GROUNDS FOR THE STRATONOVICH CALCULUS IN ANALYSIS OF WAVE PROPAGATION THROUGH THE MEDIA WITH DELTA–CORRELATED RANDOM FLUCTUATIONS IN THE DIELECTRIC CONSTANT

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The electromagnetic wave propagation through the medium with delta–correlated dielectric constant fluctuations is considered in the quasioptical approximation. Equations for the mean field and coherence functions are derived within the framework of an arbitrary stochastic calculus. It is shown that only the Stratonovich calculus is compatible with the conservation law of the radiation power.

Theoretical development on the behavior of the systems under the action of random forces is significantly simplified if this action can be proved to be a delta–correlated random process (field). In so doing quite general laws have been found which govern the behavior of the dynamic systems pertaining to different fields of natural sciences, e.g., physics, chemistry, biology, etc. (see, for example, Refs. 1–3). However, with multiplicative noise the simplification of the solution of the problem due to the delta–correlation leads to the ambiguity of going over from the stochastic equations describing the behavior of the dynamic system to the equations for the statistical moments (i.e., to the equations of the Fokker–Planck type for the probability density functions). In this case the so–called Ito–Stratonovich dilemma takes place (see, for example, Refs. 1, 3, and 4).

On the one hand, since the diffuse random process approximation is the zeroth approximation in the noise correlation time, it seems to be natural to use the Stratonovich calculus in which the delta–correlated processes can be manipulated like ordinary functions.[2] On the other hand, as shown in Ref. 5, in the case of thermodynamic systems the kinetic calculus obeying the Onsager principle corresponds neither the Ito calculus no the Stratonovich calculus (see also Ref. 4). Mathematicians developing the stochastic differential equation theory are more impressed by the Ito calculus (see, for example, Refs. 1, 3, and 6).

As is well known, the above ambiguity is associated with the extreme irregularity in the white noise behavior which for the Wiener process $W(\xi)$ is mathematically described by the relations

$$\delta W(\xi) \sim \sqrt{\delta\xi} \;, \quad (\delta W(\xi))^2 \sim C_\alpha \delta\xi \;.$$

Here the constant C_α takes different values depending on the type of calculus, in particular, in the case of the Stratonovich calculus ($\alpha = 1/2$) $C_\alpha = 0$. To eliminate the ambiguity and to choose the appropriate stochastic calculus the formal solution of the dynamic equation has to be followed by further (physical) considerations concerning the system behavior.

In this paper we discuss the problem of choosing the type of the stochastic calculus in the study of wave propagation through the media with delta–correlated fluctuations in the complex dielectric constant ε. More than 20 years ago it was shown that under certain conditions the random variations $\tilde{\varepsilon}$ could be treated as delta–correlated (see, for example, Refs. 7 and 8). However, the substantiation of choosing the type of calculus for this problem has not been performed and all the calculations have been really made default within the framework of the Stratonovich calculus. However, it is of interest to approach this problem from the two angles. First, the parabolic equation in quasioptics approximation (which governs the complex field amplitude) containing the random field of dielectric constant is the example of the dynamic equation including the multiplicative white noise and for $\mathrm{Im}(\tilde{\varepsilon}) = 0$ it can be easily observed how the unambiguity of the solution is established. Namely, in the case of $\mathrm{Im}(\tilde{\varepsilon}) = 0$ the complex field amplitude, in addition to Eq. (2), must satisfy the conservation law of the radiation power following directly from Eq. (2). Second, if the imaginary part of $\tilde{\varepsilon}$ is nonzero then, because of nonconservation of the power, it is necessary to take into account some additional considerations to establish the unambiguity. In particular, the Ito–Stratonovich dilemma is solved here under condition that the random actions of real and imaginary parts of $\tilde{\varepsilon}$ are treated within the framework of the same calculus.

