Possibilities of aiming optical beams through turbulent atmosphere

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When solving problems of optical location, communication, and energy transmission, a task appears of transporting radiant energy in the form of a light beam to an object located in a randomly inhomogeneous medium. In this case, as a rule, it is necessary to maximize the amount of energy delivered to the object. As is well known, radiation scattering by inhomogeneities of the refractive index of the medium leads to a decrease of the average intensity in the near-axial region of the light beam and to the appearance of intensity fluctuations, which taken together substantially degrade the energetic characteristics of the indicated systems. Several simple variants of the adaptive control for beam parameters, based on the principle of reciprocity of radiation propagation through an inhomogeneous medium, are proposed. The information on the medium inhomogeneties distribution along the beam propagation path is derived from the intensity distribution in the image plane of an object.

Introduction

When solving the problems of optical location, communication, and energy transmission, a radical means of preventing undesirable effects is an application of various adaptive methods, which allows one in principle to almost completely eliminate the influence of the medium inhomogeneities. The essence of these methods reduces to controlling the initial distribution of the beam field on the basis of information about the instantaneous distribution of inhomogeneities of the medium, in which the beam is propagating.

1. Use of the reciprocity principle to control the parameters of an optical beam

Introduce the Cartesian coordinate system (õ, ρ) (see Fig. 1) so that to direct the X axis along the direction of the beam propagation.

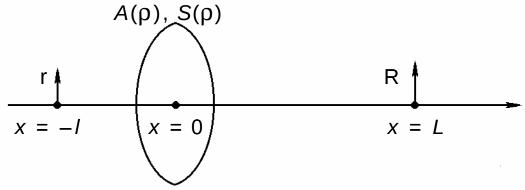

Fig. 1. Block-diagram of an adaptive system for aiming and focusing optical beams.

Let the initial distribution of the beam field $U_0(\rho)$ be prescribed in the õ = 0 plane. At the point (L, R) of the inhomogeneous medium, the field can be written in the form

$$U_\mathrm{b}(L,\mathrm{R}) = \iint d^2\rho\, U_0(\rho) G(L,\mathrm{R};0,\rho), \qquad (1)$$

where $G(L, \mathrm{R}; 0, \rho)$ is the Green's function of the problem or, equivalently, the field of the spherical wave at the point $(0, \rho)$ of the inhomogeneous medium, created by a point source located at the point (L, R). The radiant intensity at that point then has the form

$$I_\mathrm{b}(L,\mathrm{R}) = \iint d^2\rho_1 d^2\rho_2\, \Gamma_0(\rho_1,\rho_2) G(L,\mathrm{R};0,\rho_1) G^*(L,\mathrm{R};0,\rho_2), \qquad (2)$$

where $\Gamma_0(\rho_1,\rho_2) = U_0(\rho_1) U_0^*(\rho_2)$ is the initial value of the beam coherence function.

In the case of a partially coherent beam $\Gamma_0(\rho_1,\rho_2) = \langle U_0(\rho_1) U_0^*(\rho_2)\rangle$, where $\langle\ldots\rangle$ is the averaging that is carried out over the random fluctuations of the source (here it is understood that $I_\mathrm{b}(L,\mathrm{R})$ in Eq. (2) is also averaged over source fluctuations).

Let radiation from a point object located at the point (L, R), be incident upon the receiver aperture of the optical system located in the plane $x = 0$. Assuming that the medium inside the optical system is homogeneous, the field in the plane $x = -l$ can be written in the form

$$U_\mathrm{im}(-l,\rho) = W \iint d^2\rho\, G(L,\mathrm{R};0,\rho) A(\rho) \times \exp\big(iS(\rho)\big)\, G_0(0,\rho;-l,\mathrm{r}). \qquad (3)$$

Here $A(\rho)$ is the amplitude transmission coefficient of the receiving optical system, $S(\rho)$ is the phase shift introduced by the optical system (see Fig. 1), W is a constant which depends on the radiation power emitted by the point object, $G_0(x_0, \rho_0; x, \rho)$ is the

Green's function for the homogeneous medium, which has the form

$$G_0(x_0,\rho_0;x,\rho) = = \frac{k}{2\pi|x-x_0|}\exp\left\{ik|x-x_0| + i\frac{k(\rho-\rho_0)^2}{2|x-x_0|}\right\}. \quad (4)$$

Using formulas (3) and (4), we obtain the following expression for the radiant intensity in the plane $x = -l$:

$$I_{\rm im}(-l,\mathrm{r}) = \frac{W^2k^2}{4\pi^2l^2}\times$$
$$\times\iint d^4\rho_{1,2}G(L,\mathrm{R};0,\rho_1)G^*(L,\mathrm{R};0,\rho_2)A(\rho_1)A(\rho_2)\times$$
$$\times\exp\big(i[S(\rho_1)-S(\rho_2)]\big)\times$$
$$\times\exp\left\{i\frac{k}{2l}(\rho_1^2-\rho_2^2) - i\frac{k}{l}\mathrm{r}(\rho_1-\rho_2)\right\}. \quad (5)$$

By virtue of the reciprocity principle

$$G_0(x_0,\rho_0;x,\rho) = G_0(x,\rho;x_0,\rho_0), \quad (6)$$

comparing Eqs. (1) and (3), taking into account Eqs. (4) and (6), we can easily see that[1,2] upon fulfillment of the condition

$$U_0(\rho) = q\frac{k}{2\pi il}A(\rho)\exp\left(iS(\rho) + ikl + i\frac{k\rho^2}{2l} - ik\frac{\mathrm{r}\rho}{l}\right), \quad (7)$$

where q = const, the field of the coherent beam at the point (L, R) coincides to within a constant factor with the intensity of the radiation from the point source at the point r in the plane $x = -l$, i.e.,

$$U_{\rm b}(L,\mathrm{R}) = ClU_{\rm im}(-l,\mathrm{r}). \quad (8)$$

What is more, Eq. (7) does not depend on the position of the source (i.e., on either L or R) and the parameters (l, r) can be chosen quite arbitrarily on the basis of convenience.

Similarly to Refs. 3 to 6, from Eqs. (2) and (5) upon fulfillment of the condition

$$\Gamma_0(\rho_1,\rho_2) = \frac{q^2k^2}{4\pi^2l^2}A(\rho_1)A(\rho_2)\exp\{[S(\rho_1) - S(\rho_2)] +$$
$$+ i\frac{k}{2l}(\rho_1^2-\rho_2^2) - i\frac{k}{l}\mathrm{r}(\rho_1-\rho_2)\}, \quad (9)$$

we find that the beam intensity at the point (L, R) coincides to within a constant factor with the intensity of the radiation from the point source at the point r in the plane $x = -l$:

$$I_{\rm b}(L,\mathrm{R}) = C^2l^2I_{\rm im}(-l,\mathrm{r}). \quad (10)$$

Also, it is not hard to see that for a coherent beam, for which $\Gamma_0(\rho_1,\rho_2) = \langle U_0(\rho_1)U_0^*(\rho_2)\rangle$, fulfillment of condition (7) also leads to Eq. (10).

