SIGNAL-TO-NOISE RATIO OF PULSED OPTICAL SOUNDING OF TARGETS THROUGH A SCATTERING MEDIUM

A.P. Ivanov and A.I. Kolesnik

Institute of Physics, Academy of Sciences of Belorussian SSR, Minsk
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This paper presents a theoretical analysis of the signal-to-noise ratio (SNR) for pulsed optical sounding of targets in a turbid medium as a function of the primary optical parameters and type of noise, external background intensity, and the parameters of the transmitter-receiver. The calculations were made using the small-angle diffusion approximation and taking into account temporal distortions of the pulse shape in a scattering medium. Also the conditions are determined under which different types of noise become significant in forming the signal-to-noise ratio δ as a function of range. It is shown that in the case of typical values of the sounding pulse power the decrease of δ with increase in the range is caused by transmission losses stet significant increase of the initial pulse energy of the sounding pulse the decrease of δ is due to a decrease in the recorded contrast. In the former case the value of δ strongly depends on the probability of photon survival, while in the latter it depends on the assymetry of the scattering phase function in the forward direction. Simple expressions are derived for estimating the value of δ in limiting cases of short and long distances.

The signal-to-noise ratio δ is one of the basic parameters of a sounding system which characterizes its efficiency of operation, and it has been widely analyzed in the literature (see, e.g., Refs. 1–5). However, practical calculations require, as a rule, additional information on the values entering into the general relationships for δ. In the present paper we present a theoretical analysis of δ for pulsed optical sensing of targets in turbid media as a function of the optical properties of the medium, the parameters of the transmitter-receiver, and the external background level.

The idea of optical ranging is quite simple. A short light pulse is sent into a medium. A receiver located near the transmitter collects and records the intensity of light scattered by the medium. This backscattered light is frequently called the backscattering noise (BSN). Radiation reflected from a target at a certain range produces a peak in the BSN envelope. The position of the peak allows one to determine the range to the target. But if the target is too far from the sounding device, it becomes not so simple to detect the signal from it due to the influence of noise.

Let the photoelectronic recording system be a direct detection system operating in the analog regime. Since the post-detector electronics can not increase the SNR[1], we shall calculate the maximum δ values which can be achieved at the photocathode assuming that noise in the post-detector electronics are much lower. A measure of the photodetector noise can be obtained from the rms deviation of the number of photoelectrons $\sqrt{D}$ detected during the detector's rise time Δt_r^2. Here D is the variance of the number of electrons, and $\Delta t_r = 1/\Delta f$, where Δf is the frequency bandwidth of the recording electronics. Of the variety of noise types one can distinguish four groups[1,2].

The first group includes the noise sources which do not depend on the number of photocounts detected, or dark current noise. In this case $D_1 = n_d \Delta t_r$, where n_d is the counting rate of the dark current photoelectrons. In the other words, the variance of the dark noise is equal to the number of dark noise photocounts recorded during the rise time Δt_r.

The second group of noise sources involves those types of noise whose variance is proportional to the number of photocounts recorded during the rise time Δt_r, or $D_2 = \alpha n_{ph} \Delta t_r$, where n_{ph} is the mean number of electrons produced by the photons incident on the cathode per unite time. Shot noise is an example of this kind of noise. It can be approximately assumed that this noise obeys Poisson statistics, wherefore $\alpha = 1$. For other statistics α differs from unit. In our further analysis of the influence of this type of noise on δ, only shot noise will be taken into account since it is as most typical.

Let us consider now the noise sources appearing in the system as a whole, including the light source, the scattering medium, and the photodetector. These noise terms exist due to, e.g., fine temporal structure of light pulses, instabilities in optical properties of the medium along the sounding path, temporal variability of the detector sensitivity and also due to induction. Since the behavior of these noise term can be different, the derivation of an expression for 8 must be carried out separately for each particular case. To continue our noise classification we shall point out limiting conditions under which two types of noise are most important.

