ANGULAR DEPENDENCE OF THE COHERENT BACKSCATTERING PEAK IN SCATTERING MEDIA OF A LIMITED SHAPE

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We consider here the recent investigations into the coherent backscattering effect in discrete scattering media. The shape of the coherent backscattering peak for spherical scattering media is studied experimentally. It is shown that the peak shape for the spheres of small optical depth is adequately described by the Fraunhofer diffraction pattern, and for the case of optically dense spheres the above-mentioned peak is described by the convolution of the two functions, namely, the diffraction pattern for a sphere and the angular distribution within the peak for semi-infinite scattering medium with a plane boundary.

The effect of coherent backscattering at multiple scattering of waves in discrete scattering media has been predicted theoretically about twenty years ago by Dr.K. Watson, a founder of the multiple scattering theory in quantum mechanics.[1] This effect is inherent in the scattering of waves of any nature by arbitrary scatters and it has a clear physical interpretation. Namely, this effect is explained by the interference between the waves passed through the same scatters in the direct and opposite directions and having therefore the same phase at the observation point. This effect is observed experimentally as a narrow peak increasing the intensity of multiply scattered radiation in the backward direction almost by a factor of two.

From the methodical standpoint, the coherent backscattering is noteworthy because it does not appear in the fundamental equation of radiation transfer (ERT),[2] describing multiple scattering of radiation of any nature, and can be considered as a correction to this equation[1,3,4] taking into account the wave nature of radiation. This correction can also manifest itself in numerous practical methods of diagnostics of scattering media, because one of the most suitable experimental schemes for diagnostics uses the radiation scattered in the backward direction, for example, in sounding of atmospheric clouds by coherent lidars.[5]

Interest in the coherent backscattering effect has increased once its analogy with the general phenomenon of wave localization has been discovered. Localization of waves in randomly inhomogeneous media plays an important role, especially, in the theory of electric conductivity of solid bodies without the ordered crystal structure (alloys, disordered semiconductors, and so on). The analogy between the coherent backscattering effect and the wave localization lies in the following fact. It is known that strong localization of electron waves on a randomly inhomogeneous potential (the Anderson localization),[6] when the contribution of a localized electron state to the electric conductivity reduces to zero, is closely connected with the interference between the fields scattered by the potential inhomogeneities. The above interference between the wave, passed through the same scatters in forward and backward directions, results, as in the previous case, in the decrease of electron conductivity and hence can be considered as a predecessor of the Anderson localization. It has been suggested that this phenomenon be known as weak localization[7,8] as opposed to the Anderson localization.

Thus a possibility[9,10] has evolved to study the basic regularities of weak localization of electron waves in the disordered solid bodies, when modeling this phenomenon using more simple experimental instruments, measuring, for example, the backscattering peak in light scattering by water suspension of polystyrene particles one micron in diameter.

It should be noted that the effect of amplification of backscattering in the discrete scattering media is also identical to the effect of amplification of a signal reflected in backward direction in randomly inhomogeneous media like turbulent atmosphere (this effect was first noted by de Wolf[11]) as well as to the effect of amplification of a signal reflected from rough surfaces in backward direction. By now the number of papers in this field is so large that it is impossible to discuss the above problems in this paper and we note here only some reviews.[12–14]

The first experimental measurements of the backscattering peak in discrete scattering media have been carried out by the authors of Refs. 15–18, where laser radiation was scattered by a suspension of polystyrene particles of one micron size. In these papers as well as in the subsequent experimental papers[19-24] the scattering medium is taken in the form of a plane-parallel layer so that the scattering medium

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can be considered to be unlimited across when interpreting the experimental data.

As to the theoretical calculations of the shape of coherent backscattering peak, from the structure of the Feynman diagrams, describing the field multiple scattering, one can see that the coherent backscattering does not appear in the solution of the radiation transfer equation,[2] and it can readily be expressed through this solution as an integral.[1,3,4] Thus, all the difficulties with the calculation of the peak shape have amounted to difficulties occurred when obtaining either analytical expressions or numerical values for solutions of the radiation transfer equation at different parameters and geometry of scattering medium.