Relations (7)–(10) are thus an exact consequence of the reciprocity principle (6) and mean that the field from the point source located at the point (L, R) in the inhomogeneous medium having passed through an opening with amplitude–phase transmission coefficient $A(\mathrm{r})\exp(iS(\mathrm{r}))$ and being observed at the point $(-l, \mathrm{r})$ coincides with the field from a point source located at the point $(-l, \mathrm{r})$, which has passed through the same opening and is being observed at the point (L, R) of the inhomogeneous medium.[7]

The initial distribution of the beam field $U_0(\rho)$ is therewith treated as the result of the passage of a spherical wave created by a source located in the region $x < 0$ through a screen with some chosen value of the amplitude–phase transmission coefficient.

Equality (10) together with condition (9) makes it possible to obtain information about the instantaneous values of the intensity fluctuations of the beam field at some remote point of the inhomogeneous medium on the basis of intensity measurements at an appropriately chosen point located behind the optical system.

Below we consider one of the variants[7–9] of this possibility application to control order beam parameters in order to maximize the radiation intensity at the point (L, R).

1.1. Choice of the time of emission of the radiation pulse

Let us consider the simplest case[4,7] of beam parameter control – a choice of the moment of the radiation pulse emission. In this case, we pose the problem of choosing the time of emission of the laser pulse in such a way that the field intensity at the object is maximized. Here we also assume that the power of the radiation emitted by the object does not fluctuate, and the propagation time to the object and back, as well as the duration of the pulse are so short that the intensity has not time to vary to any significant extent during this time. To solve this problem, it is necessary to track the intensity of self-radiation of the object at some point $(-l, \mathrm{r})$ located behind the optical system with amplitude–phase transmission coefficient $A(\mathrm{r})\exp(iS(\mathrm{r}))$, defined by Eq. (7), and to emit the pulse at the moment when the intensity has a significant positive peak.

For a coherent beam with initial field distribution

$$U_0(\rho) = \sqrt{I_0(\rho)}\exp\left(-i\frac{k\rho^2}{2F}\right), \quad (11)$$

where $I_0(\rho)$ is the initial intensity distribution, F is the radius of curvature of the beam wavefront, for a given value of l from Eq. (7) we have

$$A^2(\rho) = I_0(\rho);$$
$$S(\rho) = \frac{k\rho^2}{2}\left[-\frac{1}{l} - \frac{1}{F}\right] = -\frac{k\rho^2}{2f}. \quad (12)$$

Here f is the focal length of the optical system, and these relations do not depend on the position of the object relative to the beam axis. For a given value of

f, it is not hard to see that when using a collimated beam ($1/F = 0$), the observation plane should coincide with the focal plane of the optical system. When aiming a focused beam ($F = +L$), $1/f = 1/l + 1/L$, i.e., the intensity should be recorded in the image plane of the object.

1.2. Aiming of an optical beam

Let us investigate the problem of aiming the beam at an object. In this case our goal is to determine the direction of the beam axis that maximizes $I_b(L, \mathrm{R})$. Varying the beam direction is equivalent to introducing a linear phase term in the initial field distribution, i.e., in the case under consideration

$$U_0(\rho) = U^0(\rho)\exp(ik\gamma\rho),$$

here γ is the direction of the beam axis, $U^0(\rho)$ is the initial field distribution independent of γ. For a given distance from the receiver aperture plane l to the observation plane we choose the amplitude–phase transmission coefficient such that

$$A(\rho)\exp\big(iS(\rho)\big) = CU^0(\rho)\exp\left(i\frac{k\rho^2}{2l}\right). \qquad (13)$$

For initial field distribution (11), the condition (13) again leads to Eq. (12). In this case the matching condition (7) is fulfilled for

$$\gamma = -\mathrm{r}/l, \qquad (14)$$

this means that the intensity distribution $I_{\mathrm{im}}(-l, \mathrm{r})$ in the observation plane will be proportional to the beam intensity at the object, if the direction of the beam axis $\gamma = -\mathrm{r}/l$. Thus, the best directions of the beam axis in the sense of maximizing the intensity at the object will be those that correspond to those r in the recording plane, at which the intensity of the radiation from the object is maximal.

Obviously, the above method of aiming the beam can be combined for the case of pulsed radiation with the above method of choosing the best time for emitting the pulse. For aiming continuous radiation at an object, it is necessary to aim the beam in the direction determined by the brightest point of the intensity distribution. By virtue of the fact that matching conditions (13) and (14) do not depend on the coordinates of the object (L, R), this aiming method also provides for automatic tracking of the motion of the object. Note also that the aiming method being in a widespread use at the present time, based on the center of gravity of the intensity distribution, does not give the best beam direction in the sense described above.

1.3. Focusing an optical beam

Let us consider the problem of aiming and focusing a bounded coherent beam. In this case it is necessary to optimize the direction and focal length of the beam from measurements of the intensity distribution in some region located behind the optical system. The initial field distribution has the form

$$U_0(\rho) = U^0(\rho)\exp\left(ik\gamma\rho - i\frac{k\rho^2}{2F}\right), \qquad (15)$$

where γ and F are, respectively, the direction of the beam axis and the focal length of the beam, both are objects of control. Assume that $U^0(\rho)$ does not contain any terms linear or quadratic in ρ that can be included in the phase factor in Eq. (15), and let $S(\rho) = S'(\rho) - l\rho^2/2f$, where $S'(\rho)$ also does not contain any quadratic terms, and f is the focal length of the receiving optical system. Upon satisfaction of the equality

$$A(\rho)\exp\big(iS'(\rho)\big) = CU^0(\rho) \qquad (16)$$

conditions (7) and (9) are fulfilled if

$$\begin{cases} \gamma = -\mathrm{r}/l; \\ \dfrac{1}{F} = \dfrac{1}{f} - \dfrac{1}{l}. \end{cases} \qquad (17)$$

Upon satisfaction of equality (16) and conditions (17)

$$I_b(L, \mathrm{R}) \approx l^2 I_s(-l, \mathrm{r}),$$

i.e., the intensity of the radiation from a point source at the point $(-l, \mathrm{r})$, located behind the optical system will be proportional to the intensity of the beam whose direction γ, and focal length F are given by relations (17).

Thus, the best focal length of the beam F and direction of its axis γ are the values defined by relations (17), i.e., there exists a point $(-l, \mathrm{r})$, at which the quantity $l^2 I_s(-l, \mathrm{r})$ is at its maximum. Instead of searching for the maximum of $I_s(-l, \mathrm{r})$ in a three-dimensional space, we can vary the focal length of the receiver system f in time with l fixed. It is not hard to show that for the case of a homogeneous medium, rules (16) and (17) ensure that the beam axis is focused and aimed at the point object. Naturally, the principle under consideration can be used only for focusing the field without aiming the beam axis. In this case, it is necessary to measure $I_s(-l, \mathrm{r})$ along the extension of the beam axis into the region $x < 0$ and to choose F on the basis of the second of conditions (17).

