A SIMPLE INTERPRETATION SCHEME FOR DATA OF POLARIZATION LASER SOUNDING OF CRYSTALLINE CLOUDS

A.A. Popov and O.V. Shefer

Mari Polytechnical Institute, Yoshkar-Ola Institute of Atmospheric Optics, Siberian Branch of the Academy of Sciences of the USSR, Tomsk

Received May 16, 1989

A theoretical study of the dependence of the backscattering coefficients of a polarization lidar on the optical properties of a polydisperse medium has been performed. In the capacity of the latter a set of oriented circular plates with complex refractive index was considered. As a result, within the scope of this model we have succeeded in relating the ratios of the backscattering coefficients through simple analytic relations with the Euler angles defining the orientation of the plates and their complex refractive index.

When the atmosphere is sounded with a laser, the measured Intensity of the backscattering signal is related by the lidar equation to two optical characteristics of the scattering volume: the backscattering coefficient β_π and attenuation coefficient α. It is clear that at a single frequency it is impossible to determine these two characteristics without using additional information, on which the dependability of interpretation schemes for interpreting the data of single-frequency laser sounding depends significantly.

In single-frequency polarization laser sounding four (the number of parameters of the Stokes vector) detection channels may be used simultaneously. But, polarization lidars are used to probe morphologically complicated atmospheric formations, such as, for instance, crystalline clouds. In this case one can expect certain difficulties to arise in the interpretation due to the lack of information. However, such apprehensions appear to be for nothing. In the present paper it is shown that in polarization laser sounding one can do without additional information. Moreover, it is possible to connect the sounding data with the optical properties of the polydisperse crystalline medium by simple relations.

Let us define the parameters of the Stokes vector for the backscattering signal by the following relations

$$I_{\pi_1} = |E_{R,\parallel}|^2 + |E_{R,\perp}|^2,\ I_{\pi_2} = |E_{R,\parallel}|^2 - |E_{R,\perp}|^2.$$

$$I_{\pi_3} = 2\,\mathrm{Re}(E_{R,\parallel}\cdot E^*_{R,\perp}),\ I_{\pi_4} = 2\,\mathrm{Im}(E_{R,\parallel}\cdot E^*_{R,\perp}),$$

where $E_{R,\parallel}$ and $E_{R,\perp}$ are the complex amplitudes of the electric components of the backscattered electromagnetic field, defined in two mutually perpendicular planes. Every parameter of the Stokes vector is proportional to the corresponding backscattering coefficient, i.e., $I_{\pi j} = D\beta_{\pi j}$ $(j = 1, 2, 3, 4)$. Note that in the given problem the entire set of coefficients $\beta_{\pi j}$ $(j = 1, 2, 3, 4)$ can be thought of as backscattering coefficients for a polarized signal, bearing in mind that in the classical sense of this term only one of them, $\beta_{\pi 1}$, is so called. The proportionality coefficient D includes as factors the point spread function, the reciprocal square of the distance from the lidar to the scattering volume, the square of the transmittance, i.e., the quantities involved in the lidar equation. Here we only note that the quantity D is the same for all four equations connecting the parameters of the Stokes vector to the backscattering coefficients. For this reason the ratios of the Stokes parameters, found by experiment,[1] are equal to the ratios of the corresponding backscattering coefficients:

$$P_{j1} = \frac{I_{\pi j}}{I_{\pi 1}} = \frac{\beta_{\pi j}}{\beta_{\pi 1}}.$$

Thus, despite the fact that the backscattering coefficients $\beta_{\pi j}$ themselves cannot be calculated from the corresponding lidar equations without use of a priori information, their ratios P_{j1} can be determined directly by experiment.

