the obtained approximation formula can be successfully used in practice in the dynamic-stochastic algorithm for the very short-range forecasting of wind field in the atmospheric boundary layer.

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