1.4. A partially coherent beam

Up until this point, the consideration has been limited to coherent beams, for which the amplitude and phase of the initial distribution $U_0(\rho)$ do not undergo uncontrolled fluctuations, and fulfillment of equalities (12), (13), and (16) is in principle possible. In the case of a partially coherent beam, the phase and/or amplitude invariably experience fluctuations, which makes it impossible to satisfy relations of the type (12), (13), and (16). Besides, it is obvious that a partially coherent beam cannot be represented as the result of diffraction of a spherical wave by an opening with regular amplitude–phase transmission. Thus, relation (9) for a partially coherent beam is not fulfilled, and the recording of the radiant intensity by the receiver system does not allow one to predict the behavior of the intensity of a partially coherent beam at a remote object. Nevertheless, this problem can be solved for one class of partially coherent beams.

Let the intensity in the $x = -l$ plane be recorded not at a point, as earlier, but with the help of a receiver of finite dimensions, described by a spatial intensity transmission coefficient $T(\mathrm{r}_0)$, whose center, as before, is located at the point r. The recorded radiant flux $I_s(-l, \mathrm{r})$ in this case, referring to formula (5), can be written as

$$I_s(-l,\mathrm{r}) = \iint \mathrm{d}^2 r_0 I_s(-l,\mathrm{r}_0) T(\mathrm{r}_0 - \mathrm{r}) =$$

$$= \frac{C^2k^2}{4\pi^2 l^2} \iint \mathrm{d}^4\rho_{1,2} G(l,\mathrm{R};0,\rho_1) G^*(l,\mathrm{R};0,\rho_2) \times$$

$$\times A(\rho_1) A(\rho_2) F(\rho_1 - \rho_2) \exp\{i[S(\rho_1) - S(\rho_2)]\} \times$$

$$\times \exp\{i\frac{k(\rho_1^2 - \rho_2^2)}{2l} - i\frac{k}{l}\mathrm{r}(\rho_1 - \rho_2)\}, \qquad (18)$$

where

$$F(\rho) = \iint \mathrm{d}^2 r_0 T(\mathrm{r}_0) \exp\left(-i\frac{k}{l}\mathrm{r}_0\rho\right). \qquad (19)$$

A comparison of Eqs. (18) and (2) leads to the matching condition

$$\Gamma_0(\rho_1, \rho_2) = \frac{q^2k^2}{4\pi^2 l^2} A(\rho_1) A(\rho_2) \exp\{i[S(\rho_1) - S(\rho_2)]\} \times$$

$$\times \exp\{i\frac{k}{2l}(\rho_1^2 - \rho_2^2) - i\frac{k}{l}\mathrm{r}(\rho_1 - \rho_2)\} F(\rho_1 - \rho_2), \quad (20)$$

the fulfillment of which leads to relation (10). The above relation is a generalization of Eq. (9) and allows us to extend the previously obtained results to the case of partially coherent beams with the coherence function

$$\Gamma_0(\rho_1, \rho_2) = \langle U_0(\rho_1) \rangle \langle U_0^*(\rho_2) \rangle P(\rho_1 - \rho_2). \qquad (21)$$

Coherence function of such a form arises if one assumes that the beam undergoes spatially homogeneous amplitude–phase fluctuations with the second moment $P(\rho)$.

As follows from Eqs. (9), (20), and (21), upon fulfillment of the additional matching condition

$$P(\rho) = \iint \mathrm{d}^2 r T(\mathrm{r}) \exp\left(-i\frac{k\mathrm{r}}{l}\rho\right) \qquad (22)$$

the solution of the above-described problems of choosing the best time of pulse emission and aiming and focusing of the beam becomes possible. In the case of an object with brightness distribution $M(r)$, expression (5) takes the form

$$I_s(-l,\mathrm{r}) = \frac{C^2k^2}{4\pi^2 l^2} \iint \mathrm{d}^4\rho_{1,2} \iint \mathrm{d}^2 R G(l,\mathrm{R};0,\rho_1) G^*(l,\mathrm{R};0,\rho_2) \times$$

$$\times M(\mathrm{R}) A(\rho_1) A(\rho_2) \exp\{i[S(\rho_1) - S(\rho_2)]\} \times$$

$$\times \exp\{i\frac{k(\rho_1^2 - \rho_2^2)}{2l} - i\frac{k}{l}\mathrm{r}(\rho_1 - \rho_2)\}. \qquad (23)$$

Here, obviously, one can only pose a task of maximizing the integral

$$\iint \mathrm{d}^2 R M(\mathrm{R}) I_b(L,\mathrm{R}). \qquad (24)$$

As was noted above, the approaches considered here can be regarded as the simplest cases of adaptive control for radiation parameters. Here we are controlling only the total tilt of the wavefront (13), (14), and / or its curvature. Currently available means of adaptive control (multi-element mirrors, deformable mirrors, etc.) allow one to control simultaneously a large number of radiation parameters (in particular, the wavefront) and to create with a high degree of accuracy and speed the required phase distribution in the radiation reflected from the mirror. The fundamental difficulty here is the problem of working out the requirements to the phase distribution to be created.

Let the phase shift introduced by the optical system receiving radiation from the point source be the quantity subject to control (with the help of active optical elements). If the realized phase shift $S_m(\rho)$ is such that the intensity of the received radiation $I_s(-l, \mathrm{r})$ at some point r in the $x = -l$ plane behind the optical system is maximized in comparison with other possible realizations $S(\rho)$, then upon fulfillment of matching condition (9), by virtue of relation (10), the beam intensity at the point object will also be maximized. At the same time, if there exists some phase distribution of the beam (for a given amplitude distribution), which maximizes the intensity at the point object, then the intensity of the radiation of the point object, at the point $(-l, \mathrm{r})$, located behind the optical system, with the same amplitude transmission coefficient, is maximized if condition (7) is fulfilled. The phase of the beam and the phase shift of the optical system differ here by the regular added term $(k\rho^2/2l - k\mathrm{r}\rho/l)$, which ensures that the beam wave is focused in the $x = -l$ plane and that its axis is directed at the point r.

Thus, the necessary and sufficient condition for maximizing the intensity of the coherent beam radiation at a point object while controlling its phase is the maximization of the intensity of the radiation received from this point object. The problem of maximizing the beam intensity at the object, which may be inaccessible, reduces to the problem of maximizing the intensity of radiation from the object at some accessible point $(-l, \mathrm{r})$.

Obviously, all previously examined problems[4–9,14] of beam aiming and focusing are simple special cases of the scheme under consideration. The foregoing generalizations of these problems to a partially coherent beam and an extended object hold for the general phase control problem.