The backscattering peak is described by the function

$$P(\theta) = I(\theta)/I^*(\theta), \tag{1}$$

where $I(\theta)$ is the intensity of multiply scattered radiation; θ is the angle between the backward direction and the direction of scattering; $I^*(\theta)$ is the radiation intensity, corresponding to the solution of the radiation transfer equation.

Since the function $I^*(\theta)$ is practically constant within the peak of backscattering, it can readily be measured experimentally as the intensity value $I(\theta)$ at the peak boundary. We consider mainly not the function $P(\theta)$ itself but its characteristic parameters, namely, the amplification factor

$$K = I(0)/I^*(0) \tag{2}$$

and the peak halfwidth θ_0, defined by the relation

$$I(\theta_0) - I^*(\theta_0) = \frac{1}{2}\,[I(0) - I^*(0)]. \tag{3}$$

At small optical depths of the medium $\tau \ll 1$ some authors[25–27] used simple analytical expressions of the double scattering approximation for theoretical description of the function $P(\theta)$. However, since the optical depth of the medium in transverse direction was taken to be unlimited in this case, some doubts are cast upon the possibility of obtaining quantitative agreement between these expressions and the experimental data. At the same time, because the amplification factor (2) is small at moderate optical depths, in this case there are only few such experimental measurements.

The experimental measurements of the backscattering peak $P(\theta)$ are usually carried out for the case of large optical depth $\tau \gg 1$. In theoretical papers, in this case, the solution of ERT in the diffusion approximation[26,28–31] is mainly used both in scalar version and taking into account the polarization of electromagnetic waves.

The theoretical papers are of primary interest, in which not the approximations but exact solution of ERT are used. Hence, in a recent paper[32] the exact analytical expression was obtained for the function $P(\theta)$ for the case of a half-space with Rayleigh scatters. As an alternative to analytical methods, the numerical methods identical to the known numerical methods of solution of ERT are used in Refs. 4 and 30 for calculating the function $P(\theta)$, and in Ref. 33 the numerical methods are used for calculation of the amplification factor (2) for different states of radiation polarization, different optical depths of the medium, and different particle size, including the particles of spheroidal shape.

We indicate the main qualitative regularities obtained in the above theoretical and experimental papers. It is shown that the amplification factor (2) takes the values in the interval $1 < K < 2$; and its value depends essentially on the state of polarization of incident radiation and on the state of polarization recorded by a receiver. Besides, the peak is an anisotropic function with the halfwidth, depending on the two angles: φ_1, the angle between the direction, determining the polarization state of the incident radiation and the scanning plane, and φ_2, similar angle for the polarization state of the radiation detected.

For small particles $a < \lambda$, where λ is the radiation wavelength and a is the particle size, the function $P(\theta)$ is a smoothly decreasing angular function. In this case the peak halfwidth at a large optical depths is determined by the relation;

$$\theta_0 = \alpha\lambda/2\pi l, \tag{4}$$

where l is the extinction length in a medium, and α is a constant of the order of unit. In particular, for a half-space of the Rayleigh particles, according to Ref. 32, at linear polarization of incident radiation with the direction of the polarization vector $\mathbf{e}$ we have

$$K_1 = 1.75 \quad \text{and} \quad \alpha_1 = 0.61, \tag{5}$$

if the direction of the radiation detector polarizer and the scanning direction coincide with the vector $\mathbf{e}$, and

$$K_2 = 1.12 \quad \text{and} \quad \alpha_1 = 1.27, \tag{6}$$

if the above directions are perpendicular to the vector $\mathbf{e}$.

