Numerical model of the formation and evolution of fogs and stratus clouds

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A numerical hydrodynamic model of the formation and evolution of fogs and stratus clouds is developed. The model is based on the equations of influx (budget) of heat, water vapor, and water content of fog. Equations for the wind velocity and the turbulence coefficient are derived within the theory of similarity. The continuity equation is involved to find the altitude dependence of the vertical velocity. Advective inflows of heat and humidity are written with the allowance for actual data, according to which they are proportional to hyperbolic tangent of the horizontal distance. As a boundary conditions near the Earth's surface, more physically justified equations of heat and humidity balance in the roughness layer are used. Fog parameters characterizing the role of various factors in fog formation and evolution are calculated.

Fogs and clouds are related to such natural phenomena, with which the change in meteorological visual range and other optical characteristics of the atmosphere is associated. They essentially affect the human economic activity, as well as agriculture, building industry, instrument-making industry, etc.

It follows already from the formula for relative humidity $f = e/E(T)$ that the saturation state ($f = 1$) and subsequent condensation can be reached due to increasing the water vapor pressure e and (or) decreasing air temperature T, to which the saturation pressure $E(T)$ is unambiguously related.

Initial equations

The equations describing the temporal and spatial variations of temperature and water vapor are:

a) equation of the budget (balance) of heat in the wet turbulent atmosphere

$$\frac{\partial T}{\partial t} = - w\left(\frac{\partial T}{\partial z} + \gamma_a\right) - \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z} k_z\left(\frac{\partial T}{\partial z} + \gamma_a\right) + k_s\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \quad (1)$$

b) equation of the budget (balance) of moisture. Following the method of invariants,[1] let us write it in the form

$$\frac{\partial s}{\partial t} = - w\frac{\partial s}{\partial z} - \left(u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y}\right) + \frac{\partial}{\partial z} k_z\frac{\partial s}{\partial z} + k_s\left(\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2}\right). \quad (2)$$

Here s is the specific water content in air:

$$s = q + \delta, \quad (3)$$

q and δ are the mass portion of water vapor and the water content of fog (cloud); u, v, and w are the components of the velocity of motion (wind) along the axes x, y, and z, respectively (z axis is directed vertically up), k_z and k_s are the vertical and horizontal turbulence coefficients, $\gamma_a = 0.98\cdot10^{-2}$ °C/m is the dry adiabatic lapse rate (at $f < 1$) replaced with the moist adiabatic γ_{ma} in saturated air ($f = 1$).

In formula (3), $\delta = 0$ and $s = q$ at $f < 1$ and $s = q_m + \delta$ at $f = 1$, where q_m is the mass portion of the saturated water vapor

$$q_m = 0.622\, E(T)/p, \quad (4)$$

where p is the air pressure determined at an arbitrary height by the static equation

$$- dp = g\rho dz. \quad (5)$$

Here g is the acceleration due to gravity; ρ is the air density related to p and T by the wet air state equation:

$$p = R_c\, \rho T_v.$$

($T_v = T(1 + 0.61q)$ is the virtual temperature, $R_c = 287$ J/(kg · K).

Successively attracting the theory of similarity, the foundations of which were laid by A.S. Monin and A.M. Obukhov,[2] we obtain[3] the formulas that describe the following quantities:

a) the coefficient k_z as a function of height

$$k_z(z) = \chi L_* u_* [1 - \exp(-z/L_*)], \quad (6)$$

b) distributions of T, q, and the absolute value of wind velocity c over height:

$$T(z) = T_0 + T_* \ln[(\eta/\eta_0 - \gamma_a (z - z_0)]; \quad (7)$$

$$q(z) = q_0 + q_* \ln(\eta/\eta_0); \quad (8)$$

$$c(z) = (u_*/\chi) \ln(\eta/\eta_0). \quad (9)$$

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Here T_0 and q_0 are the values of T and q at the roughness level z_0; T_*, q_*, and u_* are the scales of T, q, and c; L_* is the scale (characteristic thickness) of the near-ground layer:

$$L_* = T_0\, u_*^2/(\chi^2 g T_*); \quad (10)$$

$$\eta = \exp(z/L_*) - 1, \quad \eta_0 = \exp(z_0/L_*) - 1, \quad (11)$$

$\chi = 0.38$ is the Prandtle–Karmann constant.