2. Control for parameters of an optical beam based on reflected radiation

In the above problems it was assumed that the object, to which radiation is being delivered, is a self-luminant one or scatters radiation from some source off to the side, which does not pass through the inhomogeneous medium. Let now the object located at the point (L, R) be illuminated by radiation, which has passed through the layer of inhomogeneous medium. The radiation scattered by the object passes through the same layer and is received by the optical system. Following the development of Eqs. (1)–(5), we may write the radiation intensity at the point ($-l$, r) located behind the optical system in the form

$$I_R(L,\mathrm{R};-l,\mathrm{r}) = C\iint d^2\rho_1 d^2\rho_1' d^2\rho_2 d^2\rho_2' \Gamma_0(\rho_1,\rho_2)\times$$
$$\times G(0,\rho_1;L,\mathrm{R})G^*(0,\rho_2;L,\mathrm{R})G(L,\mathrm{R};0,\rho_1')G^*(L,\mathrm{R};0,\rho_2')\times$$
$$\times A(\rho_1')A(\rho_2')\exp[i(S(\rho_1') - S(\rho_2')]G_0(0,\rho_1';-l,\mathrm{r})\times$$
$$\times G_0^*(0,\rho_2';-l,\mathrm{r}), \qquad (25)$$

where C is the intensity reflection coefficient of the object, the variables' indices without primes correspond to coordinates in the initial beam, and the primed indices – to coordinates in the receiving plane. If the matching condition (9) is fulfilled, expression (25) reduces to

$$I_R(L,\mathrm{R};-l,\mathrm{r}) = \frac{C}{l^2} I_\mathrm{b}^2(L,\mathrm{R}). \qquad (26)$$

It can be seen from relation (26) that the intensity of the reflected signal fluctuates as the square of the field intensity at the object and, consequently, when the maximum of the received signal is reached, then the intensity at the object also becomes maximal.

2.1. A point object

Let the scattering object be a point and located at the point (L, 0), and let the initial beam distribution $U_0(\rho)$ be given in the $\tilde{o}$ = 0 plane. The receiving aperture of the optical system, described by the amplitude transmission function $\grave{A}(\rho)$ and phase shift $S(\rho)$, is also located in the $\tilde{o}$ = 0 plane. The intensity of the received radiation is recorded in the $x = -l$ plane behind the receiving aperture. It is assumed that the medium behind the receiving aperture (for $\tilde{o}$ < 0) is homogeneous.

The intensity of the radiation incident on the object, I_obj, can be written in the form

$$I_{\hat{\imath}\mathrm{bj}} = \iint d^4\rho_{1,2}\Gamma_0(\rho_1,\rho_2)G(0,\rho_1;L,0)G^*(0,\rho_2;L,0), \quad (27)$$

where $\Gamma_0(\rho_1, \rho_2)$ is the source coherence function; $G(\ldots)$ is the Green's function for an inhomogeneous medium. The radiant intensity at the point ($-l$, r) located behind the receiving system can be written in the form

$$I_R(-l,\mathrm{r}) = I_{\hat{\imath}\mathrm{bj}}B\iint d^4\rho_{1,2}G(L,0;0,\rho_1)G^*(L,0;0,\rho_2)\times$$
$$\times A(\rho_1)A(\rho_2)\exp[iS(\rho_1) - iS(\rho_2)]G_0(0,\rho_1;-l,\mathrm{r})\times$$
$$\times G_0^*(0,\rho_2;-l,\mathrm{r}), \qquad (28)$$

where $\hat{A}$ is the effective scattering surface of the object, $G_0(0, \rho_1;\ -l, \mathrm{r})$ is the Green's function for a homogeneous medium. For l = const, let I_R attain its maximum at r = $\mathrm{r_m}$. As can be seen from Eq. (28), this takes place when the integral in Eq. (28) reaches its maximum by r. Comparing Eqs. (27) and (28) at fulfillment of the condition

$$\Gamma_0(\rho_1,\rho_2) = CA(\rho_1)A(\rho_2)\exp[iS(\rho_1) - iS(\rho_2)]\times$$
$$\times G_0(0,\rho_1;-l,\mathrm{r})G_0^*(0,\rho_2;-l,\mathrm{r}) =$$
$$= \frac{Ck^2}{4\pi^2 l^2}A(\rho_1)A(\rho_2)\exp\{i[S(\rho_1) - iS(\rho_2)] +$$
$$+ i\frac{k}{2l}(\rho_1^2 - \rho_2^2) - i\frac{k\mathrm{r}}{l}(\rho_1 - \rho_2)\} \qquad (29)$$

and taking into account the reciprocity principle (6)

$$G(x_0,\rho_0;x,\rho) = G(x,\rho;x_0,\rho_0) \qquad (30)$$

it is not difficult to show that if r = $\mathrm{r_m}$, then the radiant intensity at the object $I_{\hat{\imath}\mathrm{bj}}$ also reaches its maximum in comparison with other values of r. Relation (29) for r = $\mathrm{r_m}$ gives a selection rule for the best (in the sense of the maximum of $I_{\hat{\imath}\mathrm{bj}}$) direction of the beam axis.

If the initial beam field distribution is given to within the axis tilt (to within terms in the phase, linear in ρ), then optimal reception is achieved when the receiving optical system ($\grave{A}(\rho)$, $S(\rho)$) is matched with the beam according to condition (29). Then the best direction of the beam axis is the direction corresponding to the maximum of $I_R(-l, \mathrm{r})$.

Analogous considerations can be employed to obtain the best focusing or, what is the same, to choose the phase front curvature (the quadratic terms in the initial phase of the beam). In this case, one must search for $l^2 I_\mathrm{rec}(-l, \mathrm{r})$ maximum over the variables l, r. If this maximum is reached at l_m, $\mathrm{r_m}$, then the form, which the initial coherence function should take, is

$$\Gamma_0(\rho_1,\rho_2) = Cl^2A(\rho_1)A(\rho_2)\exp[iS(\rho_1) - iS(\rho_2)]\times$$
$$\times G_0(0,\rho_1;-l_\mathrm{m},\mathrm{r_m})G_0^*(0,\rho_2;-l_\mathrm{m},\mathrm{r_m}) =$$
$$= \frac{Ck^2}{4\pi^2}A(\rho_1)A(\rho_2)\exp\{i[S(\rho_1) - iS(\rho_2)] +$$
$$+ i\frac{k}{2l_\mathrm{m}}(\rho_1^2 - \rho_2^2) - i\frac{k\mathrm{r_m}}{l_\mathrm{m}}(\rho_1 - \rho_2)\}. \qquad (31)$$

For this choice, the focal length and direction of the beam are optimal. Choosing U_0 and Γ_0 according to condition (31) actually changes only the linear and quadratic terms in the beam phase and does not change the field amplitude, thanks to the factor of l^2 before I_{rec}.

Note that in contrast to the case when the source is a point object, generally speaking, the problem of finding the optimal time of the pulse emission is insoluble since the magnitude of I_{obj} also fluctuates.