For the mean-sized particles both the diffusion approximation and the numerical calculations result in the conclusion that in this case the above qualitative pattern is retained when the extinction length is substituted by the transport extinction length, i.e., when introducing the supplementary cofactor

$$\beta = 1 - \mu \tag{7}$$

into the right-hand side of Eq. (4) where μ is the main cosine of the scattering angle by one particle, being close to zero for small particles and approaching unity for large particles. And, at last, for very large particles when $\mu > 1/3$ (Ref. 31) the monotony of the backscattering peak is broken and a supplementary maximum in the function $P(\theta)$ appears. This maximum is present also at small optical depths of the medium in the approximation of double scattering.[27]

The above described qualitative pattern has been obtained for scattering media, having the shape of a plane-parallel layer. One of the important problems is the investigation of the backscattering peak for scattering media of a different shape. In particular, in Ref. 34 the problem of the influence of medium limitedness on the backscattering peak is identified as the most actual problem in this field.

In this paper the backscattering peak is studied experimentally on the basis of a simple shape of scattering medium, namely, a homogeneous ball.

The schematic diagram of the experiment is presented in Fig. 1. A linearly polarized Gaussian beam from a He-Ne laser *1* with the power 30 mW and wavelength $\lambda = 0.63$ μm falls on a beam-splitting plane-parallel plate *2* which directs a beam portion (50%) to a polystyrene sphere *3*. In the experiments the spheres of different diameters ranging from 0.6 to 1.35 mm are used.

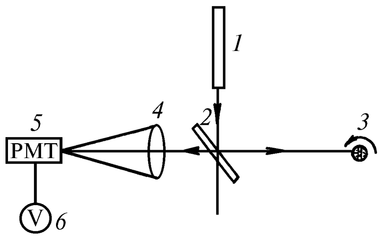

FIG. 1. Optical arrangement of the experimental setup.

Note that in the experimental studies of the backscattering peak on water suspensions of particles the interference pattern, formed as a result of the interference between the scattered waves diverging from separate particles, is automatically averaged due to the particle motion. The pattern observed corresponds to the intensity, averaged over particle positions in the medium. In this case the scattering particles are immovable, therefore instead of the mean radiation intensity we observed the chaotic speckle-structure requiring statistical averaging. As an illustration, Fig. 2 gives the measured pattern of unaveraged spatial intensity distribution when reflecting a laser beam from a face of the polystyrene plate. Note that the backscattering peak is represented here as a central spot of the speckle-structure, having a random shape.

At the same time, Fig. 2 illustrates also the basic physical regularity, considered in this paper, namely, this figure shows the influence of dimensions of the scattering medium on the shape of the coherent scattering peak. Here Fig. 2*a* corresponds to the vertical position and Fig. 2*b* corresponds to the horizontal position of the plate 1 mm thick. The central spot shows a distinct anisotropy. In this case the spot is extended in the direction perpendicular to the plate that corresponds to the influence of the transverse size of the medium on the radiation intensity coming out from the medium in directions close to the backward direction.