Thus the third group of noise terms includes the high frequency noise observed when the rise time of the predetector part of the system is much smaller than that of the receiver ($\Delta t_{pds} \ll \Delta t_r$). Here Δt_{pds} is the typical period of variation of the response of the system, which results in fluctuations in the number of recorded photons. It should be noted that not all the fluctuations are observed, due to the finite rise time of a detector. It can be shown that the variance of this type of noise can be written as follows $D_3 = K_{hf}^2 \Delta t_{pds} n_{ph}^2 \Delta t_r$, where $K_{hf} = (\bar{n}_{hf}^2 / n_{hf}^2)^{1/2}$ is the high frequency coefficient of variation of the number of photoelectrons, and $\bar{n}_{hf}^2$ is the variance of the high frequency fluctuations of the photoelectrons recorded during the rise time Δt_r.

The fourth group of noise terms include the low frequency noise observed at $\Delta t_{pds} \gg \Delta t_r$. In his case the variance $D_4 = K_{lf}^2 n_{ph}^2 \Delta t_r^2$, where K_{lf} is the low frequency coefficient of variation of the number of photoelectrons.

As was already mentioned, information about target in the medium is extracted by comparing two signals at nearby moments, i.e. the BSN signal and the signal from the target. In this situation, the variance of the i-th noise terms is given by $D_i = D_i^{BSN} + D_i^t$, where D_i^{BSN} and D_i^t are the variances corresponding to the above signals.

For statistically independent noise terms one obtains that

$$\delta = \frac{n_e \Delta t_r}{(D_1 + D_2 + D_3 + D_4)^{1/2}} = \frac{1}{[(1/\delta_1)^2 + (1/\delta_2)^2 + (1/\delta_3)^2 + (1/\delta_4)^2]^{1/2}} \quad (1)$$

where n_e is the number of electrons produced by the photons arriving from the target per second;

$$\delta_1 = \frac{n_c \Delta t_r}{\sqrt{D_1}}, \quad \delta_2 = \frac{n_c \Delta t_r}{\sqrt{D_2}}, \quad \delta_3 = \frac{n_c \Delta t_r}{\sqrt{D_3}}, \quad \delta_4 = \frac{n_c \Delta t_r}{\sqrt{D_4}}$$

are the signal-to-noise ratios due to corresponding noise terms.

It can be shown that

$$\delta_1 = \xi K A \eta \sqrt{N \Delta t_r} \quad (2)$$

$$\delta_2 = K \sqrt{A \eta N \Delta t_r} \quad (3)$$

$$\delta_3 = \frac{K}{K_{hf}} \sqrt{\frac{N \Delta t_r}{\Delta t_d}} \quad (4)$$

$$\delta_4 = K/K_{lf}, \quad (5)$$

where $\xi = \sqrt{e / 2i_d}$; N is the total number of photo recorded;

$$A = 2 W S_\lambda S_r \omega_r / e \quad (6)$$

is the energetic parameter of the transmitter receiver;

$$K = \frac{\bar{B}_{S,max} + G\bar{B}_{BSN} - \bar{B}_{BSN}}{\bar{B}_{S,max} + G\bar{B}_{BSN} + 2\bar{B}_{BGD} + \bar{B}_{BSN}} \quad (7)$$

is the contrast of a signal from the target at the photodetector;

$$\eta = \frac{\bar{B}_{S,max} + G\bar{B}_{BSN} + 2\bar{B}_{BGD} + \bar{B}_{BSN}}{2W} \quad (8)$$

is the coefficient of energy transmission; W is the energy of the sounding pulse; S_r and ω_r are the area and solid angle of the receiver respectively; i_d and S_λ are the dark current and spectral sensitivity of the photodetector; e is the electron charge; $\bar{B}_{BSN}$, $\bar{B}_{S,max}$, $\bar{B}_{BCD}$ are the brightness of backscatter, of the target at signal maximum, and of the external background, respectively, averaged over the time interval Δt_r (the detector rise time), the receiver area S_r and the receiving angle W_r; G is a coefficient characterizing the decrease of the BSN level by at the moment of maximum signal from the target due to shadowing of the sounding beam (it can be shown that G is about 0.7 for the case of an infinite screen and tends to unity for small screens).