2.2. An extended diffuse object

Let a reflecting diffuse extended object has the intensity reflection coefficient $B(r)$. This means that

$$\Gamma_{ref}(\rho_1,\rho_2) = \Gamma_{inc}(\rho_1,\rho_2)B\left(\frac{|\rho_1+\rho_2|}{2}\right)\delta(\rho_1-\rho_2). \quad (32)$$

Here, in analogy with Eqs. (27) and (28), we can write $I_R(-l, \mathrm{r})$ in the form

$$I_R(-l,\mathrm{r}) = \iint \mathrm{d}^4\rho_{1,2}\mathrm{d}^4\rho'_{1,2}\mathrm{d}^2R\,\Gamma_0(\rho_1,\rho'_1)G(0,\rho_1;L,\mathrm{R})\times$$
$$\times G^*(0,\rho'_1;L,\mathrm{R})B(\mathrm{R})G(L,\mathrm{R};0,\rho_2)G^*(L,\mathrm{R};0,\rho'_2)\times$$
$$\times A(\rho_2)A(\rho'_2)\exp\{i[S(\rho_2)-S(\rho'_2)]\}\times$$
$$\times G_0(0,\rho_2;-l,\mathrm{r})G_0^*(0,\rho'_2;-l,\mathrm{r}). \quad (33)$$

Assuming that $B(\mathrm{r})$ characterizes the position and extent of the object and is a binary function (equal to 1 on the object and 0 off it), we can write an expression for the power W_{obj} of radiation incident on the object:

$$W_{obj} = \iint \mathrm{d}^2RB(\mathrm{R})I_{inc}(\mathrm{R}) = \iint \mathrm{d}^2R\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2\Gamma_0(\rho_1-\rho_2)\times$$
$$\times G(0,\rho_1;L,\mathrm{R})G^*(0,\rho_2;L,\mathrm{R})B(\mathrm{R}). \quad (34)$$

If matching condition (29) is fulfilled, we obtain from comparing Eqs. (33) and (34)

$$I_R(-l,\mathrm{r}) = C\,W_{obj}^2, \quad (35)$$

i.e., the intensity at the receiver fluctuates as the square of the power on the object, and the maxima of these quantities are attained simultaneously. It is important that in the given case the condition (29) cannot be treated as the equation for the best direction of the beam axis. For some given direction of the beam axis, the observation point r is uniquely determined by this direction. In the given case, we can solve the problem by choosing the optimal time of emission of the pulse for fixed $\Gamma_0(\rho_1, \rho_2)$ if A, S, l, and r are chosen in accordance with $\Gamma_0(\rho_1, \rho_2)$ and the pulse is emitted at the moment when the intensity of the reflected signal at the point $(-l, \mathrm{r})$ reaches its maximum.

The search for the optimal direction of the beam axis can be realized by scanning the beam axis over the angle and simultaneous changing the points at which the reflected radiation is observed in accordance with Eq. (27). This process should be sufficiently fast so that the intensity distribution have not time to vary while the beam scanning.

The situation with choosing the beam focal length is analogous. For some problems it is of interest to maximize the energy at a given point of the receiving plane $(-l, \mathrm{r})$. In this case, the direction of the beam axis is chosen in accordance with Eq. (29). Then in the $x = -l$ plane the point $\mathrm{r_m}$ is searched for, at which I_{ref} takes its maximum. If after this we choose the direction of the beam axis in accordance with Eq. (29) for $\mathrm{r} = \mathrm{r_m}$, then, as follows from Eq. (33), $I_{ref}(-l, \mathrm{r})$ is maximal.

All of the above-enumerated problems are generalized to the case of a partially coherent beam with factorable coherence function. In this case, instead of receiving radiation at a point, it is necessary to fix the radiation, reflected from some object, at some area matched with the coherence spectrum of the initial radiation over the difference coordinate.

All these approaches to adaptive correction of distorted optical beams focused on remote objects in the atmosphere or behind it, are oriented to the adaptive optical systems, which use the intensity analyzers as the wave front sensors. A lot of approaches is available for searching for maxima for two-dimensional distributions of physical fields, among them optical distributions of laser beams propagating in a stochastically inhomogeneous medium. However, comparing them with phase sensors of wave front, for example, Shack–Hartmann sensor or shift interferometer, it must be noted that the intensity analyzer, naturally, requires a higher speed of operation and a very wide dynamical range. In particular, this is important for optical systems operating under conditions of "strong" fluctuations of intensity.

3. Computer modeling of adaptive systems operating with reflected signals

To justify the applicability[7,8,14,16] of the signal backscattered from atmospheric inhomogeneities in order to closing the feedback in adaptive optical systems, numerical calculations were performed using the following system of equations:

$$\begin{cases} 2ik\dfrac{\partial E}{\partial z} = \left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right)E + 2ik(n-1)E - 2ik\sqrt{\alpha_{ext}}E, \\ -2ik\dfrac{\partial E_{ref}}{\partial z} = \left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right)E_{ref} + \\ +2ik(n-1)E_{ref} - 2ik\sqrt{\alpha_{ext}}E_{ref} \end{cases} \quad (36)$$

under boundary conditions for the corrected field

$$E(x,y,z=0,t+\tau_a) = \sqrt{I(x,y)}\exp\left\{ik\frac{x^2+y^2}{2f} + i\varphi_c\right\}, \quad (37)$$

with the correcting phase

$$\varphi_c = A\{E_{ref}(x,y,z=0,t)\}, \quad (38)$$

and for the reference field

$$E_{\text{ref}}(x,y,z=f,t) = B(x,y,t)E(x,y,z=f,t). \quad (39)$$

Here τ_a is the time lag[4,7] in the controlling loop of the adaptive optical system as a whole; A is the operator of control for the adaptive system (algorithm); B is the reflectivity from a target or the Rayleigh coefficient of the backscattering from atmospheric inhomogeneities.

In the analysis of efficiency of some adaptive system, numerical models of the adaptive optical system are used,[4] namely:

models of wave front sensors:

a) an ideal phase sensor

$$\varphi_c = \arg\{E_{\text{ref}}(x,y,z=0,t)\};$$

b) an ideal sensor of phase difference

$$4\varphi_{i,j} - \varphi_{i+1,j} - \varphi_{i-1,j} - \varphi_{i,j-1} - \varphi_{i,j+1} =$$
$$= \Delta\varphi^x_{i-1,j} + \Delta\varphi^y_{i,j-1} - \Delta\varphi^x_{i,j} - \Delta\varphi^y_{i,j},$$

i, j = 1, 2, ..., N (N is the dimension of the calculation grid),

$$\Delta^x_{i,j} = \arg(E_{i+1,j}E^*_{i,j}), \quad \Delta^y_{i,j} = \arg(E_{i,j+1}E^*_{i,j});$$

c) the Shack–Hartmann sensor

$$g_k = \frac{1}{P_k}\iint_{A_k} I(\rho)\nabla\varphi d^2\rho =$$
$$= \frac{1}{P_k}\iint_{A_k}\{\text{Re}E\nabla(\text{Im}E) - \text{Im}\nabla(\text{Re}E)\}d^2\rho,$$

where P_k is the power passed through the aperture A_k; g_k is the measured phase gradient;

models of wave front correctors:

a) modal (Zernike) corrector

$$\varphi_k = \sum_{l=1}^{N_Z} a_l Z_l\left(2\frac{x}{D}, 2\frac{y}{D}\right),$$

D is the aperture diameter;

b) flexible mirror

$$\varphi_k = \sum_{l=1}^{N_Z}\Phi_k f\left(\frac{\rho - \rho_k}{d_1}\right), \quad f(\rho) = \exp(-\rho^2/w^2),$$

where d_1 is the space between neighboring actuators; $w = 0.575$; ρ_k is the position of the subaperture center; Φ_k is the estimate of phase in the center of kth aperture.