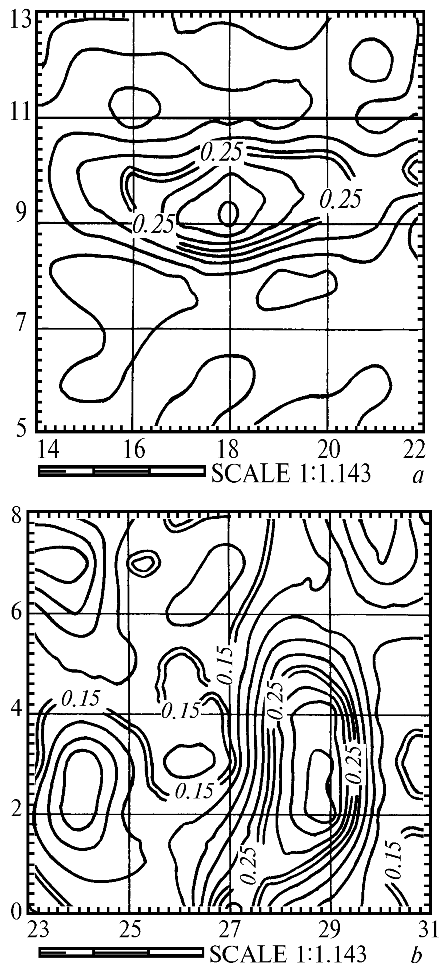

FIG. 2. Speckle-structure of backscattered radiation for the plate 1 mm *thick at vertical (a) and horizontal (b) position of scattering medium. Digits on the coordinate axes denote the conventional numbers of elements of photodiode matrix, used in the measurements.*

In the case of the spherical scattering medium, considered in detail in this paper, the averaging of speckle-pattern over the positions of scatterers in space is made by rotation of spheres relative to the vertical axis. The radiation scattered in backward direction, was focused by a lens *4* to the input diaphragm of the photomultiplier *5* (Fig. 1). The angular resolution was equal to 0.06 mrad. The photomultiplier was fixed in a light proof housing on the table with a micrometer screw shifts. Scanning of scattered light was performed in a horizontal plane. A signal from the photomultiplier was fed into an integrating voltmeter *6*. The fact that in the observation plane the light flux was small presented a real difficulty in performing the experiment. Therefore, for the increase of intensity of the radiation detected, the laser beam diameter was selected comparable with the diameter d of the sphere. In our case, small spheres (d < 0.8 mm) were illuminated by a Gaussian beam 0. 44 m in diameter (at the e^{-1} level). In this case the peaks were recorded with the lens *4* whose focal length was f = 490 mm. Large spheres (d > 0.8 mm) were illuminated by a laser beam 0.81 mm in diameter and recorded at f = 860 mm. Simultaneously the extinction length l for every sphere was estimated from separate measurements of radiation extinction in a sphere.

The points in Figs. 3 and 4 stand for the typical experimental data of the backscattering peak shape for the two spheres with different relation between the diameter d and the extinction length l. The data for all spheres, obtained in such a form, were processed based on the following physical considerations. As is seen from Ref. 3, for the spheres with small optical depth, when $d \ll l$, the backscattering peak shape is close to the Fraunhofer diffraction pattern for sphere with the diameter d, described by the well-known expression

$$P_1(\theta) = [2J_1(kd\theta/2)/(kd\theta/2)]^2\,, \tag{8}$$

where k is the wave number, and J_1 is the Bessel function. The lower curves in Figs. 3 and 4 correspond to the function (8). In this case the experimental data are in good agreement with the function (8) at lower values of the ratio d/l.

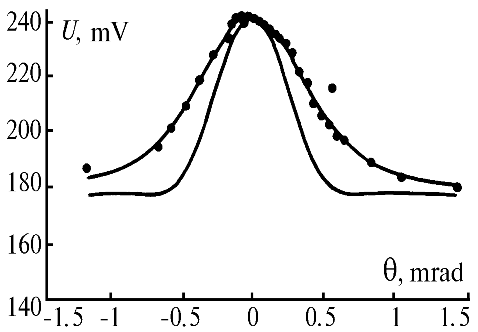

FIG. 3. Angular dependence of the backscattering peak: d = 0.8 mm; l = 0.12 mm; K = 1.41; θ_0 = 0.517 mrad; θ_l = 0.223 mrad; θ_d = 0.405 mrad.

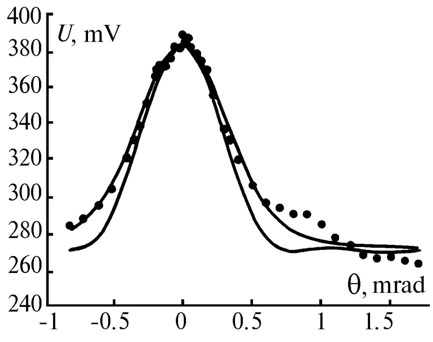

FIG. 4. Angular dependence of the backscattering peak: d = 0.7 mm; l = 1.29 mm; K = 1.47; θ_0 = 0.436 mrad; θ_l = 0.040 mrad; θ_d = 0.463 mrad.