Let us consider the electromagnetic wave propagation through the medium with fluctuations in the complex dielectric constant which varies as

$$\tilde{\varepsilon}(z, \rho) = \tilde{\varepsilon}_R(z, \rho) + i\, \tilde{\varepsilon}_I(z, \rho) \,, \tag{1}$$

where $\tilde{\varepsilon}_R$ and $\tilde{\varepsilon}_I$ are the real and imaginary parts of $\tilde{e}$, respectively. Let the conditions of the problem be such that the wave propagation is described by the parabolic equation of quasioptics[7]

$$2\, i\, \kappa \frac{\partial}{\partial z} U + \Delta_\perp U + \kappa^2 \tilde{\varepsilon}(z, \rho) U = 0 \,, \tag{2}$$

where $U(z, \rho)$ is the complex amplitude of the electric field strength, k is the wave number, and the field $\tilde{\varepsilon}$ is taken to be Gaussian, homogeneous, and delta–correlated in the preferred direction of the wave propagation (i.e., along the z axis, whereas ρ is the radius vector in the transverse direction). Note, that this approximation is adequate for the propagation of optical radiation through the turbulent atmosphere as well as for the propagation of acoustic waves in the ocean in which the sound velocity is the fluctuating

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quantity. The mean value of the imaginary part of ε is eliminated from Eq. (2) because it has no direct effect on the statistical wave characteristics and will be taken into account in the last section when analyzing the results.

First two sections of the paper deal with the derivation of the closed system of equations describing the behavior of the mean field $\langle U\rangle$ and coherence functions

$$\Gamma_{mn}(z, \{\rho_j\}_{j=1,\, n+m}) = \langle\prod_{j=1}^{n}\prod_{l=n+1}^{n+m} U(z, \rho_j)\, U^*(z, \rho_l)\rangle$$

for unspecified type of stochastic calculus. These equations are the generalization of the well–known equations, derived in Ref. 7, to the case of the wave propagation through the media with the complex dielectric constant fluctuations. In the third section the solution for the random field realization within the framework of the arbitrary calculus are represented in terms of the solution after Stratonovich. Analysis of the results and choice of the calculus are given in the last section. In the basic part of the paper the case is considered in which both the real and imaginary parts of $\tilde{\varepsilon}$ follow the same calculus. As for the problem of different calculuses for $\tilde{\varepsilon}_R$ and $\tilde{\varepsilon}_I$, it is briefly discussed at the end of the paper.

1. MEAN FIELD EQUATION

Simultaneously with Eq. (2), we hereafter will use the parabolic equation in the form

$$\delta U(z, \rho) = \frac{i}{2\kappa}\Delta_\perp U(z, \rho)\delta z + \frac{ik}{2}U(\tilde{z}, \rho)\,\delta W(z; \rho)\ . \qquad (3)$$

Here $\delta W(z; \rho) = \tilde{\varepsilon}(z, \rho)\delta z$ is the Wiener process (the term *field* would be more correct, nevertheless we follow the conventional terminology) with zero mean

$$\langle\delta W(z; \rho)\rangle = \langle\tilde{\varepsilon}(z, \rho)\rangle\,\delta z = 0 \qquad (4a)$$

and correlation functions

$$\langle\delta W(z; \rho_1)\,\delta W(z; \rho_2)\rangle = \langle\tilde{\varepsilon}(z, \rho_1)\,\tilde{\varepsilon}(z, \rho_2)\rangle\,\delta z\,\delta z =$$

$$= A_{\varepsilon\varepsilon}(\rho_1 - \rho_2)\,\delta z\ ;$$

$$\langle\delta W(z; \rho_1)\,\delta W^*(z; \rho_2)\rangle =$$

$$= \langle\tilde{\varepsilon}(z, \rho_1)\,\tilde{\varepsilon}^*(z, \rho_2)\rangle\delta z\,\delta z = A_{\varepsilon\varepsilon^*}(\rho_1 - \rho_2)\delta z\ . \qquad (4b)$$