A computer program package has been made, which allowed us to obtain a sufficiently great amount of new interesting results justifying the capability of the adaptive system to successfully operate with signals backscattered from atmospheric inhomogeneities.

4. Effect of coherence on parameters of a laser guide star

One of key elements of an adaptive system's optical scheme is a reference source. In this section, we consider some aspects of using the laser reference sources connected with the coherence of their radiation.

Using the results from Refs. 14–16, the following formula can be written for the variance in the angular jitter of centroid of the laser beam emitted vertically upward from the ground:

$$\langle(\varphi_{\text{lb}})^2\rangle =$$
$$= 4\pi^2 x\int_0^1 d\xi(1-\xi)^2\int_0^\infty d\kappa\kappa^3\Phi_n(\kappa, x\xi)\exp(-\kappa^2a^2q^2/2), \quad (40)$$

where

$$q(\xi) = [\xi^2\Omega^{-2} + (1-\xi x/f)^2]^{1/2}, \quad \Omega = ka^2/x; \quad (41)$$

x is the distance; a is the initial size of the Gaussian laser beam; f is the curvature radius of the Gaussian beam phase front. In our calculation, we use in Eq. (2) the turbulence spectrum of the form[16]:

$$\Phi_n(\kappa, x\xi) = 0.033C_n^2(x\xi)\kappa^{-11/3}\{1 - \exp[-\kappa^2/\kappa_0^2]\}, \quad (42)$$

taking into account a deviation from the Kolmogorov spectrum in the range of large scales of inhomogeneities of the refractive index of the atmosphere; κ_0^{-1} is the turbulence outer scale.

As a result, we obtain for a focused beam from Eq. (40)

$$\langle(\varphi_{\text{lb}})^2\rangle = 4\pi^2 x 0.033\frac{\Gamma(1/6)}{2^{5/6}}a^{-1/3}\int_0^1 d\xi(1-\xi)^2 \times$$
$$\times\{(1-\xi)^{-1/3} - [(1-\xi)^2 + \frac{2}{a^2\kappa_0^2}]^{-1/6}\}C_n^2(x\xi) \quad (43)$$

and under the condition $\kappa_0^{-1} \gg a$, we have for the collimated beam[10–13]:

$$\langle(\varphi_{\text{lb}})^2\rangle =$$
$$= 4\pi^2 x 0.033\frac{\Gamma(1/6)}{2^{5/6}}a^{-1/3}\int_0^1 d\xi(1-\xi)^2 C_n^2(x\xi). \quad (44)$$

The next step will be the calculation of the variance of the jitter of a secondary source image, i.e., a scattering volume illuminated from the ground (or the image of some laser reference source) in the focal plane of the objective. Since the light scattering by atmospheric inhomogeneities (molecular scattering, aerosol scattering, and stimulated emission at free atoms) is the process of light scattering by independent scatterers, then the resulting wave field will be fully incoherent.[17]

The size of the illuminated zone within the scattering layer is calculated based on conclusions of the theory of light propagation in a turbulent medium. In a series of works,[14–16] the distribution of the mean intensity of the Gaussian beam having passed through a layer of turbulent medium has been calculated as:

$$< I(\mathrm{R},\xi) >= \frac{a^2}{a_{\text{eff}}^2(\xi)}\exp(-R^2 / a_{\text{eff}}^2),$$

where

$$a_{\text{eff}}^2(\xi) = a^2\{(1-\xi / f)^2 + \Omega^{-2} + \Omega^{-2}[1 / 2D_{\mathrm{S}}(2a)]^{6/5}\}$$

is the effective size of the beam in a scattering medium; $D_{\mathrm{S}}(2a)$ is the phase structural function.

Further we will use the deductions of the coherence theory.[15–17] The van Cittert–Zernike theorem deals with propagation of the mutual coherence function of a field

$$\gamma(x;\mathrm{r}_1,\mathrm{r}_2) = \frac{< U^*(\rho_1,x)U(\rho_2,x) >}{\sqrt{I(\rho_1,x)I(\rho_2,x)}}$$

and quantitatively describes the effect of diffraction of the incoherent light at its propagation from some laser guide star (LGS) to the Earth. The modulus of complex degree of the coherence for an initially incoherent source after passing trough homogeneous layer with depth x is given by following formula:

$$\gamma(x;\mathrm{r}_1,\mathrm{r}_2) = \frac{\left|\iint \mathrm{d}^2 s I(\mathrm{s})\exp(-ik\mathrm{s}(\mathrm{r}_1-\mathrm{r}_2) / x)\right|}{\int \mathrm{d}^2 s I(\mathrm{s})}. \quad (45)$$

Thus, the van Cittert–Zernike theorem gives that the modulus of the complex degree of the coherence for an initially incoherent source of a small angular size is equal to the modulus of the normalized Fourier transform for distribution of field intensity on a source. Thus, for a circular incoherent homogeneously lighted source of d size in the initial plane, the modulus of the complex degree of coherence at the distance x is

$$|\gamma(x;\mathrm{r}_1,\mathrm{r}_2)| = \frac{2J_1(k\alpha r / 2)}{(k\alpha r / 2)}, \quad (46)$$

ãäå $\alpha = d/x$ is the angular size of the source as seen from the distance x; $r = |\mathrm{r}_1 - \mathrm{r}_2|$. As a result, the radiation spatial coherence radius $\rho_{\mathrm{c}} \approx 1.22\lambda x / d$. Naturally, these estimates are obtained for the case of radiation propagation in a homogeneous medium.

For the complex degree of coherence

$$\gamma(\mathrm{R},\rho) = \frac{\Gamma_2(\mathrm{R},\rho)}{[\Gamma_2(\mathrm{R}+\rho/2,\,0)\,\Gamma_2(\mathrm{R}-\rho/2,\,0)]^{1/2}}, \quad (47)$$

where

$$\Gamma_2(\mathrm{R},\rho) =< U(\mathrm{R},\rho)U^*(\mathrm{R},\rho) >;$$

$$\mathrm{R} = (\rho_1+\rho_2)/2; \quad \rho = (\rho_1-\rho_2),$$

the following equation was derived[14]:

$$2ik\frac{\partial\Gamma_2(\mathrm{R},\rho)}{\partial x} + 2\nabla_R\nabla_\rho\Gamma_2(\mathrm{R},\rho) + \frac{i\pi k^3}{2}H(x,\rho)\Gamma_2(\mathrm{R},\rho) = 0 \quad (48)$$

with the boundary conditions for a deterministic initial field

$$\Gamma_2(\mathrm{R},\rho) = U_0(\mathrm{R}+\rho/2)U_0^*(\mathrm{R}-\rho/2).$$

The function $H(x,\rho)$ in Eq. (48) characterizes statistical properties of fluctuations of the dielectric permittivity

$$H(x,\rho) = 8\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Phi_n(x,\kappa)[1-\cos\kappa\rho]\mathrm{d}^2\kappa. \quad (49)$$

If the initial field U_0 fluctuates, then the function $\Gamma_2^0(\mathrm{R},\rho) = \langle\langle U_0(\mathrm{R}+\rho/2)U_0^*(\mathrm{R}-\rho/2)\rangle\rangle$ must be used as the boundary condition, where the double angle brackets point to averaging over the ensemble of realizations of the source.

