Application of the phase screen method to analysis of stellar scintillations caused by refractive index fluctuations in the stratospheric layer

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Stellar scintillations during the occultation by the Earth's atmosphere are analyzed. Factors determining the accuracy of the phase screen method as applied to description of a wave propagated through a distorting layer in the case of spatial dependence of the regular component of refractive index are determined. The corresponding errors are estimated and a possible way of compensating for them is proposed.

Introduction

In observing stellar occultation by the Earth's atmosphere from onboard a space station, random variations of the intensity of radiation received occur along with the monotonic decrease in the stellar brightness.[1–3] These are associated with the scattering of stellar radiation by atmospheric inhomogeneities and carry significant information about the structure of air density fluctuations in the middle atmosphere.

The approximation of a phase screen (PS) is usually used to study scintillation spectra.[4,5] This approximation is believed to provide for good accuracy of description of the electromagnetic field when the receiver is far from the layer studied. However, this approximation neglects that the influence of refraction on the characteristics of scattered field depends on the position of inhomogeneities in the layer. These characteristics include, in particular, the scattering angle and the ratio of the spatial scale of inhomogeneities and the size of the wave tube.

The aim of this paper is to estimate the errors caused by the neglect of this dependence. In this paper, the equations are derived, in the first approximation of the method of smooth perturbations (MSP), for the spectrum of correlation function of the wave intensity after propagation through a turbulent layer with regular refraction. The errors in the PS approximation due to refraction in the Earth's atmosphere at altitudes of 25–75 km are estimated.

Scintillation spectra in the first approximation of MSP

Consider the propagation of electromagnetic radiation from a remote source through a turbulent layer with regular refraction. Assume that the source is spaced far enough from the layer $r_S >> \Delta^2/\lambda$, where r_S is the separation between the source and the layer; Δ is the transverse dimension of the layer; λ is the wavelength, so that the wave with the harmonic time dependence $(-i\omega t)$ and the wave number $k_0 = 2\pi/\lambda$, incident on the outer boundary of the layer under study, can be assumed plane. The refractive index in the layer $n_t(\mathrm{r}) = n(\mathrm{r}) + \delta n(\mathrm{r})$ is a superposition of the regular $n(\mathrm{r})$ and random $\delta n(\mathrm{r})$ components, depending on the radius vector r of a current point. Assume that the spatial scale of variation of the regular component H is large enough for the inequality $L_0 << H^2/\lambda$ to hold. Here L_0 is the separation between the receiver and the distorting layer. In the absence of refractive index fluctuations $\delta n(\mathrm{r}) \equiv 0$, this allows the field at the observation point R_0 to be described in the approximation of geometric optics (GO). The random field of refractive index inhomogeneities will be characterized by the correlation function

$$B_n(\mathrm{R}, \mathrm{r}_1 - \mathrm{r}_2) = \langle \delta n(\mathrm{r}_1)\delta n(\mathrm{r}_2)\rangle \qquad (1)$$

assuming its dependence on the variable $\mathrm{R} = (\mathrm{r}_1 + \mathrm{r}_2)/2$ to be much weaker than that on the coordinate difference $\mathrm{r}_1 - \mathrm{r}_2$. At $|\mathrm{r}_1 - \mathrm{r}_2| > H$ it becomes negligibly weak. In addition, assume that the variance of field fluctuations at the observation point is small.

The non-local character of the distorting layer will be taken into account within the framework of the first approximation of the method of smooth perturbations. With the backscattering neglected, the components of the electric field at the point r can be represented in the form[6]:

$$U(\mathrm{r}) = U_0(\mathrm{r})e^{\psi_{10}(\mathrm{r})};$$

$$\psi_{10}(\mathrm{r}) = \iiint\limits_{V'} \frac{U_0(\mathrm{r}')}{U_0(\mathrm{r})} G_0(\mathrm{r}, \mathrm{r}')\delta n(\mathrm{r}')\mathrm{d}\mathrm{r}', \qquad (2)$$

where $U_0(\mathrm{r})$ and $G_0(\mathrm{r}, \mathrm{r}')$ are the electric field and the Green's function of a point source in an unbounded medium in the absence of refractive index fluctuations. In the GO approximation, they can be presented as follows:

$$\begin{Bmatrix} U_0(\mathrm{r}) \\ G_0(\mathrm{r},\mathrm{r}') \end{Bmatrix} = \begin{Bmatrix} A_0(\mathrm{r})e^{i\Psi(\mathrm{r})} \\ A_G(\mathrm{r},\mathrm{r}')e^{i\Psi_G(\mathrm{r},\mathrm{r}')} \end{Bmatrix}. \quad (3)$$

