REGIONAL PROBABILISTIC-STATISTICAL MODEL OF CLOUDS BASED ON THE MARKOV CHAINS THEORY

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The probabilistic–statistical model of the clouds based on the Markov chains theory, allows one to take into account the features of a given region is proposed. The graphs for the probabilistic state of cloud types for different seasons of the year as applied to the North Caucasus region are plotted on the basis of the proposed model being constructed based on Tables TM–1. Recommendations on recording Tables TM–1 are given.

Solution of the different problems in applied meteorology as well as investigation of the atmosphere as the dynamic information channel for the propagation of optical signals and image transfer, optical monitoring of the environment, and optical diagnostics of the atmospheric pollution would call for the data on the spatiotemporal distribution of cloud particles, which determine the optical regime of the atmosphere in the investigated region. These data can be represented as the spatiotemporal model of clouds in this region. There are some variants of cloud models which can be classified as follows: models based on the hydrodynamics equations, statistical, and probabilistic models. The models of the first type are described in detail in Ref. 1. These models that indicate the main points of the physical processes in the atmosphere are insufficiently related to the local factors of the region and, in addition, their use is essentially difficult in calculational aspect. The statistical and probabilistic models are more suitable here. The questions pertaining to the construction of such models are discussed in detail in Ref. 2.

However, in spite of the fact that the theoretical aspects of constructing such models have been studied quite well, practice would call for the use of these models taking the essential features of the given region into account.

This paper is devoted to constructing the regional probabilistic-statistical cloud model on the basis of the Markov chains theory. The cloud characteristics obtained by the generalization of the data given in Refs. 1–7 for the visible range are shown in Fig. 1 (*a* and *b*) and in Tables I and II. For the scattering phase function we selected the function corresponding to the wide particle size distribution approximated by the log-normal law. In so doing, we proceeded from the best theoretical foundation of such distribution.

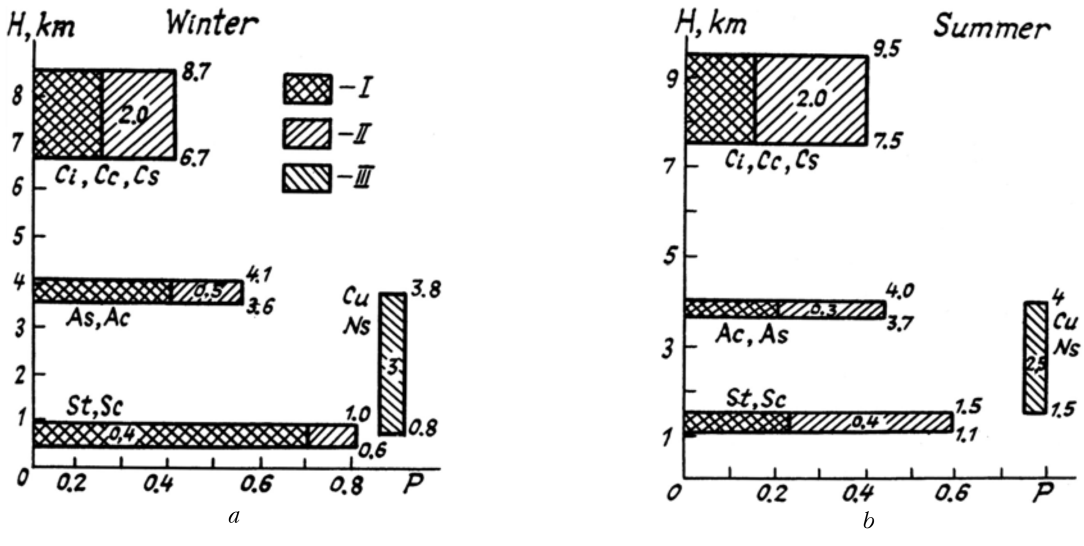

FIG. 1. Vertical distribution of clouds which is typical for the central regions of the USSR: a) winter, b) summer, I refers to low, middle and high cloud covers for index of 8 – 10; H denotes the heights of cloud boundaries and their probability; II is the same for a cloud cover index of 3 – 7; and, III represents the clouds of vertical development neglecting their probability.