As an example, present a boundary condition for a partly coherent light beam, the field of which is

$$U_0(\rho) = A(\rho)\exp(i\varphi(\rho)),$$

where $\varphi(\rho)$ is a random phase with a zero mean and, for example, the Gaussian distribution.

Let the initial beam be Gaussian:

$$A(\rho) = U_0\exp\{-\rho^2/2a^2 - ik\rho^2/2f\}.$$

In this case

$$\Gamma_2^0(\mathrm{R},\rho) = |U_0|^2\times \exp\{-R^2/a^2 - \rho^2/4a^2 - ik\rho\mathrm{R}/f - E(\rho)/2\}, \quad (50)$$

where

$$E(\rho_1-\rho_2) = \langle\langle[\varphi(\rho_1)-\varphi(\rho_2)]^2\rangle\rangle.$$

For simplicity

$$E(\rho) = \rho^2/2a_k^2.$$

Then

$$\Gamma_2^0(\mathrm{R},\rho) = |U_0|^2\times \exp\{-R^2/a^2 - \rho^2/4a^2 - ik\rho\mathrm{R}/f - \rho^2/2a_k^2\}. \quad (51)$$

Here a_k is the radius of the initial spatial coherence of the radiation source. In a random medium[14–16]:

$$\Gamma_2(x,\mathrm{R},\rho) = \frac{k^2}{4\pi^2x^2}\iint \mathrm{d}^2R'\iint \mathrm{d}^2\rho'\Gamma_2^0(\mathrm{R}-\mathrm{R}',\ \rho-\rho')\times \exp\{ik\mathrm{R}'\rho/x - \frac{\pi k^2}{4}\int_0^x H[x',\ \rho-\rho'(1-x'/x)]\mathrm{d}x'\}. \quad (52)$$

Consider the limiting case in Eq. (52), i.e., transition to a fully incoherent initial (thermal) source: then $a_k \to 0$ and

$$\Gamma_2^0(\mathrm{R},\rho) = 4\pi|U_0|^2 a_k^2 \exp\{-R^2/a^2\}\delta(\rho).$$

Note that this formula is a special case of general relationship for the coherence function of a thermal source. In the general case, if $a_k \to 0$, then $\Gamma_2^0(\mathrm{R},\rho)$ can be approximated by

$$\Gamma_2^0(\mathrm{R},\rho) = b^2 I(\mathrm{R})\delta(\rho). \quad (53)$$

It turns out in this case that $b = \lambda/\sqrt{2\pi}$, i.e., the coherence radius of the thermal source is comparable with the wavelength.

At the distance x in the turbulent medium

$$\Gamma_2(x,\mathrm{R},\rho) = \frac{U_0^2 k^2 a^2 a_k^2}{x^2}\exp\{ik\mathrm{R}\rho/x - -\frac{k^2a^2\rho^2}{4x^2} - \frac{\pi k^2}{4}\int_0^x H(x',\rho x'/x)\mathrm{d}x'\}. \quad (54)$$

For modulus of the complex degree of coherence

$$|\gamma(x,\mathrm{R},\rho)| = \exp\left\{-\frac{k^2a^2\rho^2}{4x^2} - \frac{\pi k^2}{4}\int_0^x H(x',\rho x'/x)\mathrm{d}x'\right\}. \quad (55)$$

It is seen[16,17] that there are two opposite tendencies in variation of the spatial coherence radius of the initially incoherent radiation. On the one hand, it grows proportionally to $d_0 = 2x/ka$ (due to decrease of the visible angular size $\gamma_s = a/x$ of the source), and on the other hand, it decreases because of the loss of coherence in the turbulent medium.

We can estimate the modulus of the complex degree of coherence through the phase structural function and obtain

$$D_S(x,\rho) = \frac{k^2a^2\rho^2}{2x^2} + \frac{\pi k^2}{2}\int_0^x H(x',\rho x'/x)\mathrm{d}x'. \quad (56)$$

Calculations by formula (56) with the Karman turbulence spectrum result (under the condition $\kappa_0^{-1} >> \rho$) in

$$D_S(x,\rho) = = \frac{k^2a^2\rho^2}{2x^2} + 2.91k^2\rho^{5/3}\int_0^x \mathrm{d}x' C_n^2(x')(x'/x)^{5/3}. \quad (57)$$

For a wide ($\Omega = ka^2/x \gg 1$) collimated beam the variance of the centroid jitter can be written as

$$\langle(\varphi_{lb})^2\rangle = <(\rho_{lb})^2>/x^2 = = 4\pi^2 0.033\frac{\Gamma(1/6)}{2^{5/6}}a^{-1/3}\int_0^x \mathrm{d}x'(1-x'/x)^2 C_n^2(x'), \quad (58)$$

$$4\pi^2 0.033\frac{\Gamma(1/6)}{2^{5/6}} = 4.04.$$

For a collimated wide beam, the variance of the image angular jitter is

$$\langle\varphi^2\rangle = \frac{a^2}{x^2} + 4.85D^{-1/3}\int_0^x \mathrm{d}x' C_n^2(x')(x'/x)^{5/3} + +4.04a^{-1/3}\int_0^x \mathrm{d}x' C_n^2(x')(1-x'/x)^2, \quad (59)$$

in this case, the secondary source size is a. For the focused beam the secondary source size is a/Ω; and as a result[18–21]:

$$\langle\varphi^2\rangle = \frac{a^2}{\Omega^2 x^2} + 4.85D^{-1/3}\int_0^x \mathrm{d}x' C_n^2(x')(x'/x)^{5/3} + +4.04a^{-1/3}\int_0^x \mathrm{d}x' C_n^2(x')(1-x'/x)^{5/3}, \quad (60)$$

where D is the diameter of the receiving objective; a is the size of the laser Gaussian beam forming the laser guide star.

Once again return to the formula for the structural function. If to assume a square approximation in the second term, then the coherence radius of the initial incoherent source in the turbulent medium is[16]:

$$\rho_{\mathrm{coh}} = \frac{d_0(x)}{(1 + d_0^2(x)/\rho_t^2)}, \quad (61)$$

where ρ_t is the coherence radius for a spherical wave in the turbulent medium.[15.16]

Application of laser guide stars to correction of images of the off-atmospheric objects faces a series of serious problems.[5–8,10–13,20] One of them is connected with impossibility to provide for efficient correction of the total wavefront tilt (TWFT). The correction of TWFT fluctuations for a natural star with the help of only LGS signal is known to be inefficient.[8,10–13] The tradition monostatic scheme, which uses only the aperture of the telescope itself (main), is inefficient even with the use of procedure of LGS signal optimization.[11]

In this connection, some investigators declare the need to use simultaneously both LGS and natural stars for the TWFT correction. Since the angle of spatial correlation for TWFT fluctuations significantly exceeds the isoplanatism angle for higher aberrations of phase fluctuations of the optical wave having passed through the turbulent atmospheric layer, a sufficiently remote star can be used for TWFT correction.