Here $\begin{Bmatrix} A_0(\mathrm{r}) \\ A_G(\mathrm{r},\mathrm{r}') \end{Bmatrix}$ are slowly varying amplitude factors; $\begin{Bmatrix} \Psi(\mathrm{r}) \\ \Psi_G(\mathrm{r},\mathrm{r}') \end{Bmatrix}$ are eikonals. The reasoning, similar to that given in Ref. 6 for the case of a macroscopically isotropic and homogeneous medium, allows us to restrict our consideration to zero-order terms for the amplitudes and second-order terms for the phase changes in the Taylor series of the integrand in Eq. (2) in terms of the coordinates, transverse to the beam. The substitution of this series into Eq. (2) yields

$$\psi_{10}(\mathrm{R}_0) = \int_\Sigma \mathrm{d}l A_G(\mathrm{R}_0,\mathrm{r}_0)\frac{A_0(\mathrm{r}_0)}{A_0(\mathrm{R}_0)}\times$$

$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta n(\mathrm{r}')e^{\left\{i(\mathrm{r}'-\mathrm{r}_0)(\mathrm{r}'-\mathrm{r}_0):\overline{\overline{F}}(\mathrm{R}_0,\mathrm{r}_0)\right\}}\mathrm{d}x\mathrm{d}y. \quad (4)$$

In this equation, the first integral is calculated over the trajectory Σ of the ray, passed through the point R_0; r_0, x, y are the projection of the current point r' onto the trajectory and its transverse displacement. The dyadic $\overline{\overline{F}}(\mathrm{R}_0,\mathrm{r}_0)$ characterizes the additional phase shift of the ray, which arrived at the observation point r after scattering by an inhomogeneity at the point r', with respect to the ray, which arrived at the point r along the trajectory Σ. Hereinafter the signs $\cdot$, $\times$, $:$ denote the scalar, vector, and double scalar products; the omitted sign of operation implies dyad product.

In calculating the correlation function

$$B_I(\mathrm{R}_0,\delta\mathrm{r}) = \left\langle \frac{\delta I(\mathrm{R}_0+\delta\mathrm{r}/2)}{\langle I(\mathrm{R}_0+\delta\mathrm{r}/2)\rangle}\frac{\delta I(\mathrm{R}_0-\delta\mathrm{r}/2)}{\langle I(\mathrm{R}_0-\delta\mathrm{r}/2)\rangle}\right\rangle \quad (5)$$

of the intensity fluctuations $\delta I(\mathrm{r}) \approx 2\mathrm{Re}\psi_{10}(\mathrm{r})$, we neglect variations of the amplitudes $A_{0,G}(\mathrm{r},\mathrm{r}_0)$ and the components $\overline{\overline{F}}(\mathrm{r},\mathrm{r}_0)$ at the size of inhomogeneities. It is convenient to pass on from the correlation functions to their Fourier transforms in terms of the fast variables, that is, to the three-dimensional spectrum

$$\tilde{B}_n(\mathrm{r}_0,\kappa) = \frac{1}{8\pi^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B_n(\mathrm{r}_0,\delta\mathrm{r}_0)e^{-i\kappa\delta\mathrm{r}_0}\mathrm{d}\delta\mathrm{r}_0$$

and the two-dimensional spectrum

$$\tilde{B}_I(\mathrm{R}_0,\kappa) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B_I(\mathrm{R}_0,\delta\mathrm{r})e^{-i\kappa\cdot\delta\mathrm{r}}\mathrm{d}\mathrm{p}$$

(weak dependence on the component $\delta\mathrm{r}$, parallel to the ray at the detection point, is neglected). Calculating the integrals over the directions, transverse to the ray, in the straight and reciprocal spaces, we obtain the following relation between the spectra:

$$\tilde{B}_I(\mathrm{R}_0,\kappa) = 16\pi^3 k_0^2 A_0^2(\mathrm{R}_0)\times$$

$$\times\int_\Sigma \mathrm{d}l \frac{A_0^2(\mathrm{r}_0)A_G^2(\mathrm{R}_0,\mathrm{r}_0)}{\det\overline{\overline{F}}(\mathrm{R}_0,\mathrm{r}_0)\det\overline{\overline{\mu}}(\mathrm{R}_0,\mathrm{r}_0)}\tilde{B}_n\left(\mathrm{r}_0,\kappa\cdot\overline{\overline{\mu}}^{-1}(\mathrm{R}_0,\mathrm{r}_0)\right)\times$$

$$\times\sin^2\left\{\left|\kappa\cdot\overline{\overline{\mu}}^{-1}(\mathrm{R}_0,\mathrm{r}_0)\cdot\overline{\overline{F}}^{-1/2}(\mathrm{R}_0,\mathrm{r}_0)\right|^2/4k_0\right\}, \quad (6)$$

where $\overline{\overline{\mu}}(\mathrm{R}_0,\mathrm{r}_0)$ is the matrix of affine transformation of the wave tube cross section at the transition from the point r_0 to the point R_0.