In so doing the functions

$$A_{\varepsilon\varepsilon}(\rho) = A_{RR}(\rho) + 2\,i\,A_{RI}(\rho) - A_{II}(\rho)\ ,$$

$$A_{\varepsilon\varepsilon^*}(\rho) = A_{RR}(\rho) + A_{II}(\rho) \qquad (5)$$

are expressed in terms of the three–dimensional spectral power density $\Phi_{qq'}(\kappa_z, \kappa)$ of the components of $\tilde{\varepsilon}$

$$A_{qq'}(\rho) = 2\pi\int\int d^2\kappa\,\Phi_{qq'}(0, \kappa)\,e^{i\kappa\rho}\ ,$$

where $\{q, q'\} = \{R, I\}$. The value of $U(\tilde{z}, \rho)$ in the second term in the left side of Eq. (3) is determined in the plane $z = z_i + \alpha\delta z$ (see Fig. 1), so that depending on the value of the parameter $\alpha \in [0, 1]$ we deal with different stochastic calculuses. In particular, the value $\alpha = 0$ corresponds to the solution after Ito and the value $\alpha = 1/2$ corresponds to the solution after Stratonovich which is usually used when solving Eq. (3). We do not specify the value of the parameter α so far, but generalize the known results reported in Ref. 7 for the mean field within the framework of the Stratonovich calculus to the case of the arbitrary calculus. The angular brackets denote averaging over the ensemble of realizations of the random field $\tilde{\varepsilon}$.

In accordance with Eq. (3), we write down the equation for the mean field

$$\delta\langle U(z, \rho)\rangle = \frac{i}{2\kappa}\Delta_\perp\langle U(z, \rho)\rangle\delta z + \frac{i\kappa}{2}\langle U(\tilde{z}, \rho)\,\delta W(z; \rho)\rangle. \qquad (6)$$

To determine the correlator $\langle U(\tilde{z})\delta W(z)\rangle$ we expand U in the Taylor series around the point z_i (see Fig. 1)

$$U(\tilde{z}) = U(z_i) + \alpha\delta U(z_i)$$

and making use of Eq. (5), the property of nonadvanced function within the framework of the Ito calculus, and the field characteristics $\delta W(z, \rho)$ from Eqs. (4), we obtain

$$\langle U(\tilde{z})\,\delta W(z_i)\rangle = \langle[\,U(z_i) + \alpha\delta U(z_i)]\,\delta W(z_i)\rangle =$$

$$= \frac{i\kappa}{2}\alpha\langle U(z_i, \rho)\rangle\langle\delta W^2(z_i; \rho)\rangle = \frac{i\kappa}{2}\,\alpha\,A_{\varepsilon\varepsilon}(0)\langle U(z_i, \rho)\rangle\delta z. \qquad (7)$$

Substitution of Eq. (7) into initial equation (6) gives the equation for the mean field

$$\delta\langle U(z, \rho)\rangle = \frac{i}{2\kappa}\Delta_\perp\langle U(z, \rho)\rangle\delta z - \frac{\kappa^2}{4}\,\alpha\,A_{\varepsilon\varepsilon}(0)\langle U(z, \rho)\rangle\delta z$$

or in a more usual form

$$2i\,\kappa\frac{\partial}{\partial z}\langle U\rangle + \Delta_\perp\langle U\rangle + \frac{i\kappa^3}{4}\,\alpha\,A_{\varepsilon\varepsilon}(0)\langle U(z, \rho)\rangle = 0\ . \qquad (8)$$

Equation (8) for $\alpha = 1/2$ (the Stratonovich calculus) coincides with that found previously (for $\tilde{\varepsilon}_I = 0$) by the alternative method (see, for example, Ref. 7).

To conclude, we note that the solution of Eq. (8) can be represented as

$$\langle U(z, \rho)\rangle = \langle U(z, \rho)\rangle_{1/2}\exp\left[-\frac{\kappa^2}{4}(\alpha - 1/2)\,A_{\varepsilon\varepsilon}(0)\,z\right], \qquad (9)$$

where $\langle U(z, \rho)\rangle_{1/2}$ is the solution after Stratonovich for $\alpha = 1/2$. Hereafter the notation $F_{1/2}$ (for the functions of the parameter α) has the same meaning.