For the spheres with larger optical depth $d \gg l$ it is evident that the peak shape is determined by the radiation intensity distribution in local regions with linear dimensions of the order of l and the peak shape must not depend essentially on the shape of the scattering medium. In this case the peak shape can be approximated by a simple expression known for the plane-parallel layer[26]:

$$P_2(\theta) = \theta_l^2/(\theta_l^2 + \theta^2), \tag{9}$$

where θ_l is the halfwidth of the function (9).

For the media of an arbitrary optical depth one can assume that joint action of the two factors, i.e., limitedness of the medium and the finite value of the extinction length, can be approximated by a two-dimensional convolution of the functions (8) and (9). The upper solid curve in Figs. 3 and 4 corresponds to the convolution of the function (8) with the function (9), where the free parameter θ_l is fitted using the least squares method so that the experimental points coincide best with a given convolution.

The experimental data obtained in this way for all the spheres considered have been divided into two groups. The first group is represented by the spheres with small optical depth $d < l$. Here the peak shape of coherent scattering is well described by the function (8), and the peak halfwidth θ_0 corresponds to the relationship followed from Eq. (8):

$$\theta_d = 0.514\ \lambda/d\,. \tag{10}$$

The experimental data in Fig. 5 are presented by dots, and solid curve stands for the linear dependence constructed as the best linear approximation of these experimental data. The proportionality factor obtained is equal to 0.504 that is close to theoretical value in Eq. (10).

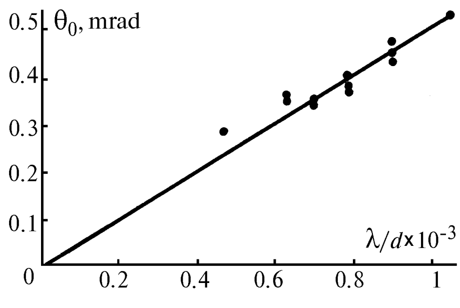

FIG. 5. Angular halfwidth of the backscattering peak for the spheres of small optical depth.

The second group is represented by the spheres with large optical depth. The data obtained here, as a result of the above inversion of the functions (8) and (9) convolution, for the peak halfwidth θ_l show their linear dependence (Fig. 6) on the parameter λ/l

$$\theta_l = 0.065\ \lambda/l\,. \tag{11}$$

similar to the above dependence (4) for the plane-parallel scattering media.

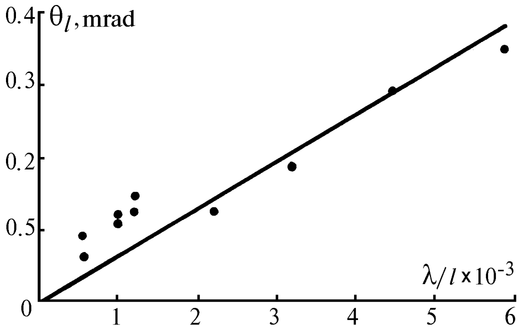

FIG. 6. Angular halfwidth of the backscattering peak for the spheres of large optical depth.

It should be noted that in Ref. 19 for plane-parallel media with the particle dimensions within 0.2–2.0 μm the measured proportionality factor in Eq. (11) equals 0.0573. Although we did not carry out special measurements estimating the particle size, the practical coincidence of these coefficients can be considered as an indirect proof of the fact that the particle dimensions in our scattering media were in the same interval.

Thus, the assumption on the influence of the shape and dimensions of scattering medium on the peak of coherent backscattering as the convolution of the two functions, having a physical meaning of the above functions (8) and (9), has been confirmed by the experimental data obtained.

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