Errors of the PS approximation

The PS approximation neglects the dependence on the coordinates of the current point r_0 in the amplitude functions $A_0(\mathrm{r}_0)$, $A_G(\mathrm{R}_0,\mathrm{r}_0)$ and dyadics $\overline{\overline{F}}(\mathrm{R}_0,\mathrm{r}_0)$, $\overline{\overline{\mu}}(\mathrm{R}_0,\mathrm{r}_0)$ and these coordinates are substituted by those at the point r_{0PS} of intersection of the ray L and the phase screen. The errors, arising in this case, can be divided into three classes.

First, the errors caused by the change of the amplitude factor

$$A_{\mathrm{eff}}(\mathrm{R}_0,\mathrm{r}_0) = \frac{A_0^2(\mathrm{r}_0)A_G^2(\mathrm{R}_0,\mathrm{r}_0)}{\det\overline{\overline{F}}(\mathrm{R}_0,\mathrm{r}_0)\det\overline{\overline{\mu}}(\mathrm{R}_0,\mathrm{r}_0)}$$

in the integrand. It should be noted that the factor $\det\overline{\overline{F}}(\mathrm{R}_0,\mathrm{r}_0)$ compensates for singularity of the Green's function, so that the amplitude factor varies quite slowly, as the receiver approaches the layer, and has no singular point. The relative error does not exceed

$$\varepsilon_1 = \frac{A_{\mathrm{eff}}(\mathrm{R}_0,\mathrm{r}_{0f}) - A_{\mathrm{eff}}(\mathrm{R}_0,\mathrm{r}_{0c})}{A_{\mathrm{eff}}(\mathrm{R}_0,\mathrm{r}_{0PS})}, \quad (7)$$

where r_{0f}, r_{0c} are the ray Σ points, farthest from and closest to the receiver. This error is independent of the spatial frequency κ, and the condition $\varepsilon_1 \ll 1$ corresponds to the approximation of an optically thin layer.

Second, the error can be attributed to variation of the wave tube cross section in the distorting layer and, consequently, to variation of the ratio between the tube size and the spatial scale of inhomogeneities. This leads to spectrum blurring and to the error

$$\varepsilon_2 = \frac{\frac{\partial}{\partial \mathrm{q}}\tilde{B}_n(\mathrm{r}_{0PS},\mathrm{q})\Big|_{\mathrm{q}=\kappa\cdot\overline{\overline{\mu}}^{-1}(\mathrm{R}_0,\mathrm{r}_{0PS})}}{\tilde{B}_n\left(\mathrm{r}_{0PS},\kappa\cdot\overline{\overline{\mu}}^{-1}(\mathrm{R}_0,\mathrm{r}_{0PS})\right)}\kappa :$$

$$:\left[\overline{\overline{\mu}}^{-1}(\mathrm{R}_0,\mathrm{r}_{0f})-\overline{\overline{\mu}}^{-1}(\mathrm{R}_0,\mathrm{r}_{0c})\right], \qquad (8)$$

which increases in the direct proportion to $|\kappa|$. When estimating this error, the main factor is how strongly the spectrum of refractive index inhomogeneities depends on the spatial frequency.

Third, the refraction in the layer leads to an increase in the angle of scattering by inhomogeneities, lying in the layer part remote from the receiver, and, consequently, in the Fresnel scale. This restricts the range of the spatial frequencies, within which the PS approximation is applicable, as follows:

$$\left|\kappa\cdot\overline{\overline{\mu}}^{-1}(\mathrm{R}_0,\mathrm{r}_{0f})\cdot\overline{\overline{F}}^{-1/2}(\mathrm{R}_0,\mathrm{r}_{0f})\right|^2-$$

$$-\left|\kappa\cdot\overline{\overline{\mu}}^{-1}(\mathrm{R}_0,\mathrm{r}_{0c})\cdot\overline{\overline{F}}^{-1/2}(\mathrm{R}_0,\mathrm{r}_{0c})\right|^2<\pi k_0. \qquad (9)$$

At higher spatial frequencies, the maxima and minima of the diffraction patterns from inhomogeneities of the same size, but lying on the ray Σ in different parts of the layer, superpose. The singularity of the diagonal components of the dyadic $\overline{\overline{F}}(\mathrm{R}_0,\mathrm{r}_0)$ at $\mathrm{R}_0=\mathrm{r}_0$ leads to quick deterioration of the accuracy of PS approximation, as the receiver approaches the layer, similarly to the case of scintillations in the absence of refraction.