The given experimental data testify that the propagation of the visible radiation is significantly affected by the middle and low clouds. This fact allows one to accept rather insufficient data on the upper clouds. Seasonal variations in the cloud thickness do not play an important role. Much more important is the presence or absence of the

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clouds at one and the same altitude interval, i.e., temporal variability of these clouds. It should be added that the temporal variability is rigidly enough related to the climatic features of every region and season of the year. The data on temporal variability of cloud types for the investigated region can be taken from Tables TM–1, including the statistics of the data of long standing on cloud cover which were obtained by the network of meteorological stations of Goskomgidromet. The row "cloud types," containing the necessary data is shown in Table III. The cloud cover index for total and low clouds as well as the cloud type distribution are given in this table in two stages. At the first stage the types which refer to the same altitude interval are distributed through the columns. Convective clouds (Cu, Cb) and nimbostratus (Ns, FrNb) are presented in the separate columns. This is due to the fact that this type of clouds is substantially different from the layered structure of St and Sc. From the standpoint of the optical properties, the difference consists in another microstructure and thickness of the cloud layer. Subsequent definition of the cloud types is performed by compliting the table by the definite decimal number corresponding to the cloud type into each column.

TABLE I. The scattering phase function of radiation at λ = 0.63 μm *corresponding to the wide–size distribution of cloud particles.*

$\gamma°$	$f(\gamma)$	$\gamma°$	$F(\gamma)$
0	$0.1367\cdot10^{5}$	80	$0.4757\cdot10^{-1}$
1	$0.1358\cdot10^{4}$	90	$0.2557\cdot10^{-1}$
2	$0.1759\cdot10^{3}$	100	$0.1964\cdot10^{-1}$
4	$0.2588\cdot10^{2}$	110	$0.2145\cdot10^{-1}$
6	$0.1276\cdot10^{2}$	120	$0.3573\cdot10^{-1}$
8	$0.9282\cdot10^{1}$	130	$0.4884\cdot10^{-1}$
10	$0.7688\cdot10^{1}$	140	$0.3318\cdot10^{-1}$
15	$0.5644\cdot10^{1}$	145	$0.1875\cdot10^{0}$
20	$0.4242\cdot10^{1}$	150	$0.1497\cdot10^{0}$
25	$0.3106\cdot10^{1}$	155	$0.1318\cdot10^{0}$
30	$0.2279\cdot10^{1}$	160	$0.1181\cdot10^{0}$
40	$0.1169\cdot10^{1}$	165	$0.1093\cdot10^{0}$
50	$0.5665\cdot10^{0}$	170	$0.1141\cdot10^{0}$
60	$0.2555\cdot10^{0}$	175	$0.1537\cdot10^{0}$
70	$0.1089\cdot10^{0}$	180	$0.6756\cdot10^{0}$

TABLE II. The scattering coefficients for clouds located in different altitude intervals with account of recurrence of cloud types.

Cloud types	σ, km^{-1}	
	Summer	Winter
Ci, Cc, Cs	2.5	2.5
Ac, As	42	32
St, Sc	54	36
Ns	38	36
Cu, Cb	100	100

After corresponding recoding Tables TM–1 with the help of the procedure which will be described below, statistical processing of the experimental data obtained for the cloud cover over the region of Rostov–on–the–Don for all the seasons of the year: spring, summer, fall, and winter in the 1978's–1982's was carried out. Formally, one can assume that the atmosphere with an account of separating the clouds located at different altitude intervals and selecting a group of convective clouds can be observed in 32 states including the state "cloudless." However, from purely physical reasons it is clear that the number of such states must be smaller because some combinations of states are impossible. For example, the state "cloudless" is antithetic to the presence of clouds located at arbitrary altitude. Thus, in reality, they are smaller in number and after statistical processing, "spring" lad the extreme number of states being equal to 22. A number of these states represents such seldom events with inadequate statistics that it was eliminated without any disadvantage of the model. In processing, the hypothesis on the possibility of representing the transition from me state to another as a simple chain of events was tested. For this, the stationarity of the chain of events was tested and the probability density functions for time intervals between the transitions from state to state were constructed. A method described in Ref. 2 was used to test the stationarity hypothesis. According to this method, $\hat{\chi}^2$ statistics was calculated in the form

$$\hat{\chi}^2 = \sum_{ij} \frac{f_{ij} g_{ij}}{f_{ij} + g_{ij}} \left(\frac{f_{ij}}{f_i} - \frac{g_{ij}}{g_i} \right), \qquad (1)$$

where $\|f_{ij}\|$ and $\|g_{ij}\|$ are the matrices of transition frequencies from state to state for the two halves of seasonal samples. This statistics has χ^2 distribution with $s(s-1)$ degree of freedom, where s is the number of states. Testing showed that stationarlty hypothesis for the chain of events when the entire sample was divided into 4 seasons could be accepted for each of four seasons ($P_i > 0.24$, $i = \overline{1,4}$.