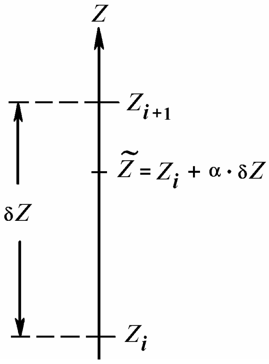

FIG. 1.

2. EQUATION FOR THE COHERENCE FUNCTIONS

Here we need to use one more unusual property of the functions of the Wiener process. [The first one consists in the ambiguity of going to the equations for the statistical moments which is a consequence of the dependence of the results of averaging, for example, in Eq. (7), on the choice of the point $\tilde{z}(\alpha)$.] In the root–mean–square limit within the framework of the Ito calculus the relation

$$\delta W(z;\ \rho_1)\,\delta W(z;\ \rho_2) = A_{;\,;}(\rho_1 - \rho_2)\,\delta z$$

holds,[1] which is generalized to the case of the arbitrary calculus

$$\delta W(z;\ \rho_1)\,\delta W(z;\ \rho_2) = (1 - 2\alpha)\,A_{\varepsilon\varepsilon}(\rho_1 - \rho_2)\delta z\ . \qquad (10)$$

As a consequence, for the differential of the product of two functions U taken at different transverse points, making use of the definition of δU in the form of Eq. (3) and retaining the terms of the first order in δz, we have

$$\delta(U(z,\ \rho_1)\,U^*(z,\ \rho_2)) =$$

$$= U(z,\ \rho_1)\delta U^*(z,\rho_2) + U^*(z,\rho_2)\delta U(z,\rho_1) + \delta U(z,\rho_1)\delta U^*(z,\rho_2) =$$

$$= U(z,\ \rho_1)\,\delta U^*(z,\ \rho_2) + U^*(z,\ \rho_2)\,\delta U(z,\ \rho_1) -$$

$$- \frac{\kappa^2}{2}(\alpha - 1/2)\,A_{\varepsilon\varepsilon^*}(\rho_1 - \rho_2)\,U(z,\ \rho_1)U^*(z,\ \rho_2)\delta z\ . \qquad (11)$$

First let us derive the equation for the coherence function of the second order

$$\Gamma_{11}(\rho_1,\ \rho_2;\ z) = \langle U(z,\ \rho_1)\,U^*(z,\ \rho_2)\rangle\ .$$

We will do that in a standard way. The equations for U_1 and U_2^* are multiplied by U_2^* and U_1, respectively, and are subtracted one from another. After averaging and using Eq. (11), we obtain

$$\delta\Gamma_{11} = \frac{i}{2\kappa}[\Delta_1 - \Delta_2]\,\Gamma_{11}\,\delta z - \frac{\kappa^2}{2}(\alpha - 1/2)\,A_{\varepsilon\varepsilon^*}(\rho_1 - \rho_2)\,\Gamma_{11}\,\delta z +$$

$$+ \frac{i\kappa}{2}\langle U_1(\tilde{z})U_2^*(\tilde{z})\,[\delta W_1(z) - \delta W_2^*(z)]\rangle\ ,$$

where

$$U_j(z) = U(z,\ \rho_j)\ ,\ \delta W_j(z) = \delta W(z;\ \rho_j)\ ,$$

$$\Delta_j = \frac{\partial^2}{\partial x_j^2} + \frac{\partial^2}{\partial y_j^2}\ ,\ \text{and}\ j = 1,\ 2.$$