For a polydisperse system of nonspherical particles, characterized by two linear dimensions a and d, the backscattering coefficients $\beta_{\pi j}$ $(j = 1, 2, 3, 4)$ can be expressed in the form

$$\beta_{\pi_j}(\lambda) = \int\limits_a \int\limits_d N(a, d)\,\sigma_{\pi_j}(a, d, \lambda)\,da\,dd, \qquad (1)$$

where $N(a, d)$ is a two-dimensional function of the particle size distribution; $\sigma_{\pi j}(a, d, \lambda)$ are the backscattering cross sections for a non-spherical particle, which depend also on the Euler angles β and γ, the complex refractive index $\tilde{n} = n + i\kappa$, and the parameters of the Stokes vector

$$I_1 = |E_\parallel|^2 + |E_\perp|^2,\ I_2 = |E_\parallel|^2 - |E_\perp|^2,$$

$$I_3 = 2\,\mathrm{Re}\,(E_\parallel,\ E^*_\perp), I_4 = 2\,\mathrm{Im}\,(E_\parallel E^*_\perp),$$

0235-6880/90/01 036–6 $02.00

of a polarized wave incident on the particle. Earlier we obtained and investigated the formulae for the backscattering cross sections $\sigma_{\pi j}$ of polarized radiation from the simplest non-sphere in the form of a round plate.[3, 4] Finally, the quantities $\sigma_{\pi j}$ are expressed as various combinations of the complex coefficients f_1, f_2, f_3, which have the following form:

$$f_1 = C_{\parallel} \cos^2\gamma - C_{\perp} \sin^2\gamma,$$
$$f_2 = C_{\perp} \cos^2\gamma - C_{\parallel} \sin^2\gamma,$$
$$f_3 = -(C_{\parallel} + C_{\perp}) \sin\gamma \cos\gamma, \qquad (2)$$

where

$$C_{\parallel} = \frac{1+\cos 2\beta}{2} (R_{\parallel} G_0(2\beta, 0) + S_{\parallel}),$$

$$C_{\perp} = \frac{1+\cos 2\beta}{2} (R_{\perp} G_0(2\beta, 0) + S_{\perp}) \qquad (3)$$

(β is the angle between the direction of propagation of the incident wave and the plate axis; γ is the angle between one of the components of the incident wave field and the normal to the plane of incidence). Fresnel's coefficients and $R_{\parallel}$ for wave reflection from the plate and the angular scattering function $G_0(2\beta, 0)$ are defined by the following relations

$$R_{\parallel} = \frac{\tilde{n}\cos\beta - \cos\tilde{\vartheta}}{\tilde{n}\cos\beta + \cos\tilde{\vartheta}}, \quad R_{\perp} = \frac{\cos\beta - \tilde{n}\cos\tilde{\vartheta}}{\cos\beta + \tilde{n}\cos\tilde{\vartheta}}, \qquad (4)$$

$$G_0(2\beta, 0) = \pi a^2 \cos\beta \frac{2J_1(\kappa a \sin 2\beta \cos\beta)}{\kappa a \sin 2\beta \cos\beta}, \qquad (5)$$

where a is the radius of the plate; $k = 2\pi/\lambda$ is the wave number, $\tilde{\theta}$ is the complex refraction angle, $J_1(z)$ Is the Bessel function of first order. The quantities $S_{\parallel}$ and $S_{\perp}$ in relations (3) define the contribution to the scattered field of beams with various multiplicity of reflection. Formulas for these quantities are given in Ref. 3. It is only to be noted that $S_{\parallel}$ and $S_{\perp}$ depend on all of the parameters of the plate, including its thickness d.

One can simplify the polydisperse integral (1) using the functional relation $d = \varphi(a) = 2.020 \cdot (2a)^{0.449}$ between the plate thickness d and its diameter $2a$.[4] As a result, integral (1) can be reduced to

$$\beta_{\pi_j}(\lambda) = \int_0^\infty N(a)\, \sigma_{\pi_j}(a, \varphi(a), \lambda)\, da. \qquad (6)$$

(In calculating $\beta_{\pi j}$ relation (6) may be used instead of (1) without loss of generality of the problem.)

Formulas (6) for calculation of the backscattering coefficients $\beta_{\pi j}$ admit further simplification. Its essence consists in the following. The contribution of the terms $S_{\parallel}$ and $S_{\perp}$ in the sums (3) is insignificant.[3] It decreases even further when calculating the polydisperse integral (6) because $S_{\parallel}$ and $S_{\perp}$ have greater oscillations with respect to variations in the dimensions of the plate a, d in comparison with G_0. However, if the quantities $S_{\parallel}$, $S_{\perp}$ are neglected, the relations for the backscattering cross sections $\sigma_{\pi j}$ simplify. In particular, they can be represented in the form of two factors, one of which depends on the scatterer's geometric dimensions, the other, on the complex refractive index. Thus, without taking internal reflections into account ($S_{\parallel} = 0$, $S_{\perp} = 0$) the relations for the backscattering cross sections can be transformed into the form