One more disadvantage of LGS application to image correction in the ground-based telescopes is the cone effect or focal nonisoplanatism. The authors of Ref. 22 propose to use more than one star to eliminate this effect. They have shown that high coherence of the laser radiation in the guide star can be achieved only if the visible star image is small enough. Therefore, almost all LGSs were formed based on the focused laser beams.

The coherence of the received radiation is always determined by two factors: the size of the LGS visible area from the measured telescope focus and the coherence of spherical wave, because the LGS initial emission is practically incoherent.

Recently, the use of wide collimated beams for formation of LGS was reported.[23] It was assumed that the resulting guide star has a plane wave front providing for elimination of the focal nonisoplanatism. However, it was ignored that the secondary source – guide star – has a significantly low spatial coherence because of incoherence of the process of light scattering by atmospheric inhomogeneities (molecular and aerosol scattering, re-emission of radiation at free atoms in the upper atmosphere), and the coherence radius of the scattered radiation ρ_{coh} turns out to be

$$\frac{1}{\rho_{coh}^2} = \frac{k^2 a^2}{4x^2} + \frac{1}{\rho_t^2}.$$

The coherence of the secondary radiation is always lower in a wide collimated beam than in a focused beam, and the above relation can be rewritten as

$$\frac{1}{\rho_{coh}^2} = \frac{k^2 \varphi^2}{4} + \frac{1}{\rho_t^2},$$

ãäå $\varphi = \grave{a}/\tilde{o}$ is the visible area of the secondary source or its part. If the secondary source is "resolved" by the receiving aperture, i.e., it is possible to observe some its parts or fragments separately, then the angle φ must be replaced by the atmospheric angular resolution of the telescope.

If it is taken into account that the angular resolution of the atmosphere–telescope system (without adaptive correction) is expressed through the ratio λ / r_0, where r_0 is the coherence radius of radiation for a plane wave having passed through the entire atmosphere, then within the telescope field of view it is possible to separately observe fractions of λ / r_0 angular size of the laser-illuminated surface of the incoherently luminous LGS. As a result, the first term characterizing the LGS radiation coherence (calculated for vacuum) will be equal to

$$\rho_{coh} = \lambda / \theta, \qquad (62)$$

where θ is the angular resolution of the telescope in the atmosphere, i.e., $\theta = \lambda / r_0$, and $\rho_{coh} = r_0 / \pi$.

In the case of a wide focused beam (when the LGS spot cannot be resolved by the telescope) we can obtain the radiation from the secondary source with the coherence radius of the size of the aperture focusing the laser radiation. Naturally, the estimate is for a homogeneous atmosphere. For conditions of the turbulent atmosphere, the coherence radius of the secondary source can be calculated by Eq. (61).

However, in some cases, it should be believed that we deal with incoherent guide stars. Incoherent LGS also can be used efficiently, for example, for real-time measurements of the atmospheric optical transfer function along the path. This function can be used in the inverse convolution algorithm for post-detector image correction.

Certainly, this approach allows obtaining more efficient correction as compared to a "blind" inverse convolution, which assumes calculation of the atmospheric transfer function based on some atmospheric model. Besides, the guide star can be formed almost in any required direction, for example, when forming the image of some extraterrestrial object in the telescope. One of restrictions on efficient application of such a star is the problem of focal nonisoplanatism because of the LGS location at some finite distance in the atmosphere, while the object is always far beyond the atmosphere. Therefore, the object and the LGS are always seen in different planes, because they have wave fronts with different curvatures. This, in its turn, causes different fluctuations for waves coming to the objective from the object and from the reference source.

As is known, any adaptive system has a finite frequency band, which causes a lag between the received and control signals. Therefore, there exist some limitations on the quality of correction of a moving object. At the same time, when forming LGS in some given direction, it is possible to partly compensate the time lag arising in any adaptive system and connected both with evolution of random inhomogeneities in the channel and the fast change of the position of the object under study. In this case, the LGS is formed in the position, which "predicts" the future position of the object.

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References

1. V.P. Linnik, Opt. Spektrosk. 25, No. 4, 401–402 (1957).
2. J.W. Hardy, Proc. IEEE, 66, No. 6, 651–697 (1978).
3. V.P. Lukin and B.V. Fortes, *Adaptive Formation o Beams and Images in the Atmosphere* (Nauka, Novosibirsk, 1999), 212 pp.
4. V.P. Lukin, *Atmospheric Adaptive Optics* (Nauka, Novosibirsk, 1986), 286 pp.
5. V.P. Lukin and V.F. Matyukhin, Soviet J. Quant. Electron., 13, No. 12, 2465–2473 (1983).
6. V.P. Lukin, in: *Proc. ISO-16, "Active and Adaptive Optics,"* 521–524 (1993).
7. V.P. Lukin and M.I. Charnotskii, Soviet J. Quant. Electron. 9, 952–958 (1982).
8. R. Fugate, Opt. Photon. News 4, No. 6, 14–19 (1993).
9. V.M. Orlov, I.V. Samokhvalov, G.G. Matvienko, M.L. Belov, and A.N. Kozhemyakov, *Elements of Light Scattering Theory and Optical Detection and Ranging* (Nauka, Novosibirsk, 1982), 225 pp.
10. V.P. Lukin, Appl. Opt. 37, No. 21, 4634–4644 (1998).
11. V.P. Lukin, Adaptive Optics-96, Technical Digest Series 13, 150–152 (1996).
12. V.P. Lukin and B.V. Fortes, Atmos. Oceanic Opt. 10, No. 1, 34–41 (1997).
13. V.P. Lukin, Atmos. Oceanic Opt. 11, No. 11, 1076–1080 (1998).
14. V.I. Klyatskin, *Statistical Description of Dynamic Systems with Fluctuating Parameters* (Nauka, Moscow, 1975), 239 pp.

15. V.I. Tatarskii, *Wave Propagation in a Turbulent Medium* (McGraw Hill, New York, 1968), 357 pp.
16. A.S. Gurvich, A.I. Kon, V.L. Mironov, and S.S. Khmelevtsov, *Laser Radiation in the Turbulent Atmosphere* (Nauka, Moscow, 1976), 277 pp.
17. S. Enguehard and B. Hatfield, *Incoherence and Multiple Laser Guide Stars. NOAO Workshop on the Reduction of Gemini AO Data* (Tuscon, USA, 2001).
18. V.P. Lukin, in: *Abstracts of Papers at V All-Union Symposium on Laser Radiation Propagation in the Atmosphere*, Tomsk (1979), Part II, pp. 33–36.
19. V.P. Lukin, Soviet J. Quant. Electron. 7, 1270–1279 (1980).
20. V.P. Lukin and B.V. Fortes, *Computer Modeling of Adaptive Optics and Sites for Telescope Design*, OSA Techn. Digest 23, 192–194 (1995).
21. V.L. Mironov, V.V. Nosov, and B.N. Chen, Izv. Vyssh. Uchebn. Zaved., Radiofizika 25, No. 12, 1467–1471 (1982).
22. E. Viard, M. Le Louran, and N. Hubin, Appl. Opt. 41, No. 1, 11–20 (2002).
23. D. Busher, G. Love, and R. Myers, Opt. Lett. 27, No. 3, 149–151 (2002).