After the subsequent calculations similar to those given in Sec. 1 for the mean field we arrive at the equation for the coherence functions of the second order in the case of the arbitrary calculus

$$2i\kappa\frac{\partial}{\partial z}\Gamma_{11} + [\Delta_1 - \Delta_2]\Gamma_{11} + \frac{i\kappa^3}{4}[A_{\varepsilon\varepsilon}(0) + A_{\varepsilon\varepsilon}^*(0) - 2A_{\varepsilon\varepsilon^*}(\rho_1 - \rho_2)]\Gamma_{11} +$$

$$+ \frac{i\kappa^3}{4}(\alpha - 1/2)[A_{\varepsilon\varepsilon}(0) + A_{\varepsilon\varepsilon}^*(0)]\,\Gamma_{11} = 0. \qquad (12)$$

This equation for $\alpha = 1/2$ also coincides with the known equation for Γ_{11} (at $\tilde{\varepsilon}_I = 0$) obtained previously within the framework of the Stratonovich calculus.[7] Its solution may be represented in the form

$$\Gamma_{11}(\rho_1,\ \rho_2;\ z) = \Gamma_{11}(\rho_1,\ \rho_2;\ z)_{1/2}\ \times$$

$$\times\ \exp\left\{-\frac{\kappa^2}{4}(\alpha - 1/2)\,[A_{\varepsilon\varepsilon}(0) + A_{\varepsilon\varepsilon}^*(0)]\,z\right\}\ . \qquad (12a)$$

By performing the analogous calculations, Eq. (12) is easily generalized to the case of the $(n + m)$th field moment. Let us write down the final result without derivation

$$2i\kappa\frac{\partial}{\partial z}\Gamma_{nm} + \left[\sum_{j=1}^{n}\Delta_j - \sum_{j=n+1}^{n+m}\Delta_j\right]\Gamma_{nm} +$$

$$+ \frac{i\kappa^3}{4}(\alpha - 1/2)\,[n\,A_{\varepsilon\varepsilon}(0) + m\,A_{\varepsilon\varepsilon}^*(0)]\,\Gamma_{nm} +$$

$$+ \frac{i\kappa^3}{4}\left[\sum\sum_{j,\ l=1}^{n}A_{\varepsilon\varepsilon}(\rho_j - \rho_l) + \sum\sum_{j,\ l=n+1}^{n+m}A_{\varepsilon\varepsilon}^*(\rho_j - \rho_l) -\right.$$

$$\left.- 2\sum_{j=1}^{n}\ \sum_{l=n+1}^{n+m}A_{\varepsilon\varepsilon^*}(\rho_j - \rho_l)\right]\Gamma_{nm} = 0\ . \qquad (13)$$

Equation (13) within the framework of the Stratonovich calculus ($\alpha = 1/2$) in the case of radiation propagating through the transparent medium ($\mathrm{Im}\tilde{\varepsilon}_I = 0$) is also equivalent to that obtained in Ref. 7. Its solution by analogy with Eqs. (9) and (12a) can be written down as

$$\Gamma_{nm}(z,\ \{\rho_j\}_{j=1,\ n+m}) = \Gamma_{nm}(z,\ \{\rho_j\}_{j=1,\ n+m})_{1/2}\ \times$$

$$\times\ \exp\left\{-\frac{\kappa^2}{4}(\alpha - 1/2)\,[n\,A_{\varepsilon\varepsilon}(0) + m\,A_{\varepsilon\varepsilon}^*(0)]\,z\right\}. \qquad (14)$$

It can be seen from Eqs. (9), (12a) and (14) that the solution of Eq. (2) for the random realization of the complex field amplitude within the framework of the arbitrary calculus is expressed in terms of the solution after Stratonovich

$$U(z,\ \rho) = U(z,\ \rho)_{1/2}\ \exp\left\{-\frac{\kappa^2}{4}(\alpha - 1/2)\,A_{\varepsilon\varepsilon}(0)\,z\right\}. \qquad (15)$$

Now let us demonstrate that this statement immediately follows from Eq. (2).

