On determination of optical characteristic profiles of troposphere aerosol from signals of Raman lidar and single-angle nephelometer

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Received July 8, 2007

A method for retrieving troposphere aerosol coefficients of extinction ε_a and backscattering $\beta_{a,\pi}$ from signals of elastic scattering at λ_0 = 355, 532, and 1060 nm, as well as Raman scattering by the atmospheric nitrogen (λ_R = 387, 607 nm) is proposed. The method does not require differentiation of the nitrogen atmospheric density logarithm and eliminates the uncertainty in the choice of the spectral function $\varepsilon_a(\lambda)$ through measuring aerosol light-scattering coefficients $\beta_{a,\theta}$ at an angle of 33° and wavelengths λ_0. The choice of angle of 33° is based on the analysis of multiple regressions between spectral values of ε_a and $\beta_{a,\theta}$ for the ground atmospheric aerosol. Examples of retrieval of $\beta_{a,\pi}$ and ε_a vertical profiles are presented; the effect of measurement errors and atmosphere temperature fluctuations on the accuracy in $\beta_{a,\pi}$ and ε_a determination is estimated.

Introduction

Investigation of distribution of aerosol optical and microphysical parameters along vertical and horizontal paths is of importance for solving problems of the atmospheric radiation balance, physics and chemistry of aerosol, as well as pollution transfer. The lidar sensing is the most efficient instrument in such investigations, since it provides for long-term continuous measurements with a good spatial and temporal resolution. Data of multiwave lidar measurements contain information not only about aerosol optical parameters, but also allow one to determine its microphysical parameters, namely, the concentration, size distribution function, and refractive index of the aerosol matter.

Recent improvement of the lidar technique has resulted in appearance of combined lidars, based on one of the basic atmospheric gases (nitrogen, for example), which combine possibilities of multiwave sensing and receiving Raman signals (RS). Among such Raman lidars, comparatively small systems, built on Nd:YAG lasers with frequency transformation into second and third harmonics (λ_0 = 1060, 532, and 355 nm), are of the most interest.[1-3] They allow 5-wave measurements with the use of two Raman signals (nitrogen, 387 and 607 nm) or 4-wave measurements with the use of only one Raman channel, for example, at λ = 607 nm.

The lidar equation for laser waves λ_0 can be written in the form[4,5]

$$P_{\lambda_0}(z)=\frac{A_0}{z^2}T_{\lambda_0}^2\left[\beta_{\mathrm{m},\pi}(\lambda_0,z)+\beta_{\mathrm{a},\pi}(\lambda_0,z)\right]\times$$
$$\times\exp\left\{-2\int\limits_{z_0}^{z}\left[\varepsilon_{\mathrm{m}}(\lambda_0,z')+\varepsilon_{\mathrm{a}}(\lambda_0,z')\right]\mathrm{d}z'\right\}, \qquad (1)$$

where $P_{\lambda_0}(z)$ is the signal power, stipulated by molecular and aerosol scattering; z_0 is the minimal distance from the lidar, where full overlap of the lidar field of view and the sent signal is reached; A_0 is the instrumental constant; $\beta_{\mathrm{m},\pi}(\lambda_0,z)$ and $\beta_{\mathrm{a},\pi}(\lambda_0,z)$ are the coefficients of molecular and aerosol backscattering; $\varepsilon_{\mathrm{m}}(\lambda_0,z)$ and $\varepsilon_{\mathrm{a}}(\lambda_0,z)$ are the coefficients of molecular and aerosol extinction at λ_0 wavelength;

$$T_{\lambda_0}=\exp\left\{-\int\limits_{0}^{z_0}\left[\varepsilon_{\mathrm{a}}(\lambda_0,\xi)+\varepsilon_{\mathrm{m}}(\lambda_0,\xi)\right]\mathrm{d}\xi\right\}$$

is the transparency of the lidar shadow zone. For the Raman wavelength λ_R, the lidar signal intensity, stipulated only by the molecular scattering, has the form[4,5]:

$$P_{\lambda_R}(z)=\frac{A_R}{z^2}T_{\lambda_R}N_R(z)\frac{\mathrm{d}\sigma_{\lambda_R}(\pi)}{\mathrm{d}\Omega}\times$$
$$\times\exp\left\{-\int\limits_{z_0}^{z}\left[\varepsilon_{\mathrm{m}}(\lambda_0,z')+\varepsilon_{\mathrm{a}}(\lambda_0,z')+\varepsilon_{\mathrm{m}}(\lambda_R,z')+\varepsilon_{\mathrm{a}}(\lambda_R,z')\right]\mathrm{d}z'\right\}, \qquad (2)$$

where

$$T_{\lambda_R}=\exp\left\{-\int\limits_{0}^{z_0}\left[\varepsilon_{\mathrm{a}}(\lambda_0,\xi)+\varepsilon_{\mathrm{a}}(\lambda_R,\xi)+\varepsilon_{\mathrm{m}}(\lambda_0,\xi)+\varepsilon_{\mathrm{m}}(\lambda_R,\xi)\right]\mathrm{d}\xi\right\};$$