$$\sigma_{\pi_j}(a, d, \lambda) \approx \sigma^0_{\pi_j}(a, \lambda) = F_\pi(a, \lambda) \cdot A_j, \qquad (7)$$

where

$$F_\pi(a, \lambda) = \frac{\kappa^2}{\pi} \left\{ \frac{1+\cos 2\beta}{2} G_0(2\beta, 0) \right\}^2; \qquad (8)$$

$$A_1 = \frac{|R_{\parallel}|^2 + |R_{\perp}|^2}{2} + \frac{I_2}{I_1} \frac{|R_{\parallel}|^2 + |R_{\perp}|^2}{2} \cos 2\gamma - \frac{I_3}{I_1} \frac{|R_{\parallel}|^2 - |R_{\perp}|^2}{2} \sin 2\gamma \qquad (9)$$

$$A_2 = \frac{|R_{\parallel}|^2 - |R_{\perp}|^2}{2} \cos 2\gamma + \frac{I_2}{I_1} \left\{ \frac{|R_{\parallel}|^2 + |R_{\perp}|^2}{2} \cos^2 2\gamma - \mathrm{Re}(R_{\parallel} R^*_{\perp}) \sin^2 2\gamma \right\} - \frac{I_3}{I_1} \left\{ \frac{|R_{\parallel}|^2 + |R_{\perp}|^2}{2} + \mathrm{Re}(R_{\parallel} R^*_{\perp}) \right\} \frac{\sin 4\gamma}{2} + \frac{I_4}{I_1} \mathrm{Im}(R_{\parallel} R^*_{\perp}) \sin 2\gamma; \qquad (10)$$

$$A_3 = \frac{|R_{\parallel}|^2 - |R_{\perp}|^2}{2} \sin 2\gamma + \frac{I_2}{I_1} \left\{ \frac{|R_{\parallel}|^2 + |R_{\perp}|^2}{2} + \mathrm{Re}(R_{\parallel} R^*_{\perp}) \right\} \frac{\sin 4\gamma}{2} + \frac{I_3}{I_1} \left\{ -\frac{|R_{\parallel}|^2 + |R_{\perp}|^2}{2} \sin^2 2\gamma + \mathrm{Re}(R_{\parallel} R^*_{\perp}) \cos^2 2\gamma \right\} - \frac{I_4}{I_1} \mathrm{Im}(R_{\parallel} R^*_{\perp}) \cos 2\gamma; \qquad (11)$$

$$A_4 = \frac{I_2}{I_1} \mathrm{Im}(R_{\parallel} R^*_{\perp}) \sin 2\gamma + \frac{I_3}{I_1} \mathrm{Im}(R_{\parallel} R^*_{\perp}) \cos 2\gamma + \frac{I_4}{I_1} \mathrm{Re}(R_{\parallel} R^*_{\perp}). \qquad (12)$$

As a result, the backscattering coefficients $\beta_{\pi j}$ can be written as

$$\beta_{\pi_j}(\lambda) \approx \beta^0_{\pi_j} = A_j I(\lambda), \qquad (13)$$

where

$$I(\lambda) = \int_0^\infty N(a) F_\pi(a, \lambda)\, da. \qquad (14)$$

The function F_π, defined above by relation (8), depends on the geometric parameters of the scatterer and is independent of its refractive index. As a result, the polydispersion integral $I(\lambda)$ is also independent of the optical properties of the scatterers. Note that the polydispersion integral is a common factor for all the $\beta^0_{\pi j}$. Finally, we obtain the following relations for the desired quantities P_{j1}

$$P_{j1} = \frac{\beta_{\pi j}}{\beta_{\pi_1}} \approx \frac{\beta^0_{\pi j}}{\beta^0_{\pi_1}} = \frac{A_j}{A_1}, \; j = 2,\ 3,\ 4. \tag{15}$$

Thus, to first approximation the measured quantities P_{j1} depend only on the Euler angles β and γ and the complex refractive index of the plates $\tilde{n}$.