Before analyzing the behavior of $\delta(\tau)$, we will consider some methodological questions.

A general description of the expressions for B_{BSN} and $B_{S,max}$ entering into Eq. (7) and Eq. (8) can be found, in Ref. 6. Formulas for B_{BSN} have been derived in Ref. 7 and for B_S in Ref. 8 using the results in Ref. 9 and the reciprocity theorem (Ref. 10). As has been shown in Ref. 6, the temporal behavior of a signal reflected from a target B_S can be described by the sum of three gamma-distributions, designated as $R_1(t)$, $2\eta_1 R_2(t)$ and $\eta_\perp^2 R_3(t)$. The physical meaning of these three terms are as follows. The first term describes the light directly transmitted over the path transmitter-target-receiver, the second term describes that portion of the light scattered by the medium on its way to the target which upon reflection by the target then reaches the receiver directly, as well as that portion of light which reaches target directly and is then scattered on its way to the receiver, and the third term describes the portion of light scattered on both shoulders of the path. Analysis shows that the maximum of the function $2\eta_1 R_2(t)$ is always lower than the maxima of the other two functions, and that at $\tau > 5$ $\eta_1^2 R_{3,max} > R_{1,max}$. Thus in the range $\tau > 5$, which is of practical importance for pulsed ranging, we shall describe the signal reflected from a target by the term $\eta_1^2 R_{3,\max}$.

We can begin our analysis by noting that the influence of the characteristics of the receiver and transmitter parameters which enter into the energetic constant A on δ is quite obvious from expressions (2) and (3). These expressions also involve the parameter Δt_r, whose increase (in other words, the decrease of the frequency transmission band of the photodetector) results in an increase, to a certain extent, in the value of δ, due to the concomitant decrease in the dark and shot noise. But further increase of Δt_r lowers the value of the contrast.

Therefore there exists a certain optimal value of Δt_r which provides for maximum signal-to-noise ratio. Calculations show that under the condition

$$\Delta t_{tr} \leq \Delta t_r \approx \Delta t_m \tag{9}$$

the value δ reaches its extremum. Here Δt_m is the characteristic response time of the medium to δ-pulse initiation for the case when the light is first scattered by the medium, is then reflected by the target, and is then collected by the receiver. This response time is due to the differences in the path lengths which the photons travel from the transmitter to the receiver via the target. The finitude of Δt_m is due to the spread of times of passage of the photons in the medium, wherefore $\Delta t_m = D_m^{1/2} / \varepsilon c$, where D_m is the variance of the photon paths and c is the velocity of light. As follows from expression (9) in order to obtain the highest value of the signal-to-noise ratio one has to record all the energy arriving from the target at the receiver aperture S_r within the receiving solid angle ω_r. Using the results in Ref. 11 one can calculate Δt_m for media with different τ and other optical properties such as Λ and $\overline{\gamma}^2$ and for different target radii R. Here $\Lambda = \sigma/\varepsilon$ is the probability of photon survival, σ is the scattering coefficient,

$$\overline{\gamma}^2 = \int_0^\infty x(\gamma)\gamma^2\gamma d\gamma \Big/ \int_0^\infty x(\gamma)\gamma d\gamma$$

is the mean square scattering angle, and $x(\gamma)$ is the scattering phase function. Table 1 presents the values $\sqrt{D_m} = \varepsilon c \Delta t_m$ calculated for R = 15 cm and R $\to \infty$ for different situations. It is seen from the table that the duration of a light pulse reflected from a target increases with the increase of any of the parameters τ, Λ or $\overline{\gamma}^2$, which is obvious from a physical point of view.

Additional calculated results were obtained for cases satisfying condition (9). However, we should like to note that violation of the relationship $\Delta t_r = \Delta t_m$ does not have any essential effect on the value of δ.

Table 1.