If write the formulas (7)–(9) up to some level z_1, at which T_1, q_1, and c_1 are known (measured), then, solving them relatively to their scales, we obtain

$$T_* = \frac{(T_1 - T_0) + \gamma_a\,(z_1 - z_0)}{\ln(\eta_1/\eta_0)}; \quad (12)$$

$$q_* = (q_1 - q_0)/\ln(\eta_1/\eta_0); \quad (13)$$

$$u_* = \chi c_1/\ln(\eta_1/\eta_0), \quad (14)$$

where $\eta_1 = \exp(z_1/L_*) - 1$.

We assume the scale L_* involved in all formulas to be equal to the height of the upper boundary (h) of the near-ground layer $L_* = h$.

The turbulence coefficient k_z above the near-ground layer practically does not depend on height: $k_z(z) = k_z(h)$. Its value $k_z(h)$ is determined by formula (6) at $z = L_*$. The vertical distribution of the wind velocity at $z > h$ is described by the known Ekmann formulas, which are the solution of equations of the settled motion of the atmosphere.

To determine the vertical velocity involved into formulas (1) and (2), the equation of continuity is used.[4] It follows from the formulas obtained by means of integrating it over height that the vertical velocity averaged over a circle (cyclone) of the radius R depends on the Rossby number $\mathrm{Ro} = c_g/(z_1\omega\sin\varphi)$, the ratios c_1/c_g, z_0/z_1, h/z_1, and $c_g z_1/R$ (c_g is the geostrophic wind velocity; c_1 is the wind velocity at the level z_1; z_0 is the roughness parameter; h is the height of the near-ground layer; $\omega = 7.29\cdot10^{-5}\ \mathrm{s}^{-1}$ is the angular rate of the Earth rotation; φ is the geographic latitude). Vertical velocity in cyclone is positive and increases with height from zero at the ground surface up to the maximum value w_H reached near the upper boundary H of the boundary layer of the atmosphere:

$$w_H = \frac{c_g\, z_1}{R}\, G, \quad (15)$$

where

$$G = \sqrt{\frac{c_1}{c_g} D \mathrm{Ro}}\left[1 - B\frac{c_1}{c_g} - (\cos\alpha_0 - \sin\alpha_0)\right], \quad (16)$$

$$A = \frac{0.24h}{z_1\ln(\eta_1/\eta_0)}, \quad B = \frac{0.54 - \ln\eta_0}{\ln(\eta_1/\eta_0)}.$$

The angle of deviation (α_0) of the wind velocity from an isobar (geostrophic wind) in the near-ground layer is determined by the equation

$$\cos\alpha_0 = \frac{1 + B^2(c_1/c_g)^2 - N\,\mathrm{Ro}(c_1/c_g)^3}{2B(c_1/c_g)}, \quad (17)$$

$$N = 0.30\, z_1/h\, [\ln((\eta_1/\eta_0)]^{-3}.$$

The dynamic characteristics, wind velocity, and its components u, v, and the vertical velocity w, the turbulence coefficient k_z, geostrophic velocity c_g, as well as the roughness parameter z_0 and the height of the near-ground layer h do not change in time during a few hours, while fog and cloud are formed. But, naturally, they change from one situation to another. One of the purposes of this paper is to show how these variations affect the formation and development of a fog or a cloud.

Advective influxes of heat and moisture

These influxes are presented by the second terms in the right-hand sides of formulas (1) and (2). The effect of advection is most significant over the areas with large horizontal gradients of T and s: frontal zones, near the boundary of ground surfaces with different optical and thermal properties (land–water, field–forest, hill–wetland, etc.).