Thus, fulfillment of the condition of optically thin layer is insufficient for the use of the PS approximation. At least two more conditions, restricting the studied range of the spectrum and the structure of refractive index inhomogeneities, should be fulfilled as well.

PS approximation at occultation observations of scintillations

Let us assess the validity of using the PS approximation to the investigation of stellar scintillation spectra when the stars are occulted by the Earth's atmosphere. The geometry of occultation observation is shown in Fig. 1.

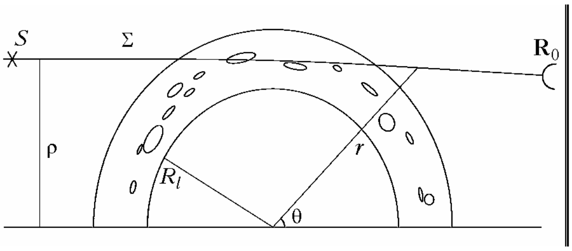

Fig. 1. Geometry of the problem.

Propagation of the electromagnetic field will be described in the spherical system of coordinates (r, θ, φ) with the origin at the center of the Earth, and the ray $\theta = \pi$ corresponds to the direction toward the source. The regular part of the refractive index $n(r) = 1 + N(r)$, which is assumed independent of the angular coordinates, leads to the deflection of the ray, while the random component $\delta n(\mathrm{r})$ leads to the development of fluctuations in the electromagnetic field of the wave. The dependence of regular component on the wavelength λ_0 and the mean temperature $\langle T(\mathrm{r})\rangle$ (K) and pressure $\langle P(\mathrm{r})\rangle$ (mbar) is described by the well known equation[7]:

$$N(r)=7.76\cdot10^{-6}\frac{\langle P(r)\rangle}{\langle T(r)\rangle}\left(1+\frac{\Lambda^2}{\lambda_0^2}\right), \qquad (10)$$

where the parameter Λ = 87 μm characterizes the atmospheric dispersion in the wavelength range $\lambda_0\in 0.3$–20 μm. The ratio of the mean pressure and temperature decreases with height by the close-to-exponential law with the spatial scale H, lying in the range $6\cdot10^3$–$8\cdot10^3$ m, which allows the radial dependence of the regular component to be approximated as:

$$N(r) = N_0\exp[(r-R_l)/H], \qquad (11)$$

where $R_l\approx 6.4\cdot10^6$ m is the height of the bottom boundary of the studied atmospheric layer; $N_0\approx 2\cdot10^{-5}$ is the refractive index at this boundary. It can be seen that for waves from the optical region the condition $L_0 \ll H^2/\lambda$ is fulfilled, if the distance between the layer and the receiver L_0 does not exceed several thousand kilometers. This allows us to use the geometric-optics description for the regular component of the field. The ray Σ lies in the plane φ = const and is determined as

$$\theta(r,\rho) = \theta_p(\rho) \pm \Delta\theta(r,\rho),$$

$$\Delta\theta(r,\rho)=\int\limits_{h_p(\rho)}^{r}\frac{\mathrm{d}r}{r\sqrt{n^2(r)r^2/\rho^2-1}}, \qquad (12)$$

where ρ is the impact parameter (the distance between the ray and the axis $\theta = \pi$ before entering the atmosphere), the height of the perigee point $h_p(\rho)$ is the solution of the equation $n(h_p)h_p = \rho$, and its angular coordinate is specified as:

$$\theta_p(\rho)=\pi-\int\limits_{h_p(\rho)}^{\infty}\frac{\mathrm{d}r}{r\sqrt{n^2(r)r^2/\rho^2-1}}. \qquad (13)$$

Let the receiver be at the point $\mathrm{R}_0 = (R_0, \Theta, 0)$. Then the impact parameters of the ray, arriving at the detection point, can be found from the equation $\Theta = \theta(R_0, \rho)$. Assume that the aperture is small enough as compared to both the correlation length of radiation intensity fluctuations and the spatial scale of variations in the amplitude of the regular component and neglect its integrating effect.