Values $\sqrt{D_m}$ */

Λ	$\overline{\gamma}^2$	R=0.15 m, τ = 5	10	20	40	$R\to\infty$, τ = 5	10	20	40
0.6	0.03	4.0-3	2.5-2	3.1-1	8.7-1	1.3-1	4.5-1	1.32	2.61
	0.06	5.7-3	3.5-2	5.5-1	1.7	2.5-1	8.1-1	1.96	3.23
	0.1	7.24-3	4.4-2	7.5-1	2.0	3.9-1	1.2	2.4	3.7
0.7	0.03	5.6-3	4.3-2	3.3-1	1.0	1.48-1	5.4-1	1.62	3.31
	0.06	7.9-3	6.2-2	6.7-1	1.8	2.9-1	9.7-1	2.4	4.1
	0.1	1.0-2	7.8-2	1.1	2.5	4.6-1	1.4	3.1	4.8
0.8	0.03	7.7-3	7.1-2	3.6-1	1.2	1.7-1	6.3-1	2.0	4.5
	0.06	1.1-2	1.1-1	6.9-1	2.2	3.3-1	1.2	3.2	5.7
	0.1	1.4-2	1.4-1	1.1	3.2	5.4-1	1.8	4.1	6.6

*/ 4.0-3 signifies $4.0\cdot10^{-3}$

Since in our case the detector records all the light energy arriving from a target within S_r and ω_r, the value of $\overline{B}_{S,\max}$, which enters into formulas (7) and (8) is found by solution of a similar problem with a continuous light source emitting the light flux F, i.e.

$$\overline{B}_{S,\max} = B_{S,\mathrm{cont}}\,\Delta t_s/\Delta t_r$$

where $B_{S,\mathrm{cont}}$ is the brightness of the target, illuminated by continuous wave radiation, as seen by the receiver. This value has been calculated in Ref. 8 using the small-angle diffusion approximation.

Let us assume that the optical axes of the receiver and transmitter coincide and are perpendicular to the screen, and that the viewing angles of the transmitter and receiver are equal to 0.5°.

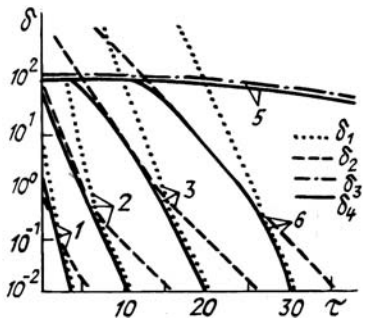

Fig. 1. Data illustrating the influence of different types of noise on δ. Here the value of A is 3.5 m^2 *(curves 1);* 3.5×10^4 m^2 *(curves 2);* 3.5×10^8 m^2 *(curves 4); and $A \to \infty$ (curves 5).*

As a rule, the contributions of different types of noise to the total noise level are not equivalent and only one of them determines the resulting value of δ. Below, on the basis of the data presented in Fig. 1, we carry out a qualitative estimation of the conditions under which one of the noise terms can dominate over the others. We consider a medium for which $\Lambda = 0.7$ and $\overline{\gamma}^2 = 0.06$. An infinite screen is assumed with brightness coefficient $\rho = 0.2$, $K_{\mathrm{lf}} = 0.01$, $\xi = 0.009$ c$^{1/2}$. The case is considered of no external background. Since the behavior of $\delta_3(\tau)$ and $\delta_4(\tau)$ is similiar, assuming that $K_{\mathrm{hf}} = K_{\mathrm{lf}}$ and taking into account that for high frequency fluctuations $N\Delta t_{\mathrm{r}} \gg \Delta t_{\mathrm{m}}$, for high light power levels one has $\delta_3 \gg \delta_4$ and $\delta = \delta_4$. Therefore here and below for the case of unbounded increase of the energy of the light source we shall consider only the value $\delta = \delta_4$, because, normally, only low frequency noise determines the range at which a target can be detected. The solid curves in the Figure represent the behavior of the resulting δ value for different A values. The other curves represent the δ_1 values. In the regions where $\delta = \delta_1$ two close curves are plotted, from Fig. 1 it is seen that the behavior of the curves $\delta_1(\tau)$ are different. The quantity δ_1 has the most rapid falloff with growth of τ. The lowest decrease with τ is observed for δ_4. This is explained by the fact that δ_1, δ_2, and δ_4 are proportional to $K\eta$, $K\sqrt{\eta}$ and K, respectively, were K and η decrease with increasing optical depth. As to the contrast, it remains almost constant in the region of small τ values, while the energy transmission coefficient rapidly decays. For this reason δ_4 is also constant at small τ values, while δ_4 varies strongly.