3. SOLUTION FOR THE FIELD REALIZATION

The solution of Eq. (2) is represented in the form of the Huygens–Kirchhoff integral

$$U(z,\ \rho) = \int\int d^2\rho_0 U_0(\rho_0)\ G(\rho,\ z\,|\,\rho_0,\ 0)\ , \qquad (16)$$

where $G(\rho,\ z\,|\,\rho_0,\ z_0)$ is the Green's function of Eq. (2) satisfying the initial condition

$$G(\rho,\ z\,|\,\rho_0,\ z_0)\big|_{z=z_0} = \delta(\rho - \rho_0)$$

and obeying the group property

$$G(\rho,\ z\,|\,\rho_0,\ z_0) = \int\int d^2\rho_1\ G(\rho,\ z\,|\,\rho_1,\ z_1)\ G(\rho_1,\ z_1\,|\,\rho_0,\ z_0), \qquad (17)$$

whose repeated application provides the basis for the representation of the Green's function $G(\rho,\ z\,|\,\rho_0,\ z_0)$ in the form of the Feynman path integral.[9] Examples of using the functional integration in problems of wave propagation through the random media can be found in Refs. 8 and 10. Here we make use of the idea from Ref. 9 and treat the solution of Eq. (2) in the elementary interval $z \in [z_i + \delta z, z_i]$ (see Fig. 1) in which the transverse change in $\tilde{\varepsilon}$ can be neglected (ρ in $\tilde{\varepsilon}(z, \rho)$ is taken to be the fixed parameter). Then we go over from Eq. (2) to the equation for the Fourier transform of the function $G(\rho, z_i + \delta z \,|\, \rho_0, z_i)$

$$\tilde{G}(\kappa, z_i + \delta z \,|\, \rho_0, z_i) = \frac{1}{(2\pi)^2} \int\int d^2\rho\, G(\rho, z_i + \delta z \,|\, \rho_0, z_i)\, e^{-i\kappa\rho},$$

which is given by

$$\delta\tilde{G}(\kappa, z \,|\, \rho_0, z_i) = -\frac{i}{2k}\kappa^2\, \tilde{G}(\kappa, z)\delta z + \frac{i\,k}{2}\tilde{G}(\kappa, \tilde{z})\,\delta W(z; \rho)\,,$$

$$\tilde{G}(\kappa, z \,|\, \rho_0, z_i)\big|_{z=z_i} = \frac{1}{(2\pi)^2}\, e^{-i\kappa\rho_0}\,. \quad (18)$$

The solution of Eq. (18) within the frameworks of both the Ito and Stratonovich calculuses is given in Ref. 1 and can be easily generalized to the case of the arbitrary calculus making use of relation (10) for the Wiener process. Finally, taking the inverse Fourier transform of the Green's function in the elementary interval δz, we obtain

$$G(\rho, z_i + \delta z \,|\, \rho_0, z_i) = G(\rho, z_i + \delta z \,|\, \rho_0, z_i)_{1/2} \times$$

$$\times \exp\left\{-\frac{k^2}{4}(\alpha - 1/2)\, A_{\varepsilon\varepsilon}(0)\, d z\right\}.$$

Furthermore, by analogy with Ref. 9, repeatedly employing the property in the form of Eq. (17), we finally arrive at the Green's function

$$G(\rho, z \,|\, \rho_0, z_0) = G(\rho, z \,|\, \rho_0, z_0)_{1/2} \exp\left\{-\frac{k^2}{4}(\alpha - 1/2) A_{\varepsilon\varepsilon}(0) z\right\}$$

This relation simultaneously with Eq. (16) leads to Eq. (15).

4. CHOICE OF STOCHASTIC CALCULUS

To eliminate the ambiguity and to determine the value of the parameter α, we make use of the additional condition for the solution of Eq. (2). For this purpose we take into account the fact that in the transparent medium ($\tilde{\varepsilon}_I = 0$) the radiation power, which is defined by the relation

$$P(z) = \int\int d^2\rho\, I(z, \rho)\,,$$

has to be preserved,[7] where $I(z, \rho)$ is the radiation intensity.