To determine the functions, entering into Eq. (6), let us use the results of Ref. 8, valid for a spherically symmetric distorting layer:

$$A_{\text{eff}}(\mathrm{R}_0,\mathrm{r}) =$$

$$= \left|\frac{8\pi k_0^2 \theta_r(r,\rho)\theta_\rho(R_0,\rho)\sin^2\Theta}{n(r)n(R_0)\theta_\rho(r,\rho)\theta_r(R_0,\rho)\sin^2\theta(r,\rho)\cos\left[\theta(r,\rho)-\Theta\right]}\right|, \tag{14}$$

$$\mu_\perp(\mathrm{R}_0,\mathrm{r}) = \frac{n(R_0)R_0}{n(r)r}\frac{\theta_r(R_0,\rho)\theta_\rho(r,\rho)}{\theta_r(r,\rho)\theta_\rho(R_0,\rho)},$$

$$\mu_\parallel(\mathrm{R},\mathrm{r}) = \frac{r\sin\theta(r,\rho)}{R_0\sin\Theta}, \tag{15}$$

$$\gamma_\perp(\mathrm{R}_0,\mathrm{r}) = 2k_0\frac{n^2(R_0)R_0{}^2\theta_r^2(R_0,\rho)\theta_\rho(r,\rho)}{\rho^2\theta_\rho(R_0,\rho)\left[\theta_\rho(r,\rho)-\theta_\rho(R_0,\rho)\right]},$$

$$\gamma_\parallel(\mathrm{R}_0,\mathrm{r}) = \frac{2k_0\rho\sin^2\theta(r,\rho)}{R_0{}^2\sin^2\Theta\tan(\theta(r,\rho)-\Theta)}, \tag{16}$$

and

$$\overline{\overline{\mu}}(\mathrm{R}_0,\mathrm{r}) = \mu_\perp(\mathrm{R}_0,\mathrm{r})\frac{\mathrm{e}_\varphi\times\mathrm{k}(\mathrm{R}_0)}{|\mathrm{k}(\mathrm{R}_0)|}\frac{\mathrm{e}_\varphi\times\mathrm{k}(\mathrm{R}_0)}{|\mathrm{k}(\mathrm{R}_0)|} +$$

$$+\mu_\parallel(\mathrm{R}_0,\mathrm{r})\mathrm{e}_\varphi\mathrm{e}_\varphi,$$

while

$$\overline{\overline{\mu}}^{-1}(\mathrm{R}_0,\mathrm{r})\cdot\overline{\overline{F}}^{-1/2}(\mathrm{R}_0,\mathrm{r})\,/\,4k_0 =$$

$$= \gamma_\perp{}^{-1/2}(\mathrm{R}_0,\mathrm{r})\frac{\mathrm{e}_\varphi\times\mathrm{k}(\mathrm{R}_0)}{|\mathrm{k}(\mathrm{R}_0)|}\frac{\mathrm{e}_\varphi\times\mathrm{k}(\mathrm{R}_0)}{|\mathrm{k}(\mathrm{R}_0)|} + \gamma_\parallel{}^{-1/2}(\mathrm{R}_0,\mathrm{r})\mathrm{e}_\varphi\mathrm{e}_\varphi.$$

The subscripts of the function $\theta(r,\rho)$ denote differentiation with respect to the corresponding variable.

The use of cumbersome equations (14)–(16) is reasonable only in the case, when the simpler equations, obtained in Refs. 4 and 5 within the framework of the phase screen method, give a significant error. Therefore, it is important to determine the applicability domain of the assumptions, which lie in the foundation of this method, namely:

1) The trajectory Σ can be thought a straight-line in the height range $r \in [h_\mathrm{p}, h_\mathrm{p} + 2H]$, which is the major contributor to the formation of intensity fluctuations.

2) The correlation function of inhomogeneities of air density can be calculated by the equations for the Cartesian coordinate system.

3) The PS approximation is applicable to the description of scintillations.

The curvature length of the trajectory Σ takes its minimum value $R_\mathrm{c} \cong H / N(h_\mathrm{p}) \geq 3\cdot 10^8$ m at the perigee point, which is 100 times larger than the height R_l. The neglect of the trajectory deflection leads to the error in determination of the height of the current point as large as $3HR_\mathrm{l}/R_\mathrm{c} \cong 300$ m. At such scales, the relative changes of the amplitude factor (14), as well as the functions (15) and (16), do not exceed 1–2%, which is much smaller than their relative variation along the ray (~10–15%). Therefore, the deflection can be neglected, and the ray part inside the atmospheric layer studied can be described by the following approximate equations:

$$\theta(r,\rho) \approx \theta_\mathrm{p}(\rho) \pm \arccos\frac{h_\mathrm{p}(\rho)}{r}, \tag{17}$$

$$\theta_r(r,\rho) \approx \pm\frac{h_\mathrm{p}(\rho)}{r\sqrt{r^2 - h_\mathrm{p}^2(\rho)}}, \tag{18}$$

$$\theta_\rho(r,\rho) \approx \frac{\mathrm{d}}{\mathrm{d}\rho}\theta_\mathrm{p}(\rho) \times$$

$$\times\frac{Hr}{\left[H\rho - h_\mathrm{p}(\rho)\rho + h_\mathrm{p}^2(\rho)\right]\sqrt{r^2 - h_\mathrm{p}^2(\rho)}}, \tag{19}$$

where the upper sign corresponds to the ray, having passed through the perigee point; otherwise, the lower sign should be taken.