$N_R(z)$ is the concentration of nitrogen molecules at the height z; $\mathrm{d}\sigma_{\lambda_R}(\pi)/\mathrm{d}\Omega$ is the backscattering

0235-6880/08/02 119-06 $02.00

differential cross section for nitrogen molecules; A_R is the instrumental constant.

Joint solution of (1) and (2) for spectral values of $\varepsilon_a(\lambda_0, z)$ and $\beta_{a,\pi}(\lambda_0, z)$ requires *a priori* assumptions or additional measurements.

First, it is necessary to know the relative spectral behavior of the light extinction by the aerosol component. Traditionally, the assumption $\varepsilon_a(\lambda) \sim \lambda^{-k}$ is used, where k, depending on the aerosol type, changes between 0 and 2 [Ref. 4]. When solving a system of equations relative to $\varepsilon_a(\lambda_0, z)$ and $\beta_{a,\pi}(\lambda_0, z)$, including one of the Raman signals (for example, at $\lambda_R = 607$ nm) and signals of elastic scattering at $\lambda_0 = 355$, 532, and 1060 nm, the uncertainty in the spectral behavior of $\varepsilon_a(\lambda)$ (in the absence of information on k) can result in significant errors in determining $\varepsilon_a(\lambda_0)$ at elastic scattering wavelengths of 355 and 1060 nm (at $\lambda_R = 607$ nm), and, consequently, in determining $\beta_{a,\pi}(\lambda_0)$. Thus, it is shown in Ref. 6 that when using one Raman channel (607 nm), the errors in retrieval of the profile $\beta_{a,\pi}(\lambda_0, z)$ at $\lambda_0 = 355$ and 1060 nm due to k uncertainty can exceed 100%.

Second, the information on the temperature profile is required (it can be set by some model of the atmosphere or received from aerologic measurements), because the relation for $\varepsilon_a(\lambda_0, z)$ determination includes operations of numerical differentiation of both the signals and logarithm of nitrogen density ratio,[4] calculated from profiles of the atmospheric temperature and pressure. As is known,[7] the solution of the numerical differentiation problem is not stable. This leads to increasing errors, caused by the difference between the actual temperature profile and the profile used in calculating $N_R(z)$, up to some physically absurd result.

Below, a method for retrieving the profiles of troposphere aerosol optical characteristics at wavelengths of the elastic scattering with the use of two Raman channels is described. The method does not require the calculation of nitrogen density derivative and excludes the uncertainty in the choice of spectral dependence of the aerosol extinction coefficient.

Method

To eliminate the necessity in calculation of the nitrogen density derivative, it is possible to use the ratio of Raman signals $P_{387}(z)$ and $P_{607}(z)$ at wavelengths of 387 and 607 nm, respectively, corrected with respect to the molecular extinction:

$$S_{\lambda_R}(z) = P_{\lambda_R}(z)\exp\left\{\int_{z_0}^{z}[\varepsilon_m(\lambda_0,\xi)+\varepsilon_m(\lambda_R,\xi)]\mathrm{d}\xi\right\}.$$

In this case, however, to retrieve the profiles of $\varepsilon_a(\lambda_0, z)$ ($\lambda_0 = 355$ and 532 nm), it is required to know relations connecting spectral values of $\varepsilon_a(\lambda_i, z)$ at $\lambda_i = 355$, 387, 532, and 607 nm, which in the very general form can be written as

$$\varepsilon_a(\lambda_i,z) = C_{\lambda i}\varepsilon_a(\lambda_0,z), \tag{3}$$

where $C_{\lambda i}$ are the bonding coefficients (the way of their determination will be described below). Supposing that the coefficients $C_{\lambda i}$ do not depend on z, the relation for $\varepsilon_a(\lambda_0, z)$ can be easily obtained from the ratio of Raman signals (2) at $\lambda_R = 387$ and 607 nm. So, at $\lambda_0 = 532$ nm