For small values of the energetic parameter A, the value of δ is determined by the dark noise. For larger A values (solid curve 2) first shot noise dominates at small optical depths, and then at larger τ dark noise become more important. Further growth of the sounding pulse energy leads to a situation in which the resulting function δ(τ) (solid curves 3, 4) is described by δ_4 in its initial portion, then by δ_2 in the middle τ range and by δ_1 in the range of large τ values. All this can be explained by the fact that energy arriving at the receiver from a target decreases with increase of the distance to it. The alteration of the roles of the different types of noise explains the growth of the rate of falloff of δ(τ) at larger τ values. Solid curve 5 in Fig. 1 corresponds to $A \to \infty$ when $\delta = \delta_4$ at any optical depth. This curve shows the maximum possible signal-to-noise ratios achievable for the medium and target under consideration. It is pertinent to note that in the case of low dark currents (this is the situation with $\xi = 0.009$ c$^{1/2}$) $\delta = \delta_1$ only if the signal is lower than the noise. Since in most cases the probability of target detection must be quite high, which is possible only if $\delta > 1$, in our further discussion the value δ_1.

Let us now consider the influence of a screen (target) radius, its brightness coefficient ρ, the optical properties (Λ, $\overline{\gamma}^2$) of the medium, and the optical depth of the path from the transmitter to the screen on the values δ_2 and δ_4.

It can be shown that at small distances, when $\bar{B}_{\mathrm{BSN}}$ □ $\bar{B}_{\mathrm{S,max}}$,

$$\delta_2^{\mathrm{BGD}} = \sqrt{K}\,\delta_2 \tag{11}$$

where δ_2^{BGD} and δ_2 are signal-to-noise ratios with external background light and without it, respectively. If the screen size is much smaller than the sounding beam, blurring due to scattering, when the useful signal is determined only by direct radiation reflected from the screen during the interval $\Delta t_{\mathrm{r}} = \Delta t_{\mathrm{s}}$, then

$$\delta_2 = e^{-\tau}\sqrt{2WN\rho S_\lambda \omega_r/\pi e} \tag{12}$$

$$K = \left[1+2\bar{B}_{\mathrm{BGD}}/\bar{B}_{\mathrm{S,max}}\right]^{-1} = \left[1+e^{2\tau}S_r\bar{B}_{\mathrm{BGD}}\Delta t_s\pi/W\rho\right]^{-1} \tag{13}$$

These formulas were obtained assuming that direct radiation from source is intercepted by the screen. Expressions (11) to (13) show that δ does not depend upon the optical characteristics Λand $\overline{\gamma}^2$ of the medium. As the contrast diminishes with the growth of τ the difference between δ_2 and δ_2^{BGD} becomes stronger.

In the case when the contribution of scattered light to the illumination of the screen is quite high one can write down the following expression for δ_2 based on the results in Ref. 8:

$$\delta_2 = \left\{\frac{WS_r\omega_r S_\lambda\rho\varepsilon^2}{4\pi^2 e(f_1+f_2)}\exp\left[-2(1-\Lambda)\tau\right]\times\left[1-\exp\frac{-R^2\varepsilon^2}{f_1+f_2}\right]\right\}^{1/2}, \tag{14}$$

and for $\bar{B}_{\mathrm{S,max}}$

$$\bar{B}_{\mathrm{S,max}} = \frac{F\varepsilon^2\rho}{4\pi^2(f_1+f_2)}\exp\left[-2(1-\Lambda)\tau\right]\left[1-\exp\frac{-R^2\varepsilon^2}{f_1+f_2}\right]\Big\}, \tag{15}$$

where $f = \varepsilon^2 r^2/4$, r is initial radius of the sounding beam, and $f_2 = \tau^3\Lambda\bar{\gamma}^2/6$. These formulas show that Λ and $\bar{\gamma}^2$ determine the values δ_2 and $\delta_2^{\rm BGD}$. Since δ_4 coincides to a constant factor $1/K_{\rm lf}$ with K, expressions (13) and (15) clearly demonstrate the dependence of δ_4 on all the parameters at low τ values.