When air mass has passed from one such surface to another, temperature and humidity of air undergo the strongest temporal changes, while the wind velocity changes essentially less (according to the estimates, not more by 10–20%). Distribution of T and s along x-axis normal to the boundary of the surfaces, is described with satisfactory accuracy by the following formulas:

$$T(x) = (T'' + T')/2 + (T'' - T')/2\,\tanh(x/D); \quad (18)$$

$$s(x) = (s'' + s')/2 + (s'' - s')/2\,\tanh(x/D). \quad (19)$$

Here T'' and T' are the temperatures of warm and cold air masses at a large distance from the boundary (front), s'' and s' are the respective values of the water vapor content; $\tanh(x/D)$ is the hyperbolic tangent

$$\tanh(x/D) = \frac{\exp(x/D) - \exp(-x/D)}{\exp(x/D) + \exp(-x/D)}, \quad (20)$$

D is the parameter (of the dimensionality of length) determining the width of the zone, in which principal change of T and s along x occurs (when passing from $x = -D$ to $x = D$ temperature changes by ΔT which is about 90% of the difference $T'' - T'$). The advective terms in the dependence of T and s on x determined by formulas (18) and (19) take the form

$$-u\frac{\partial T}{\partial x} = -u\frac{T'' - T'}{2D}[1 - \tanh^2(x/D)], \quad (21)$$

$$-u\frac{\partial s}{\partial x} = -u\frac{s'' - s'}{2D}[1 - \tanh^2(x/D)]. \quad (22)$$

The relationships (18), (19), (21), and (22) at $x = 0$ at the boundary of the surfaces or air masses take the form

$$T(0) = (T'' + T')/2; \quad s(0) = (s'' + s')/2; \quad (23)$$

$$\left(-u\frac{\partial T}{\partial x}\right)_0 = -u\frac{T'' - T'}{2D}, \quad \left(-u\frac{\partial s}{\partial x}\right)_0 = -u\frac{s'' - s'}{2D}. \quad (24)$$

The values $T(0)$ and $s(0)$ involved into Eq. (23) are the sought temperature T and water content s, which are involved into formulas (1) and (2), and should be simulated by means of them. The values T'' and s'' at advection of heat or T' and s' at advection of cold in formula (23) are known (set).

Excluding first T'' and s''

$$T'' = 2T - T', \quad s'' = 2s - s'$$

and replacing T'' and s'' according to these relationships, we reduce formula (24) in the case of advection of cold to the form

$$\left(-u\frac{\partial T}{\partial x}\right)_0 = -u\frac{T - T'}{D}, \quad \left(-u\frac{\partial s}{\partial x}\right)_0 = -u\frac{s - s'}{D}. \quad (25)$$

These formulas have analogous form in the case of advection of heat

$$\left(-u\frac{\partial T}{\partial x}\right)_0 = -u\frac{T - T''}{D}, \quad \left(-u\frac{\partial s}{\partial x}\right)_0 = -u\frac{s - s''}{D}. \quad (26)$$

Boundary conditions

Since Eqs. (1) and (2) are differential equations of second (in z) order, the sought functions T and s should fulfill two boundary conditions. The condition at the upper boundary (z) is formulated most simply. Any level situated in the middle or upper troposphere can be taken as the last. This supposition is justified by that the changes in the values observed at the level z^* do not essentially affect the processes in the lower troposphere, where formation of fogs and low clouds is considered. So the conditions at the level z^* are set in the form

$$z = z^*: T(z^*, t) = T(z^*, 0) = \text{const};$$
$$s(z^*, t) = s(z^*, 0) = \text{const}. \quad (27)$$

It is more difficult to formulate the conditions for T and s near the rough ground surface. The average (not pulsation) wind velocity decays to zero in the roughness layer (from $z = 0$ to z_0). However, turbulent pulsations of the wind velocity are observed in this layer. They favor equalizing of temperature and other values along the vertical direction. Let us denote by T_0 the mean temperature in the roughness layer, equate the change in the heat content of this layer during unit time to the sum of the heat and radiation fluxes coming to it from the atmosphere and the ground

$$c_l\, z_0 \frac{\partial T}{\partial t} = R_* - Q_T - LQ_q + Q_{\rm m}, \quad (28)$$

where c_l is the volume thermal capacity of the roughness layer, $Q_{\rm m}$ is the molecular heat flux from the ground at $z = 0$, Q_T and Q_q are turbulent fluxes of heat and moisture; R_* is the radiation budget at the level z_0; L is the specific heat of vapor formation.