$$\varepsilon_a(532,z) = \frac{\mathrm{d}}{\mathrm{d}z}\ln F_{532}(z_0,z) =$$

$$= \frac{\mathrm{d}}{\mathrm{d}z}\ln\left[\frac{S_{387}(z)S_{607}(z_0)}{S_{387}(z_0)S_{607}(z)}\right]^{\frac{1}{1-C_{355}-C_{387}+C_{607}}}, \tag{4}$$

where

$$C_{\lambda i} = \varepsilon_a(\lambda_i)/\varepsilon_a(532); \quad \ln F_{532}(z_0,z) = \int_{z_0}^{z}\varepsilon_a(532,\xi)\mathrm{d}\xi$$

is the aerosol optical thickness within $[z_0, z]$ of the sensing path at $\lambda = 532$ nm (hereinafter the subscript at $S_{\lambda i}$, $F_{\lambda i}$, and $C_{\lambda i}$ corresponds to λ_i). The expression, similar to Eq. (4), can be easily obtained for $\varepsilon_a(355, z)$ as well. It will only differ in values of the coefficients $C_{\lambda i} = \varepsilon_a(\lambda_i)/\varepsilon_a(355)$ in the exponent in Eq. (4), equal in this case to $(C_{607} + C_{532} - C_{387} - 1)^{-1}$. It is seen from Eq. (4) that the accuracy in retrieving the $\varepsilon_a(\lambda_0, z)$ profile is independent of the nitrogen concentration profile; it is determined by Raman signal measurement errors and spatial variations of $C_{\lambda i}$. The resistance of Eq. (4) to $C_{\lambda i}$ scatter is considered below.

The relative spectral dependence of $\varepsilon_a(\lambda)$ is determined by the size distribution function, normalized by unit, and the complex index of refraction of the scattering particle matter, but not by concentration of particles. Data of numerous measurements of the tropospheric aerosol vertical structure[8] show that changes in parameters of the size distribution functions and complex index of refraction for particles of individual fractions are at the level of their random variations, while changes in aerosol optical characteristics are caused primarily by variations of particle concentrations. All this justifies the supposition on the invariance of $C_{\lambda i}$ values in troposphere.

As for the presence of noises in signals, many algorithms have been worked out to the present time, which allow an efficient smoothing of the measured signals and maximally eliminate effects of random errors on results of differentiating.[9–11]

Taking into account Eqs. (3) and (4), it is easy to obtain from Eq. (1) expressions for backscattering coefficients at elastic scattering wavelengths $\lambda_0 = 355$, 532, and 1060 nm:

$$\beta_{a,\pi}(350,z)=[\beta_{a,\pi}(355,z_k)+\beta_m(355,z_k)]\times$$

$$\times\frac{S_{355}(z)}{S_{355}(z_k)}\left[\frac{F_{532}(z_0,z)}{F_{532}(z_0,z_k)}\right]^{2C_{355}}-\beta_m(355,z),$$

$$\beta_{a,\pi}(532,z)=[\beta_{a,\pi}(532,z_k)+\beta_m(532,z_k)]\times$$

$$\times\frac{S_{532}(z)}{S_{532}(z_k)}\left[\frac{F_{532}(z_0,z)}{F_{532}(z_0,z_k)}\right]^{2}-\beta_m(532,z), \qquad (5)$$

$$\beta_{a,\pi}(1060,z)=[\beta_{a,\pi}(1060,z_k)+\beta_m(1060,z_k)]\times$$

$$\times\frac{S_{1060}(z)}{S_{1060}(z_k)}\left[\frac{F_{532}(z_0,z)}{F_{532}(z_0,z_k)}\right]^{2C_{1060}}-\beta_m(1060,z),$$

where

$$S_{\lambda_0}(z)=P_{\lambda_0}(z)\exp\left\{2\int_{z_0}^{z}\varepsilon_m(\lambda_0,\xi)d\xi\right\};$$

z_k is the point of calibration, where the value of $\beta_{a,\pi}(\lambda_0, z_k)$ is set *a priori*. Usually, the supposition is used that in the top troposphere layers the relation $\beta_a(\lambda_0,z) \ll \beta_m(\lambda_0,z)$, is true for visible and UV spectral ranges, therefore, the value of $\beta_{a,\pi}(\lambda_0, z_k)$ in Eq. (5) at $\lambda_0 = 355$ and 532 nm can be zero. The choice of $\beta_{a,\pi}(\lambda_0, z_k)$ value in the IR range ($\lambda_0 = 1060$ nm) is more complicated, because there the aerosol backscattering can significantly exceed the molecular one throughout the sensing path. In this case, the only way to determine $\beta_{a,\pi}(1060, z_k)$ is interpolation of spectral values for $\beta_{a,\pi}(355, z)$ and $\beta_{a,\pi}(532, z)$, calculated by Eq. (5).