Numerical calculations, carried out for various wavelengths, plates' optical constants and orientations, showed that it is possible to replace the ratios of the polydispersion integrals $\beta_{\pi j}/\beta_{\pi 1}$ by the corresponding approximate analytical ratios A_j/A_1 with an error of not more than 2%. This means that it is possible to set up a correspondence between the measured ratios of the Stokes vector parameters $I_{\pi j}/I_{\pi 1}$ and the analytical ratio A_j/A_1 within an error of 2% in the interpretation of sounding data.

For circular polarization of the incident field ($I_2 = I_3 = 0$, $I_4/I_1 = -1$) we obtain the polarization characteristic

$$\frac{A_4}{A_1} = -\frac{2\,\mathrm{Re}\,(R_\parallel R^*_\perp)}{|R_\parallel|^2 + |R_\perp|^2}. \tag{16}$$

In this case the characteristic A_4/A_1 is independent of the angle γ. Plots of A_4/A_1 as a function of the other Euler angle β, which defines the plate orientation, are shown in Fig. 1. Note that for fixed $n = \mathrm{Re}(\tilde{n})$ the quantity A_4/A_1 stands in one-to-one correspondence with the angle β. Even though when the condition $n - 1 \gg \kappa$ holds the dependence of A_4/A_1 on $\kappa = \mathrm{Im}(\tilde{n})$ may be neglected, nevertheless two unknowns n and β are left, related to one measured quantity $P_{41} \approx A_4/A_1$. In this case the interpretation scheme should be additionally defined involving information from other measurements.

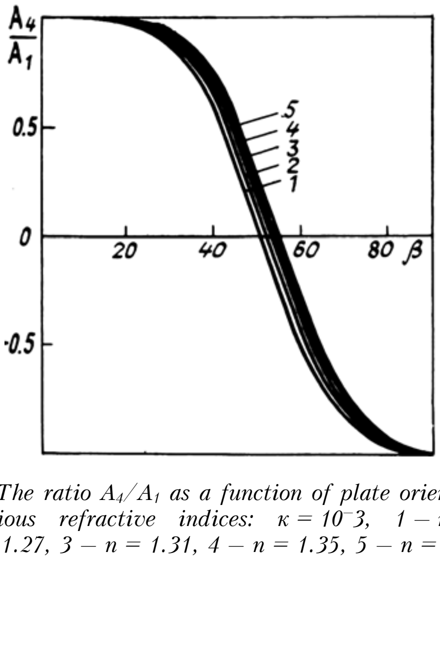

FIG. 1. The ratio A_4/A_1 as a function of plate orientation β for various refractive indices: $\kappa = 10^{-3}$, 1 – n = 1.23, 2 – n = 1.27, 3 – n = 1.31, 4 – n = 1.35, 5 – n = 1.39.

For linear polarization of the incident field ($I_3 = I_4 = 0$, $I_3/I_1 = 1$) we obtain the polarization characteristic A_2/A_1:

$$\frac{A_2}{A_1} = \frac{(|R_\parallel|^2\cos^2\gamma - |R_\perp|^2\sin^2\gamma)\cos 2\gamma - \mathrm{Re}\,(R_\parallel R^*_\perp)\sin^2 2\gamma}{|R_\parallel|^2\cos^2\gamma + |R_\perp|^2\sin^2\gamma}. \tag{17}$$

As could be expected, the ratio A_2/A_1 contains more information than does the ratio A_4/A_1. For this reason the characteristic $A_2/A_1 \approx \beta_{\pi 2}/\beta_{\pi 1}$ has found wider application in experiment.[1]

Plots of A_2/A_1 versus the angle γ for various β and n are shown in Figs. 2 and 3. At certain angles $\gamma_{min} = \gamma_{min}(\beta, \tilde{n})$ all the curves have minima. The value γ_{min} is determined by the equation

$$\frac{d}{d\gamma}\left(\frac{A_2}{A_1}\right) = 0.$$

After some obvious transformations we find the desired angle γ_{min}

$$\gamma_{min} = \arccos\sqrt{\frac{|R_\perp|}{|R_\perp| + |R_\parallel|}}.$$

(In an experiment the variation of the angle γ can be realized by rotating the lidar about its axis). The desired values β and n will be associated with a certain curve, rather than a single point, as in the previous case. As a result, an experimental dependence of $I_{\pi 2}/I_{\pi 1}$ on γ can always be put in correspondence with a theoretical dependence of A_2/A_1 on γ with parameters β and n, the latter determined by the method of least squares. The curves shown in Figs. 2 and 3 do not cross, which ensures that β and n are defined uniquely. Note that the processing of the experimental data in real time implies the realization of the interpretation algorithms on a minicomputer, with a minimum number of operations. The proposed formula (17) satisfies this criterion. If there is no opportunity to turn the lidar about its axis, the angle γ in relation (17) becomes an unknown parameter. In this case it is more expedient to use the circular polarization of the wave since relation (16) does not include the angle γ, all other things being equal.