Turbulent fluxes of heat and water vapor near the ground surface[4] have the form

$$Q_T = \chi^2 c_p\, \rho_0\, c_1 \frac{(T_1 - T_0) + \gamma_{\rm a}(z_1 - z_0)}{[\ln(\eta_1/\eta_0)]^2}, \quad (29)$$

$$Q_q = -\chi^2 \rho_0\, c_1 \frac{q_1 - q_0}{[\ln(\eta_1/\eta_0)]^2}. \quad (30)$$

Here T_1, q_1, and c_1 are the known (measured) values of T, q, and c at the level z_1 (for example, 10 m). Let us write the molecular heat flux from the ground in the form

$$Q_{\rm m} = \lambda\,(T_* - T_0)/z_*, \quad (31)$$

where T_* is temperature at depth z_*; λ is the soil thermal conductivity coefficient (the flux $Q_{\rm m} > 0$ when it is directed upwards from soil to the roughness layer). Radiation budget over land in the nighttime and over water both in the nighttime and daytime includes only the effective radiation of the ground surface: $R_* = -B^*$.

Comparison of the Angström and Brent formulas has shown that the Brent formula is more sensitive to the influence of water vapor content on B^*:

$$R_* = -B^*; \; B^* = \sigma T_0^4\,(0.45 - 2.48\sqrt{q_0}\,). \quad (32)$$

In Eq. (32) $\sigma = 5.670 \cdot 10^{-8}$ W/(m$^2 \cdot$ K^4) is the Stephan–Boltzmann constant; q_0 is the mass portion of water vapor at the level z_0, g/kg.

The water content of the roughness layer changes in time under the effect of turbulent flux of moisture from the atmosphere ($Q_{\rm s}$) and the flux of water vapor from soil ($Q'_{\rm m}$).

$$\rho_l^* = z_0 \frac{\partial s_0}{\partial t} = -Q_{\rm s} + Q'_{\rm m}, \quad (33)$$

where ρ_l^* is the density of the roughness layer.

The turbulent flux of water content is

$$Q_{\rm s} = -\chi^2 \rho_0\, c_1\,(s_1 - s_0)/[\ln(\eta_1/\eta_0)]^2. \quad (34)$$

Let us estimate the flux $Q'_{\rm m}$ on the basis of the heat flux from soil ($Q_{\rm m}$). Water vapor near the boundary between soil and the roughness layer is close to the saturation state. So the Clausius–Clapeyron equation is fulfilled here

$$\frac{{\rm d}E}{E} = \frac{L{\rm d}T}{R_{\rm w}T^2}, \quad (35)$$

where $R_{\rm w}$ is the gas constant of water vapor equal to 461.5 J/(kg·K). Taking into account this equation, we can write the flux $Q'_{\rm m}$ in the form

$$Q'_{\rm m} = 3.37 \cdot 10^3 \frac{E_0 \lambda\,(T_* - T_0)}{p_0\, T_0^2\, z_*}, \quad (36)$$

where E_0 is the saturated water vapor pressure at temperature T_0; p_0 is air pressure at the level z_0.

If water vapor at the soil–air boundary has not been saturated, the right-hand side of Eq. (36) is multiplied by relative humidity f_0. Thermal conductivity of solid parts of soil involved in Eqs. (31) and (36) varies depending on the type of soil from 0.08 to 0.27 J/(m^3·s·K). The volume thermal conductivity of the solid parts of soil varies in a comparatively narrow range and is $2.1 \cdot 10^6$ J/(m^3·K). However, both these characteristics vary in wide ranges depending on porousness and humidity of soil.