A general picture of δ2(τ) and δ4(τ) is presented in Fig. 2. Here solid lines represent δ under conditions of no background illumination, and dashed lines represent 6 for the case of a background with $\bar{B}_{\rm BGD}$ = 0.02 W m–2. In the calculations of δ2 the parameter A = 3.5×1012 m2 was used, which, which corresponds to W = 1 J, $S\lambda$ = 6.5 × 10–2 A W–1, Sr = 0.07 m2, and ωr = 3.6 × 10–5sr. Albeit only for this A value and small optical depths δ = δ4, the functions δ2(τ) are plotted starting at τ = 0 in order to present a complete picture. In the calculations of δ4 we used the value Klf = 0.01. Since in this case $A \to \infty$, the background radiation does not affect the signal-to-noise ratio.

The influence of the screen size on the SNR is illustrated by the curves plotted in Fig. 2a and 2a′. It is seen from these figures that a screen with R ~ 15 m can be considered to be infinite in extent. Since at low optical depths the signal power is greater than that of the backsground, δ2 = $\delta_2^{\rm BCD}$. However, under the conditions under consideration, differences between δ2 and $\delta_2^{\rm BCD}$ become noticeable at τ ~ 10 and increase with further increase of τ, the relative difference between them being almost independent of R.

Figures 2b and 2b′ represent the function δ = f(τ) plotted for different values of the screen brightness coefficient. In a wide range of δ the magnitude of δ2 is proportional to $\sqrt{\rho}$ since K is practically constant (close to unity), while δ2 is determined by $\sqrt{\eta}$ which in turn is proportional to $\sqrt{\rho}$. On the other hand, δ4 more strongly depends on ρ since δ4 ~ K, which decreases proportionally to the decrease of the albedo of the reflecting screen when the reflected signal is comparable with the level of backscattering noise.

The influence of the optical properties of the medium can be also seen from the data presented in Figs. 2c and 2c′. Thus, for example, the growth of the probability of photon survival results in an increase in δ2 and δ4 since the fraction of light arriving from the screen increases. The same occurs with increase of asymmetry of the scattering phase function $x(\gamma)$ in the forward direction (decrease of $\bar{\gamma}^2$). At the same time it should be noted that $\bar{\gamma}^2$ has a greater influence on δ4, while Λ has a greater effect on δ2. This can be explained by the fact that δ4 is determined by the contrast which is mainly determined by the shape of the scattering phase function, i.e., by the relationship between the fluxes scattered in the forward and backward directions. When the shot noise becomes more significant, δ2 depends significantly on the energetic parameter, which in turn depends more strongly on Λ than on $\bar{\gamma}^2$.