As is seen from Eqs. (4) and (5), to determine profiles of aerosol optical characteristics, additional measurements for finding relative spectral behavior of $\varepsilon_a(\lambda)$ (values of $C_{\lambda i}$) are required. Spectral values of $\varepsilon_a(\lambda)$ can be found with the use of integral nephelometer, located close to the lidar. However, if there is an unambiguous connection between $\beta_{a,\theta}$ at angle θ and ε_a, then to find the latter, it is possible to use measurements of the scattered light at the same θ. In this case, $\varepsilon_a(\lambda)$ values can be determined immediately on the sensing pulse way (sensing path). To do this, the signals, scattered at the angle θ, should be measured in the presence of additional photoreceiver, located immediately close to the lidar.

Thus, a sufficiently good correlation between ε_a and $\beta_{a,\theta}$ is found theoretically and experimentally for $\lambda = 500$ nm. The mean square deviation for $\rho = \beta_a(45°)/\varepsilon_a$ does not exceed 15% of its mean value of 1.45 [Ref. 12]. Similar regression dependences for other wavelengths are unavailable in the literature. In particular, of interest is a possibility to obtain $\varepsilon_a(\lambda_i)$ values at $\lambda_i = 355$, 387, 532, 607, and 1060 nm, used in sensing the atmosphere by lidars operating at frequencies of elastic and Raman scattering.

Formulate the problem: express the $C_{\lambda i}$ values from Eqs. (4) and (5), determining $\varepsilon_a(\lambda)$, as functions of several parameters, being easy to measure, i.e., spectral values of $\beta_{a,\theta}(\lambda)$. The term "function" is meant here as statistical dependence (not determinate one).

Suppose that we deal with three aerosol fractions: fine, moderate, and coarse, which are designates by indices $j = 1, 2, 3$, respectively. Each fraction is characterized by some particle concentration in a unit volume N_j, complex index of refraction m_j, and the size distribution function $f_j(a)$, normalized by the unit. With the known light wavelength λ, we can calculate volume coefficients of extinction and directed scattering by Mie formulae[13]:

$$\varepsilon_a(\lambda)=\pi\sum_{i=1}^{3}N_j\int_0^{\infty}K_\varepsilon(m_j,a,\lambda)a^2f_j(a)da;$$

$$\beta_{a,\theta}(\lambda)=\pi\sum_{i=1}^{3}N_j\int_0^{\infty}K_\theta(m_j,a,\lambda)a^2f_j(a)da, \qquad (6)$$

where $K_\varepsilon(m_j, a, \lambda)$ and $K_\theta(m_j, a, \lambda)$ are the efficiency factors of extinction and scattering at the angle θ by a particle of the radius a.

Thus, the problem of determining $C_{\lambda i}$ in our case (the Nd:YAG laser as a radiation source) is reduced to obtaining relations of multiple regressions between $\varepsilon_a(\lambda_i)/\varepsilon_a(\lambda_0)$ (at $\lambda_0 = 355$ and 532 nm, $\lambda_i = 355$; 387; 532; 607; 1060 nm, $\lambda_i \neq \lambda_0$) and $\beta_{a,\theta}(355)/\beta_{a,\theta}(532)$ and $\beta_{a,\theta}(532)/\beta_{a,\theta}(1060)$, calculated by Eq. (6). To do this, the relation of the linear orthogonal mean-square multiple regression was used[14]:

$$\ln C_{\lambda i}=\ln\frac{\varepsilon_a(\lambda_i)}{\varepsilon_a(\lambda_0)}=a_{i0}+a_{i1}\ln\frac{\beta_{a,\theta}(355)}{\beta_{a,\theta}(532)}+a_{i2}\ln\frac{\beta_{a,\theta}(532)}{\beta_{a,\theta}(1060)}, \qquad (7)$$

where coefficients a_{ik} were determined theoretically through variations of fraction concentrations, real and imaginary parts of the refractive index, and parameters of the size distribution function. To find numerical values of a_{ik}, three-fraction model of the near-ground aerosol[15] by Andreev–Ivlev with varying values of meteorological visibility range S_m and humidity q within ranges of their values 5–100 km and 10–99%, respectively, was used. Since different combinations of microphysical parameters can correspond to the same S_m and q, while the spectral dependence $\varepsilon_a(\lambda)$ is determined uniquely by the latter, then, in addition to uniform varying of S_m and q, a 15% random scatter is imposed on all microphysical model parameters (distribution functions, indices of refraction, weight contributions of fractions). This allows to some extent accounting for errors and scatter of experimental data, which form the basis of the model.