Formula (28) contains the volume thermal conductivity c_l of the roughness layer including both air and solid elements of roughness. The volume thermal conductivity of air is $c_p\rho_0 \approx 1.25 \cdot 10^3$ J/(m^3·K). As the ratio of air and solid elements of roughness can change, then the ratio $c_p\rho_0/c_l$ also changes, as well as the ratio of the densities ρ_0/ρ_l. In making calculations, these ratios were assumed to be equal to $5 \cdot 10^{-2}$.

Vertical distributions of temperature and humidity at initial moment

The vertical distributions of T and q in the near-ground layer of the atmosphere at the initial time moment are described by formulas (7) and (8) at T_* and q_* determined by the relationships (12) and (13) and set T_0 and T_1.

Linear dependence is accepted for temperature vertical distribution above the near-ground layer:

$$T(z, 0) = T_h(0) - \gamma(z - h), \quad h \le z \le z^*, \qquad (37)$$

where $T_h(0)$ is the temperature at the upper boundary (h) of the near-ground layer calculated by formula (7); γ is the vertical gradient of T; z^* is the upper boundary of the layer. Vertical distribution of q is described by the relationship:

$$q(z, 0) = f(0)\, q_{\mathrm{m}}(T, p), \quad z_0 \le z \le z^*, \qquad (38)$$

where $f(0)$ is relative humidity of air at the initial moment, which does not depend on height; q_{m} is the mass portion of water vapor at $T(z, 0)$ and pressure $p(z)$. Vertical distributions of temperatures T' and T'' and water content s' and s'' introduced above are described by the same formulas as (7), (8), (37), and (38). At $t = 0$ s' and s'' are equal to q' and q'', respectively, the values of relative humidity $f(0)'$ and $f(0)''$ can differ from the values $f(0)$.

The sought functions T, s, and water content of fog δ were calculated by formulas (1)–(3) by explicit scheme at fulfillment of the inequality between increases of independent variables Δt and Δz providing for stability of the scheme

$$\Delta t < \Delta z^2/(2k_{\min}), \qquad (39)$$

where $k_{\min}$ is minimum value of the vertical profile of the turbulence coefficient.

Derivatives with respect to height are approximated by central differences, the step Δz accepted for the near-ground layer increases with height $\Delta z_{j+1} = \beta \Delta z_j$ ($1.1 \le \beta \le 1.5$).

Results of modeling

The initial equations (1)–(3), boundary and initial conditions include significant number of parameters (factors), on which the fact of formation, water content and thickness of fog depend. The principal purpose of numerical modeling is to quantitatively estimate the role of these factors in the formation and change of the characteristics of fog.

The calculations presented below were carried out for the height of the near-ground layer h = 100 m, the width of the zone of the quick change in temperature D = 50 km, parameter of roughness z_0 = 0.1 m, cyclone radius R = 300 km, soil layer thickness z_* = 10 cm, soil thermal conductivity coefficient λ = 15 kJ/(m·s·K). Different values of the other parameters were taken.

The vertical velocity w and the turbulence coefficient k_z take the following values at the dynamic parameters presented in Table 1 (Ro = $6 \cdot 10^4$, z_0 = 0.1 m, c_{10} = 0.5 m/s):

z, m	0.1	10	21	33	60	113	313	1113	2713
w, cm/s	0.0	0.04	0.08	0.12	0.23	0.43	1.20	4.20	6.40
k_z, m/s	0.1	0.15	0.29	0.43	0.70	1.10	1.50	1.55	1.60

The data given in Table 1 make it possible to estimate the effect of different factors on the water content (intensity) of fog. If there were no advection of heat and water vapor ($T'(z, 0) - T(z, 0) = 0$, $f' = 0.9$), fog is formed and its water content increases in time, since temperature decreases due to the losses of heat for evaporation (second term in equation for $\partial T_0/\partial t$), and the water content in air increases (due to the influx of water vapor from soil).