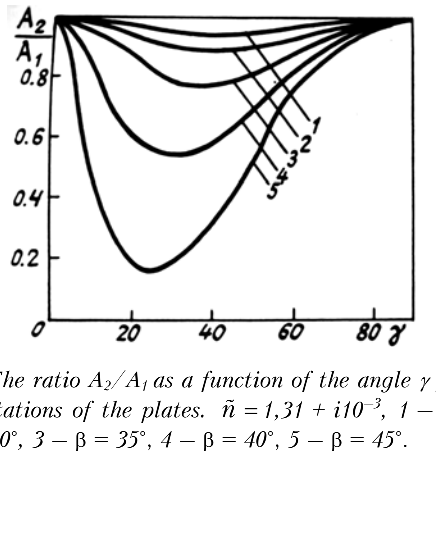

FIG. 2. The ratio A_2/A_1 as a function of the angle γ for various orientations of the plates. $\tilde{n} = 1,31 + i10^{-3}$, 1 – β = 25°, 2 – β = 30°, 3 – β = 35°, 4 – β = 40°, 5 – β = 45°.

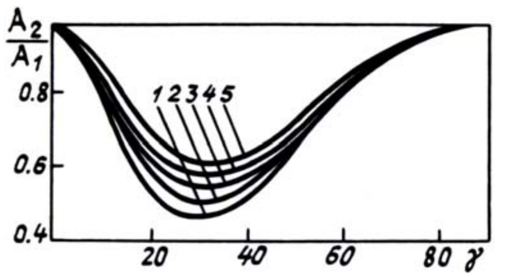

FIG. 3. The ratio A_2/A_1 as a function of the angle γ for various refractive indices: $\beta = 40°$, $\kappa = 10^{-3}$, *1 – n = 1.23, 2 – n = 1.27, 3 – n = 1.31, 4 – n = 1.36, 5 – n = 1.39.*

The analysis performed above shows that if polarization sounding data are available one can pose the problems of determining the crystal orientation β, its refractive index $n = \mathrm{Re}(\tilde{n})$, or both of these values simultaneously, without using additional information.

For a more detailed investigation of crystalline formations in the atmosphere it is necessary to determine the particle size distribution function $N(a)$, which is possible only with the help of a multifrequency lidar and the method of multifrequency laser sounding.[S] The unknown function $N(a)$ can be found from integral equation (14). In addition, it follows from relation (13) that in formulating the inverse problem of multifrequency laser sounding one has to know the coefficients A_j and, consequently, the refractive index and crystal orientation must be known a priori.

Thus, with the help of a single-frequency polarization lidar without using additional information one can determine the orientation and refractive index of lamellar crystals. It is worth using a multifrequent lidar only in cases where the refractive index and particle orientation are known from other measurements. Simultaneous use of both lidars enables one to obtain complete information about a system of nonspherlcal particles.

REFERENCES

1. V.S. Shamanaev and A.I. Abramochkln, *Airborne Polarization Lidar "Svetozar-3"*. Preprint No. 15, Institute of Atmospheric Optics, Siberian Branch of the Academy of Sciences of the USSR, Tomsk, (1984).
2. A.A. Popov. Opt. Atmos. **1**, No. 5, **19** (1988).
3. A.A. Popov and O.V. Shefer, *On Polarization Laser Sounding of Crystalline Clouds: A Simple Optical Model of a Particle.* Preprint No. 65, Institute of Atmospheric Optics. Siberian Branch of the Academy of Sciences of the USSR. Tomsk. (1988).
4. A. Auer and D. Veal, J. Atm. Sci. **27**, No. 6, 919 (1970).
5. I.E. Naats, *Theory of Multifrequency Laser Sounding of the Atmosphere* (Nauka, Novosibirsk, 1980).