Making use of the definition of $P(z)$, Eq. (2) for $U(z, \rho)$, and relation (11) for the differential of the product of two functions, we obtain the equation for $P(z)$ within the framework of the arbitrary calculus

$$\frac{d}{dz}P(z) = -k\left\{\bar{\varepsilon}_I + \frac{k}{4}(\alpha - 1/2)\,[A_{\varepsilon\varepsilon}(0) + A^*_{\varepsilon\varepsilon}(0)]\right\} P(z) -$$

$$- k\int\int d^2\rho\, \tilde{\varepsilon}_I(z, \rho)\, I(z, \rho)\,,\quad P(z)\big|_{z=0} = P_0\,, \quad (19)$$

where $\bar{\varepsilon}_I$ is the mean value of the imaginary part of the complex dielectric constant of the medium.

Let us next consider the wave propagation through the transparent medium, when $\varepsilon_I = 0$. Then the solution of Eq. (19) becomes

$$P(z) = P_0 \exp\left\{-\frac{k^2}{4}(\alpha - 1/2)\, A_{RR}(0)\, z\right\}.$$

It can be seen from this relation that the only calculus satisfying the conservation of the beam power in the problems of wave propagation is the Stratonovich calculus for $\alpha = 1/2$. In other words the simultaneous solution of Eqs. (2) and (19) allows one to eliminate the ambiguity connected with the choice of the stochastic calculus.

Considering that the fluctuations of the real and imaginary parts of the complex dielectric constant are subject to the same calculus, we can evidently generalize this result to the attenuating medium. Since the parameter α is independent of the components of $\tilde{\varepsilon}$, the result must remain unchanged in the case of the complex dielectric constant.

In conclusion, we consider briefly the case when the components of $\tilde{\varepsilon}$ are subjected to different stochastic calculuses. For example, this is the case of the strongly absorbing particles suspended in the turbulent transparent gas medium (in Ref. 11 the problem of a laser beam propagation in rain, formulated in a similar manner, was solved) providing that the random variations of the components of $\tilde{\varepsilon}$ are uncorrelated. Then the certain parameter α_q ($q = \{R, I\}$) corresponds to each component and relation (10) is transformed to

$$\delta W(z; \rho_1)\, \delta W(z; \rho_2) = (1 - 2\alpha_R)\, A_{RR}(\rho_1 - \rho_2)\delta z -$$

$$- (1 - 2\alpha_I)\, A_{II}(\rho_1 - \rho_2)\, \delta z\,. \quad (20)$$

Meanwhile, the result obtained in the case of simultaneous solution for fluctuations of $\tilde{\varepsilon}_R$ remains valid, i.e., $\alpha_R = 1/2$, and taking into account Eq. (20), Eq. (19) for the power becomes

$$\frac{d}{dz}P(z) = -k\left\{\bar{\varepsilon}_I - \frac{k}{4}(\alpha_I - 1/2)\, A_{II}(0)\right\}P(z) -$$

$$- k\int\int d^2\rho\, \tilde{\varepsilon}_I(z, \rho)\, I(z, \rho)\,. \quad (21)$$

Now we make use of the fact that the parameter α_I is independent of the path length and write down the solution of Eq. (21) in the zeroth approximation in the small parameter a_{ef}/L_0 (where a_{ef} and L_0 are the effective radius of a beam and the external scale of the turbulence, respectively)

$$P(z) = P_0 \exp\left\{-k\left[\bar{\varepsilon}_I - \frac{k}{2}(\alpha_I - \frac{1}{2})A_{II}(0)\right]z - k\int_0^z d\xi\,\tilde{\varepsilon}_I(\xi, 0)\right\}.$$

This law of the power variation corresponds to the lognormal distribution of the probability of the random power variations. In this case only the mean power depends on the type of stochastic calculus and in principle any calculus is noncontradictory. Only returning to the real processes with finite correlation length and taking into account the fact that the effects engendered by the approximation of the delta–correlated process must be negligible, one can set the value of the parameter $\alpha_I = 1/2$.

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