Table 1. Water content of fog (g/kg) near the ground surface at $T_0(0)$ = 15°C, $f(0, z)$ = 0.9; T_* = 20°C, $T_{10}(0) - T_0(0)$ = – 0.06°C, z_0 = 0.1 m, c_{10} = 0.5 m/s, Ro = $6 \cdot 10^4$, D = 50 km

$T'(z,0) - T(z,0)$, °C	$f'(0,z)$	Time, hours				
		1	2	3	5	7
−5	0.9	0.110	0.422	0.690	1.120	1.400
	0.8	0.090	0.345	0.540	0.786	0.891
	0.7	0.070	0.270	0.390	0.470	0.450
−2	0.9	0.072	0.290	0.440	0.650	0.780
	0.8	0.048	0.200	0.250	0.270	0.240
	0.7	0.023	0.110	0.070	0.000	0.000
−1	0.9	0.060	0.250	0.364	0.500	0.590
	0.8	0.030	0.155	0.168	0.100	0.010
	0.7	0.009	0.060	0.000	0.000	0.000
0	0.9	0.050	0.213	0.290	0.363	0.392
	0.8	0.022	0.112	0.080	0.000	0.000
	0.7	0.000	0.010	0.000	0.000	0.000
2	0.95	0.046	0.204	0.273	0.322	0.326
	0.90	0.031	0.146	0.154	0.075	0.000
	0.85	0.015	0.089	0.361	0.000	0.000
	0.80	0.001	0.032	0.000	0.000	0.000
5	0.95	0.029	0.138	0.126	0.000	0.000
	0.90	0.010	0.068	0.000	0.000	0.000

The water content at a fixed moment in time (for example, 2 hours) varies in a wide range depending on the difference $T'(0, z) - T(0, z)$ and relative humidity $f'(0, z)$ of the flowing air mass. The smaller is the difference $T' - T$, the larger is the water content of fog at f'=const: temperature T increases and water content decreases at advection of heat ($T' - T > 0$) and vise versa at advection of cold: T decreases and water content increases.

The water content of a fog at prescribed advection ($T' - T$ = const) increases as f' increases due to advection of water vapor.

The heat and water vapor fluxes included in the boundary conditions depend on the temperature difference $T_{10} - T_0$. Naturally, the conditions of formation of fog depend on this difference. Comparison of the values of water content in Tables 1 and 2 at the same $T' - T$ and f' makes it possible to conclude the following: the passage from decrease to increase (inversion) of temperature with height is accompanied by a significant decrease of the fog intensity. Fog was not formed in 1 hour in the presence of inversion. In all other cases the water content of fog at inversion was essentially lower than at the decrease of temperature with height. Then, at $T' - T = 2$°C and $f' = 0.95$, 0.90 and 0.85 the water content of fog in $t = 2$ h is, respectively, 0.142, 0.084, and 0.026 g/kg at inversion and 0.204; 0.146, and 0.089 g/kg at decrease of temperature.

According to data from Table 2, the dependence of water content on the difference $T' - T$ proportional to the advection of heat and f' is the same as that according to data from Table 1: the increase of this difference and decrease of f' are accompanied by the decrease of intensity (water content) of fog.

Table 2. Water content of fog (g/kg) near ground at $T_{10}(0) - T_0(0) = 0.1$°C and the same parameters as in Table 1

$T'(z,0) - T(z,0)$, °C	$f'(0,z)$	Time, hours			
		2	3	5	7
0	0.95	0.204	0.338	0.538	0.671
	0.90	0.153	0.230	0.320	0.364
	0.85	0.102	0.130	0.100	0.057
2	0.95	0.142	0.212	0.272	0.290
	0.90	0.084	0.093	0.025	–
	0.85	0.026	–	–	–
5	0.95	0.071	0.056	–	–
	0.90	0.001	–	–	–
	0.85	–	–	–	–

The calculated results on the water content of fog for the same values of the parameters as in Table 1 are shown in Table 2, except one parameter – the difference $T_{10}(0) - T_0(0)$: in Table 1 this difference is equal to −0.06°C (temperature decreases with height), while in Table 2 $T_{10}(0) - T_0(0) = 0.1$°C (temperature increases with height).