TABLE III. Fragment of Table TH–1.

	Cloud types, code				
1	Ci	Ac	Cu	st	
2	Cc	As	Cb	Sc	Ns
3	Cs				Fr Nb
4	Ci Cc	Ac As	Cu Cb	St Sc	
5	Ci Cs				
6	Cc Cs				Ns Fr Nb
7	Ci Cc Cs				
8	Cannot be determined				
9	Cloudless				

Cloud cover index			Altitude of the cloud base, code
43, 44	45, 46	47, 48, 49, 50, 51	52, 53
		Example	
Result of recoding		2 0 1 0 0	Initial code
		1 0 1 0 0	
		0 4 0 2 3	
		0 1 0 1 1	
		8 8 8 1 0	
		0 1 0 1 0	

In addition to testing the stationarity, the hypothesis about the exponential character of empirical probability density function for time intervals between the transitions from state to state was tested on the basis of χ^2 criterion. The test showed the hypothesis fits fairly good the statistical data. Proceeding from these results with an account of the known theoretical premises about the distribution of the time intervals within the simplest flux, which converts the system from state to state following an exponential law, the authors considered it possible to represent such a process of the transition in the atmosphere as four marked graphs of states. The above-indicated graphs are given in Figs. 2*a–d*. Each state is characterized by a binary five-digit code (Table III) obtained by recoding the data presented in Tables TM–1, probability, and average time of the continuous existence. Figures given near the arrows, which indicate the direction of transition, characterize transition intensity. The system of differential equations can be obtained for each of the given graphs. This system has the following form:

$$\begin{aligned} \frac{dP_1}{dt} &= -P_1(\lambda_{12}+\lambda_{13}+\ldots+\lambda_{1M})+P_2\lambda_{21}+\ldots+P_N\lambda_{N1}, \\ \frac{dP_N}{dt} &= -P_N(\lambda_{N1}+\lambda_{N2}+\ldots+\lambda_{NN-1})+P_1\lambda_{1N}+\ldots+P_{N-1}\lambda_{N-1}, \end{aligned} \tag{2}$$

where P_1 is the probability of the atmospheric state, λ_{ij} is the intensity of the transition from ith to jth state. With the given initial conditions $P_1 = 1$ and $P_{j\neq i} = 0$, the system may be solved and as a result and the probability of the cloud state may be determined at every given moment in time.