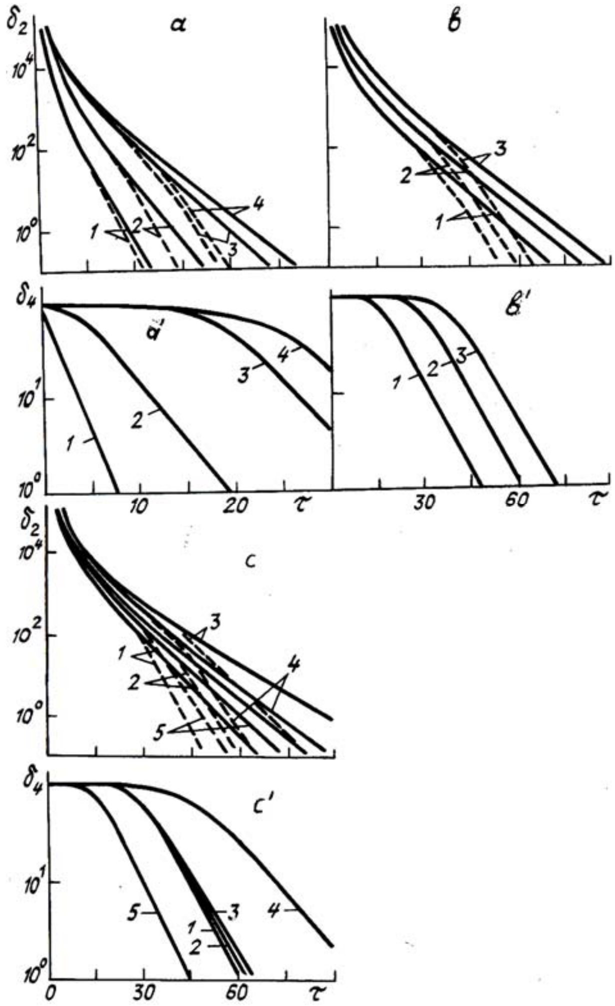

FIG. 2. The functions δ(τ) and δ₄(τ) calculated for different properties of the screen and medium.

FIGs. 2a and 2a′ represent the data calculated using $\bar{\gamma}^2$ = 0.06, ρ = 0.2, Λ = 0.7, R = 0.15 m (curves 1), R = 1.5 m (curves 2), R = 15 m (curves 3) and R → ∞ (curves 4).

FIGs. 2b and 2b′ are calculated for R → ∞ $\bar{\gamma}^2$ = 0.06, Λ = 0.7; ρ = 0.03, 0.2 and 1 for curves 1,2 and 3 respectively δ = f(τ)

FIGs. 2c and 2c′ are for R → ∞ ρ = 0.2, Λ = 0.6, $\bar{\gamma}^2$ = 0.06 (curves 1); Λ = 0.7, $\bar{\gamma}^2$ = 0.06 (curves 2); Λ = 0.8, $\bar{\gamma}^2$ = 0.06 (curves 3); Λ = 0.7, $\bar{\gamma}^2$ = 0.03 (curves 4); Λ = 0.8, $\bar{\gamma}^2$ = 0.1 (curves 5).

Thus the results obtained in this paper allow one to estimate the limiting optical depths at which targets can be detected in turbid media.

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REFERENCES

1. S.B. Gurevich, *Theory and Design of Non-broadcasting TV systems* [in Russian], Energia, Leningrad (1970).
2. V.A. Smirnov, *Introduction to Optical Radio-electronics* [in Russian], Sov. Radio, Moscow (1973).
3. V.I. Vorob'ev, *Optical Ranging for Engineers* [in Russian], Radio i Sviaz, Moscow (1983).
4. I.M. Levin, Izvestia Akad. Nauk SSSR, Ser. Fiz. Atmosfery i Okeana **5**, 62 (1969).
5. D.M. Bravo-Zhivotovskii, L.S. Dolin, I.M.Levin, A.G. Luchinin, and V.A. Savel'ev, Izvestia Akad. Nauk SSSR, Ser. Fiz. Atmosfery i Okeana **7**, 1143 (1971).
6. A.I. Kolesnik and A.P. Ivanov, Izvestiya Akad. Nauk BSSR, Fiz. Matem., 53 (1986).
7. E.P. Zege, A.P. Ivanov, I.L. Katsev, et al., Izvestia Akad. Nauk SSSR, Sen. Fiz. Atmosfery i Okeana **9**, 1051 (1973).
8. A.I. Kolesnik and A.P. Ivanov, Dokl. Akad. Nauk BSSR **30**, 601 (1986).
9. L.S. Dolin, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **26**, No .3, 300 (1988).
10. A.P. Ivanov, *Optics of Scattering Media* [in Russian], Nauka i Tekhnika, Minsk (1969).
11. E.P. Zege, A.P. Ivanov and I.L. Katsev, Image Transfer in Scattering Media [in Russian], Nauka i Tekhnika, Minsk (1985).