At the same time, the neglect of the change in the direction of the wave vector in the argument of the spectrum $\hat{B}_n(\mathrm{r},\mathrm{q})$ can, in principle, lead to significant inaccuracy in the case of strong anisotropy of the correlation function of the refractive index inhomogeneities. Taking into account that the angle between the ray and the radius vector of the current point is $\alpha = \arcsin\frac{\rho}{n(r)r}$, the second argument of the spectrum $\hat{B}_n(\mathrm{r},\mathrm{q})$ should be presented as follows:

$$\mathrm{q} = \frac{q_\parallel}{\mu_\parallel(\mathrm{R}_0,\mathrm{r})}\mathrm{e}_\varphi - \frac{q_\perp\rho}{\mu_\perp(\mathrm{R}_0,\mathrm{r})n(r)r}\mathrm{e}_r \pm$$

$$\pm\frac{q_\perp}{\mu_\perp(\mathrm{R}_0,\mathrm{r})}\sqrt{1-\frac{\rho^2}{n^2(r)r^2}}\mathrm{e}_\theta, \tag{20}$$

where the sign is selected in the same way as in Eqs. (17)–(19).

In Refs. 2 and 3 it has been shown that the large-scale inhomogeneities of the refractive index at the heights of 25–75 km are oblate in the radial direction, whereas the small-scale component is isotropic. The spectrum of inhomogeneities is a superposition of the functions of the form

$$\hat{B}_n(\mathrm{R},\delta\mathrm{r}) = F(\mathrm{R},\mathrm{K}),$$

$$\mathrm{K} = \sqrt{\delta\mathrm{r}\delta\mathrm{r} : \left\{\mathrm{e}_\theta\mathrm{e}_\theta + \mathrm{e}_\varphi\mathrm{e}_\varphi + \eta^2\mathrm{e}_r\mathrm{e}_r\right\}} \tag{21}$$

with different values of the anisotropy parameter η. For the isotropic component, it is equal to unity, and the large-scale component is characterized by the values $\eta > 30$.

In the case of neglect of the ray deflection, the deviation of the ellipsoid, at which the function of the form (21) takes equal values, from the sphere, whose radius h is equal to the ellipsoid major semi-axis, at the edges of the studied layer is $h\left\{1-\left(1/\sqrt{1+\eta^2\varepsilon^2}\right)\right\}$, where $\varepsilon = 2\sqrt{Hh_p(\rho)}/R_c$ is the angle of the ray turn in the layer with respect to its straight-line trajectory. Assuming $h = l_0$, where l_0 is the characteristic spatial scale of fluctuations in the plane $(\mathbf{e}_\varphi, \mathbf{e}_\theta)$, we obtain the estimate for the upper boundary of the anisotropy parameter:

$$\eta_{\max} = R_c/\sqrt{Hh_p(\rho)} \sim 10^3.$$

The ray deflection can be neglected, if the anisotropy parameter of spatial inhomogeneities does not exceed $\eta_{\max}$.

It is also necessary to estimate possible distortions associated with the neglect of changes in the unit vectors of the spherical coordinate system within the spatial scale of inhomogeneities. Assume that, as the current point shifts to distances not exceeding l_0, the error in determination of the parameter K is within l_0 and find the upper boundary for the anisotropy parameter in the form $\eta_{\max} = R_0/l_0 \sim 10^4$. The distortions will be significant at $\eta > \eta_{\max}$.