The data of Table 3 make it possible to estimate the effect of relative humidity $f_0(0)$ at the initial moment on the water content of fog near ground in addition to the vertical distribution of temperature: the water content at a fixed difference $T_{10} - T_0$ increases as $f_0(0)$ increases, the increase is more significant at consequent time moments (5–7 h) in comparison with the first (1–2 h) hour. However, the difference $T_{10}(0) - T_0(0)$ stronger affects the water content than $f_0(0)$: the water content at a fixed $f_0(0)$ decreases (especially sharply in 1 and 2 hours) at passage from the decrease of temperature with height to inversion.

Table 3. Water content of fog (g/kg) near the ground surface at $T_0(0) = 15$°C; $T_* = 20$°C, $c_{10} = 0.5$ m/s, Ro = $4 \cdot 10^3$, $z_0 = 0.1$ m at the absence of advection

$T_{10}(0) - T_0(0)$, °C	$f'(0,z)$	Time, hours				
		1	2	3	5	7
−0.06	0.95	0.087	0.349	0.559	0.958	1.340
	0.90	0.085	0.330	0.505	0.775	1.048
	0.85	0.084	0.310	0.451	0.636	0.751
0.10	0.95	0.029	0.285	0.495	0.886	1.288
	0.90	0.028	0.265	0.441	0.722	0.994
	0.85	0.026	0.245	0.387	0.583	0.711

According to Table 4, roughness of the ground surface essentially affects the water content of fog: the increase of the parameter z_0 is accompanied by the decrease of the water content of fog. Then, at $t = 3$ h the water content decreases from 0.957 to 0.069 g/kg as z_0 increases from 0.1 to 0.4 m. It is explained by the fact that the emission of water vapor from the ground surface increases as roughness increases.

The revealed dependence of the water content on z_0 makes it possible to indicate one of the factors of decrease of the frequency of occurrence of fogs and hazes in a city in comparison with surrounding rural area. The parameter of roughness in big cities is essentially greater than that in rural area (0.3–1.0 m against 0.01–0.1 m).

Table 4. Water content of fog (g/kg) near the ground surface at $T_0(0) = 15$°C; $T_* = 21$°C, $f(0, z) = 0.9$, $T_{10}(0) - T_0(0) = 0.2$°C, Ro = $4 \cdot 10^3$, $T'(0, z) - T(0, z) = -5$°C and different values of the roughness parameter z_0

Time, hours	Parameter z_0, m					
	0.1	0.2	0.3	0.4	0.5	0.6
1	0.308	–	–	–	–	–
2	0.669	0.214	–	–	–	–
3	0.957	0.485	0.233	0.069	–	–
5	1.402	0.817	0.536	0.355	0.226	0.128
7	1.619	0.957	0.650	0.453	0.312	0.207

Let us present a different estimate of the effect of soil properties on the intensity of fogs. According to Table 5, temperature distribution in soil plays an essential role in the formation of fogs. Ground surface is cooled under the effect of the molecular flux of heat at temperature decrease with depth [$T_* < T_0(0)$] and is heated at its increase [$T_* > T_0(0)$].

Table 5. Water content of fog (g/kg) near the ground surface at different $T_* - T_0(0)$ and fixed $T_0(0)$ = 15°C; $f(0, z)$ = 0.9, $T_{10}(0) - T_0(0)$ = 0.1°C, Ro = 6 · 10^4, $T'(0, z) - T(0, z)$ = −2°C, z_0 = 0.1 m, λ = 15 kJ/(m·s·K)

$T_* - T_0(0)$, °C	Time, hours				
	1	2	3	5	7
−5	0.802	0.785	0.706	0.622	0.579
0	0.473	0.430	0.374	0.296	0.256
5	0.141	0.092	0.038	−	−

Hence, the water content of the fog formed in the first case is significantly greater than in the second. Thermal conductivity of soil, which depends not only on the type but also on humidity and porousness of soil, has the same influence.