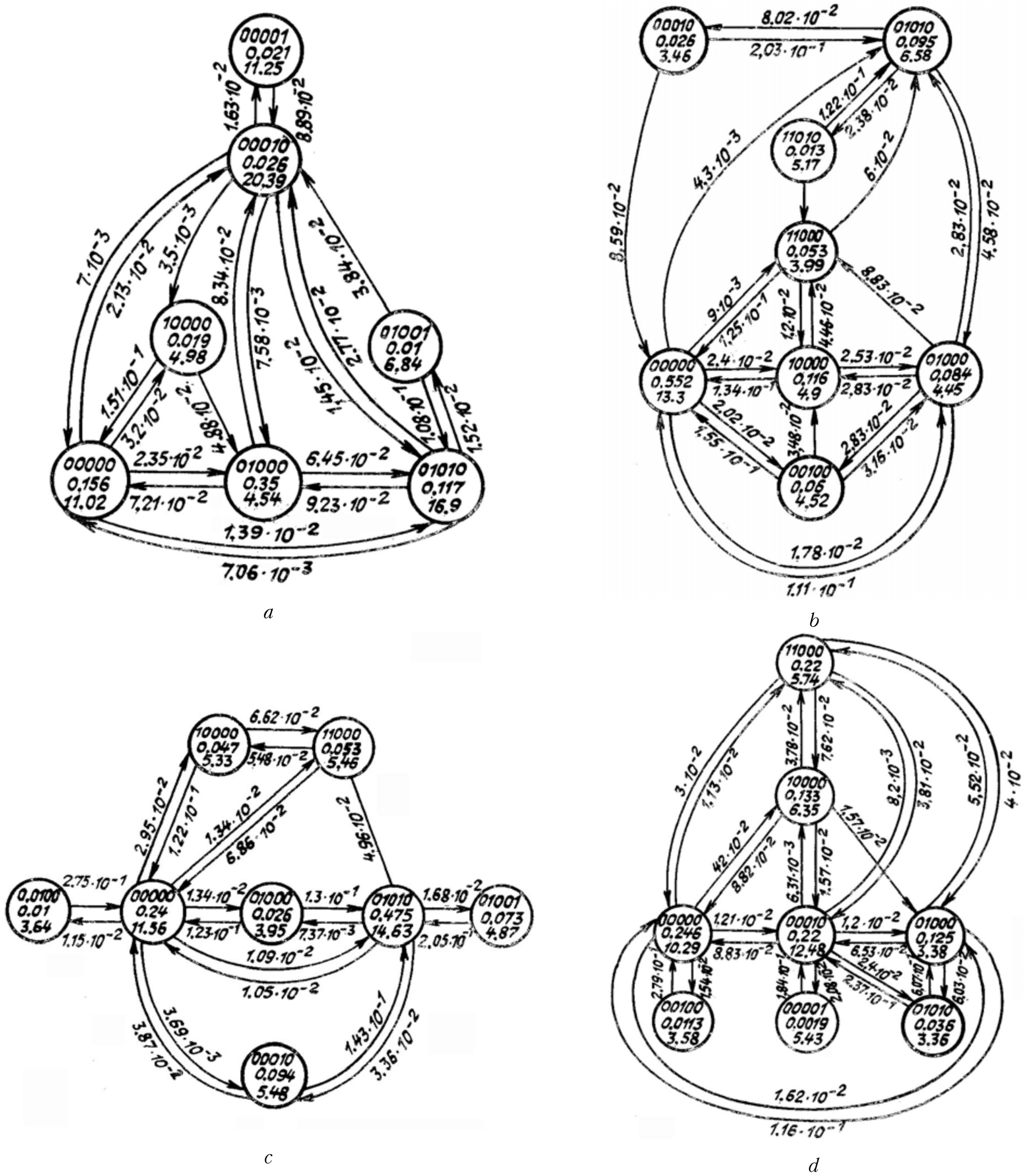

FIG. 2. Marked graphs of the cloud cover state: a) winter, b) summer, c) spring, and, d) fall.

To construct the graphs for the states of the atmosphere it is necessary to perform recoding of data shown in Tables TM-1 (see Table III). When recoding the decimal places in the Tables changed to "1" if the cloud at a given altitude interval is presented, and to "0" in the opposite case. Thus, the atmospheric state is represented by a binary code which contains five positions. However, in recoding the uncertainty may appear. For example, in completing Table TM–1 for the first time, when observation of the definite cloud types was difficult for some reasons (high cloud cover index of the low clouds, fog, snowstorm, and so on). In this case, according to the existing rule of coding Tables TM–1, figure "8" is put in a corresponding column. This fact introduces the uncertainty when forming the five–digit binary code representing the atmospheric state. This uncertainty is proposed to be resolved in the following way. If the code combination includes two or three "8" in the leading digits, this testifies to high cloud cover index (9 – 10) of the low clouds, which makes it impossible to observe higher clouds. Then taking into account the existing relation between the cloud cover index and the average thickness of total clouds $n = c\Delta H^2$, where n is the cloud cover index, ΔH is the total thickness of clouds, and c is the constant, we may assume the existence of the middle clouds with high degree of probability and replace the existing combination of figures "8" by the code "010*LL*, " where "*LL*" denotes the code of the state of the low clouds. The existence of the upper clouds is neglected, since an account of these clouds would not virtually affect the combination of optical characteristics of the total cloud mass.

If the code combination of the table consists of five figures "8" (fog, snowstorm, and so on), then when forming the binary code it is assumed that the cloud types that existed before starting the phenomenon, which made the observation difficult, remained unchanged. This is explained by the fact that the average time of continuous existence of the cloud types exceeds the time of existence of these atmospheric phenomena. This fact is verified by the statistics of observation of Rostov Gidromettsentr.

Conclusions. The developed regional probabilistic-statistical model for the clouds based on the Markov chains theory may be widely used for short-range forecasting as a source of the initial data in atmospheric optics calculations and in order to justify the position of different optical system as well as to model the operational conditions of this systems with an account of the optical weather of the given region. In so doing the forecast is obtained as a result of solving system (2) for possible states combination and their probabilities at a given moments in time. The proposed model is especially efficient for planning the operation of different optical means. The model structure enables us to realize it on a personal computer with the database and standard software designed for solving the system of differential equations.

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