It is convenient to analyze the distortions connected with the PS approximation, assuming that the spectrum of fluctuations of the refractive index is an isotropic delta function $\tilde{B}_n(\mathbf{r}_0,\mathbf{q}) = \delta(|\mathbf{q} - q_0|)$. The PS approximation keeps its singularity in the form of the one-dimensional delta function, but transforms the region, in which this function is nonzero, into an ellipse:

$$S_{\mathrm{PS}} = \left\{\frac{q_\perp^2}{\mu_{\perp\mathrm{PS}}^2} + \frac{q_\parallel^2}{\mu_{\parallel\mathrm{PS}}^2} = q_0^2\right\}, \tag{22}$$

whereas the allowance for the extension of the distorting layer removes the singularity, distributing it over the region

$$S_{\mathrm{MSP}} = \left\{\bigcup_{\mathbf{r}\in\Sigma}\frac{q_\perp^2}{\mu_\perp^2(\mathbf{R}_0,\mathbf{r})} + \frac{q_\parallel^2}{\mu_\parallel^2(\mathbf{R}_0,\mathbf{r})} = q_0^2\right\}, \tag{23}$$

where Σ is the ray part within the layer. Thus, the refraction leads to blurring of the spectrum with the characteristic scale in the reciprocal space, proportional to the spatial frequency q_0. Its relative value is different for the longitudinal $\mathbf{e}_\varphi$ and transverse $\mathbf{e}_\varphi \times \mathbf{k}(\mathbf{R}_0)$ directions:

$$\delta_\parallel = 1 - \frac{\sin\left[\theta_p(\rho) + \Delta\theta(r_{\max},\rho)\right]}{\sin\theta_p(\rho)},$$

$$\delta_\perp = 2\left|\frac{\Delta\theta_p(r_{\max},\rho)}{\partial\theta_p(\rho)/\partial\rho}\right| \Bigg/ \left(1 + \left|\frac{\Delta\theta_p(r_{\max},\rho)}{\partial\theta_p(\rho)/\partial\rho}\right|\right), \tag{24}$$

where $r_{\max} = h_p(\rho) + 3H$ is the height of the top boundary of the layer. The estimation of these parameters for the height of 30 km (perigee point) above the Earth's surface gives $\delta_\parallel \sim 1\%$, $\delta_\perp \sim 10\%$.

The assumption that the amplitude factor (14) is constant on the integration path gives the relative error of the order of l/L_0, where l is the distance between the perigee point and the point of the trajectory Σ on the boundary of the region that actively affects the formation of the field of fluctuations ($r = h_p + 2H$). The error does not exceed 10–20% when interpreting the results of observation from onboard a space station at $L_0 > 1000$ km (Fig. 2).

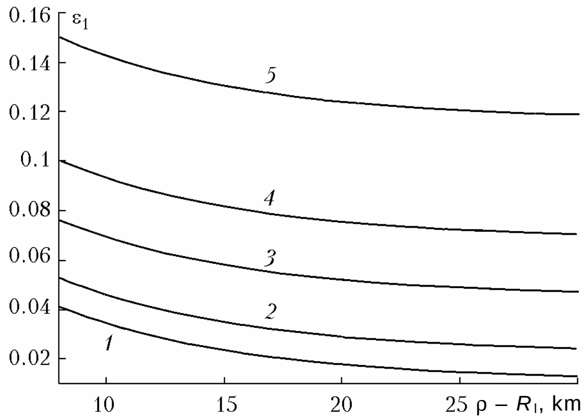

Fig. 2. Relative error in the amplitude factor; $\lambda = 0.7$ μm; $L_0 = 500$ (*1*), 1000 (*2*), 2000 (*3*), 3000 (*4*), 5000 km (*5*).

Of particular interest is the behavior of the functions (16). The condition (9) transforms into the equation

$$\frac{q_\perp^2}{\gamma_\perp(\mathbf{R}_0,\mathbf{r})} + \frac{q_\parallel^2}{\gamma_\parallel(\mathbf{R}_0,\mathbf{r})} - \frac{q_\perp^2}{\gamma_\perp(\mathbf{R}_0,\mathbf{h}_p)} - \frac{q_\parallel^2}{\gamma_\parallel(\mathbf{R}_0,\mathbf{h}_p)} \le \pi, \tag{25}$$

from which we can find the estimates for the boundary values of the spatial frequencies $q_\parallel$, $q_\perp$:

$$q_{\parallel\max} = \frac{1}{R_0\sin\Theta}\sqrt{2\pi k_0\rho\left[\frac{\cot\left[\theta_p(\rho) + \Delta\theta(r_{\max},\rho) - \Theta\right]}{\sin^2\left[\theta_p(\rho) + \Delta\theta(r_{\max},\rho)\right]} - \frac{\cot\left[\theta_p(\rho) - \Delta\theta(r_{\max},\rho) - \Theta\right]}{\sin^2\left[\theta_p(\rho) - \Delta\theta(r_{\max},\rho)\right]}\right]}, \tag{26}$$

$$q_{\perp\max} = \frac{R_0}{\rho}\left|\frac{\theta_r(R_0,\rho)}{\theta_p(R_0,\rho)}\right|\sqrt{\frac{\pi k_0}{2}\frac{\left(\partial\theta_p(\rho)/\partial\rho\right)^2 - \Delta\theta_p^2(R_0,\rho)}{\Delta\theta_p(r_{\max},\rho)}}. \tag{27}$$