The calculated results on the vertical distribution of water content of fog are shown in Table 6 for two values of the thermal conductivity coefficient. As soil temperature decreases with depth ($T_* - T_0$ = −4°C), the increase of λ means the increase of the heat flux deep into soil and more significant decrease of temperature of the ground surface. Hence, the water content of fog at larger values λ is a few greater than at smaller λ. The λ values taken for calculation of Table 6 are related to solid components of soil. The value λ sharply decreases and the effect of this parameter on the water content of fog increases as air (porousness) or water (humidity) in soil composition increase, whose thermal conductivity is, respectively, 100 and 5 times less than that of solid components.

Table 6. Vertical distribution of the water content of fog (g/kg) at two values of the thermal conductivity coefficient λ and fixed $T_0(0)$ = 6°C, $f(0, z)$ = 0.9, $T_{10}(0) - T_0(0)$ = 0.1°C; Ro = 6 · 10^4, z_0 = 0.1, $T_* - T_0(0)$ = − 4°C; $T'(0, z) - T(0, z)$ = −2°C; λ = 0.08 (*a*) and 15 J/(m·s·K) (*b*)

Height, m	T = 1 h		t = 2 h		t = 5 h	
	a	*b*	*a*	*b*	*a*	*b*
0.1	0.494	0.557	0.813	0.857	1.266	1.273
11	−	−	0.314	0.338	0.813	0.816
23	−	−	0.163	0.180	0.672	0.674
36	−	−	0.082	0.095	0.594	0.596
51	−	−	0.030	0.040	0.544	0.547
67	−	−	−	0.004	0.509	0.510
85	−	−	−	−	0.484	0.485

Tables 1–4 are calculated taking into account the effect of different factors except for the effective radiation of the ground surface. Comparison of Tables 5 and 6, in which the effect of the latter factor also was taken into account with the data of Tables 1–4 shows that radiation certainly affects the formation of fogs. However, first, the fog is also formed without taking into account radiation (and its water content is comparable with that observed in nature) and, second, effective radiation of the ground surface sharply decreases as the water content of a fog increases. The results of calculation of the near-ground water content of fog at the time moment t = 2 h are presented in Table 7 assuming that the effective radiation has decreased to the value $B^*(1) = \alpha B^*(0)$ in comparison with the initial moment due to fog formed by the moment t = 1 h. The water content decreases even at α = 0.50 by 3 to 5 times in comparison with its value which would be observed at 2 h if the initial effective radiation have decreased. Fog remains at α = 0.4 at advection of cold and without it. Radiation in this case practically does not show any effect, because the values of water content are the same as at complete exclusion of the radiation effect.

Table 7. Near-ground water content of fog (g/kg) at t = 2 h, different values of the parameter $\alpha = B^*(1)/B^*(0)$ and fixed $T_{10} - T_0$ = 0.1°C, $T_* - T_0$ = −5°C, $f(0, z)$ = 0.9, Ro = 6 · 10^4, c_{10} = 0.3 m/s

$T'(0, z) - T(0, z)$, °C	α			
	1.00	0.50	0.40	0.35
−2	1.864	0.494	0.238	0.110
0	0.772	0.365	0.104	−
2	1.696	0.255	−	−

For conclusion let us note some general peculiarities of fog formation. The following factors affect the water content, time of the beginning of formation, and thickness of fog (in order of significance):

− advective influxes of heat and water vapor;

− vertical turbulent fluxes of heat and water vapor coming to the roughness layer from the atmosphere;

− influxes of heat into the roughness layer from soil;

− radiative fluxes of heat from the ground surface.

One should especially emphasize that fog formation depends on temporal variations of not only temperature of the underlying surface, but also of the water content of the roughness layer (it was assumed in all earlier papers that the water content is constant in time). Although many factors affect the water content of fog, however, fog can be formed and become stronger only at combination of several factors. Since the probability of such combination is low, then the probability of formation of fogs is not high. It is essentially lower than the probability of formation of clouds, for example.

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