The parameters $q_{\perp,\|\max}$ are the upper boundaries of the region of spatial frequencies, in which the change in the radius of the first Fresnel zone inside the layer can be neglected. As the distance between the receiver and the layer increases, the behavior of the boundaries $q_{\perp,\|\max}$ becomes opposite. In this case, $q_{\|\max}$ increases quickly and becomes infinite at the caustic $\Theta = 0$. For the distances $L_0 \sim 1000$ km, characteristic of observation from onboard a space station, $q_{\|\max} \sim \sim 200\ \text{m}^{-1}$. The boundary $q_{\perp\max}$, on the contrary, decreases slowly as the distance from the layer increases. It is ~7 m^{-1} at the distance of 1000 km from the layer (Fig. 3). The presence of the upper boundary is connected with the fact that the angle of scattering from the inhomogeneities, lying before the perigee point, changes under the effect of regular refraction, and the relative change increases with the decrease in the size of the inhomogeneities.

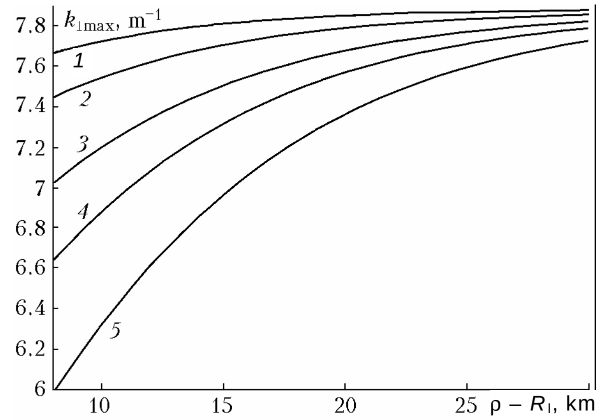

Fig. 3. The upper boundary of the applicability range of the PS approximation for the transverse spatial frequency; $\lambda = 0.7$ μm, $L_0 = 500$ (*1*), 1000 (*2*), 2000 (*3*), 3000 (*4*), 5000 km (*5*).

Upon the propagation through the atmosphere, the rays scattered by identical inhomogeneities, located in different parts of the layer, diverge at different angles, which gives rise to errors, when the receiver is far from the layer. It should be noted that this effect is significant for spatial frequencies, exceeding the Fresnel scale, and the relative difference between the upper boundary of the PS applicability range and the Fresnel scale is inversely proportional to the change in the wave tube cross section inside the layer. The behavior of the boundaries $q_{\perp,\|\max}$ as a function of the distance to the receiver is mostly determined by the transformation of the wave tube due to refraction. The compression of the spectrum in the direction $\mathbf{e}_\varphi \times \mathbf{k}(R_0)$ determines the decrease of $q_{\perp\max}$, as well as the expansion of the spectrum along $\mathbf{e}_\varphi$ causes the growth of $q_{\|\max}$.

Conclusions

In this paper, we have considered the causes for the decrease in the accuracy of the PS approximation in the analysis of intensity fluctuations of an electromagnetic wave transmitted through the turbulent layer of the medium with the variable (in space) mean refractive index. It has been found that the following factors are most significant:

1. The dependence of the amplitude of the secondary waves on the position of their source;
2. The change in the amplitude of the primary wave over the layer due to refraction;
3. The change in the wave tube cross section inside the layer;
4. The change in the angle of scattering from inhomogeneities due to refraction.

Their influence on the characteristics of the transmitted wave is quite various. The allowance for the first and second factors implies the appearance of the error, whose relative value is independent of both the spatial frequency and the shape of the spectrum of refractive index inhomogeneities. As the distance between the observation point and the distorting layer increases, this error does not vanish in contrast to the case of layers without refraction. A consequence of the third factor is blurring of the spectrum of the refractive index inhomogeneities, whose scale is directly proportional to the spatial frequency. The strength of this effect depends on how sharp is the dependence of the spectrum of refractive index inhomogeneities on the spatial frequency. The influence of the fourth factor consists in blurring of oscillations in the spectrum of intensity fluctuations at the spatial frequencies, exceeding the Fresnel scale. The obtained estimates of the errors of the PS approximation as applied to the analysis of stellar scintillation spectra observed during star occultation by the Earth's atmosphere indicate that there is no sense in using much more cumbersome equations of the method of smooth perturbations.

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