The mean value for $\rho = \beta_a(45°)/\varepsilon_a = 1.39$ at $\lambda = 500$ nm, obtained by us on the base of model[15] and differing by only 4% from the reference one[12] (1.45), justifies the use of model from Ref. 15 for calculation of correlation bonds between extinction and directed scattering coefficients.

To choose the optimal angle (in terms of accuracy in finding $\varepsilon_a(\lambda_i)$) for receiving scattered radiation, the errors in determining $C_{\lambda i}$ from Eq. (7) were estimated for θ from the range 1÷180°. The coefficients a_{ik} were found by the least-squares method for each θ:

$$\min\sum_{n=1}^{S}\left[\ln\frac{\varepsilon_a^{(n)}(\lambda_i)}{\varepsilon_a^{(n)}(\lambda_0)}-a_{i0}-a_{i1}\ln\frac{\beta_{a,\theta}^{(n)}(355)}{\beta_{a,\theta}^{(n)}(532)}-a_{i2}\ln\frac{\beta_{a,\theta}^{(n)}(532)}{\beta_{a,\theta}^{(n)}(1060)}\right]^2,$$

where $n = 1, 2,\ldots, S$ is a realization of microphysical aerosol parameters with corresponding optical characteristics $\varepsilon_a^{(n)}(\lambda_i)$ and $\beta_{a,\theta}^{(n)}(\lambda_i)$; $S = 1.5\cdot 10^3$ is the sample volume.

Calculation of maximal and sample-volume mean errors in retrieving $\varepsilon(\lambda_i)$ from Eq. (7) shows the least errors to correspond to $\theta = 33°$. The coefficients in Eq. (7), corresponding to this angle at $\lambda_0 = 355$ and 532 nm, are shown in Tables 1 and 2, respectively.

Thus, to determine $\varepsilon_a(\lambda_i)$, it is necessary to measure scattering coefficients at angle 33° at wavelengths 355, 532, and 1060 nm and calculate $C_{\lambda i}$ from Eq. (7), using the corresponding coefficients a_{i0} from Tables 1 and 2.

Table 1. Coefficients of the regression equation (7) at λ_0 = 355 nm

λ_i, nm	a_{i0}	a_{i1}	a_{i2}
387	−0.0265	−0.1725	0.0004
532	−0.0703	−0.5563	−0.1423
607	−0.0846	−0.6498	−0.2456
1060	−0.2098	−0.5621	−0.8349

Table 2. Coefficients of the regression equation (7) at λ_0 = 532 nm

λ_i, nm	a_{i0}	a_{i1}	a_{i2}
355	0.0703	0.5563	0.1423
387	0.0438	0.3838	0.1427
607	−0.0143	−0.0935	−0.1033
1060	−0.1394	−0.0059	−0.6926

The accuracy in retrieving $\varepsilon_a(\lambda_i)/\varepsilon_a(\lambda_0)$ at λ_0 = 532 nm from spectral values of the scattering coefficient at angle 33° can be seen in the error histograms $\delta C_{\lambda i}$ in Fig. 1, where $N_{\lambda i}$ is the number of realizations of some set of microphysical model parameters, at which $C_{\lambda i}$ is found with the error $\delta C_{\lambda i}$.

The histograms were obtained with accounting for the random errors in $\beta_{a,\theta}(\lambda_i)$ calculation within 5%. Figure 1 presents also values of mean square errors $<\delta C_{\lambda i}>$. In this case, as is seen in Fig. 1, errors in determination of $C_{\lambda i}$ do not exceed the total error in measuring $\beta_{a,\theta}(\lambda_i)$, equal to 15%.

Discussion

To analyze the efficiency of the proposed method, we have conducted numerical experiment on multiwave sensing of the atmosphere, results of which are presented in Fig. 2.

Profiles of $\varepsilon_m(z, \lambda_i)$ were obtained with the use of calculation formulae of Rayleigh's theory[16] from the given pressure profiles, corresponding to the standard model of the gas atmosphere,[13] and temperature profile (Fig. 2*a*). To calculate $N_R(z)$, the same profiles were used. To estimate the influence of atmospheric temperature variations on the accuracy of retrieving aerosol optical characteristics, the temperature profile, corresponding to calculated signals, was set in such a way as to differ from the standard one[13] (maximal difference is 5° at $z \approx 1.6$ km).

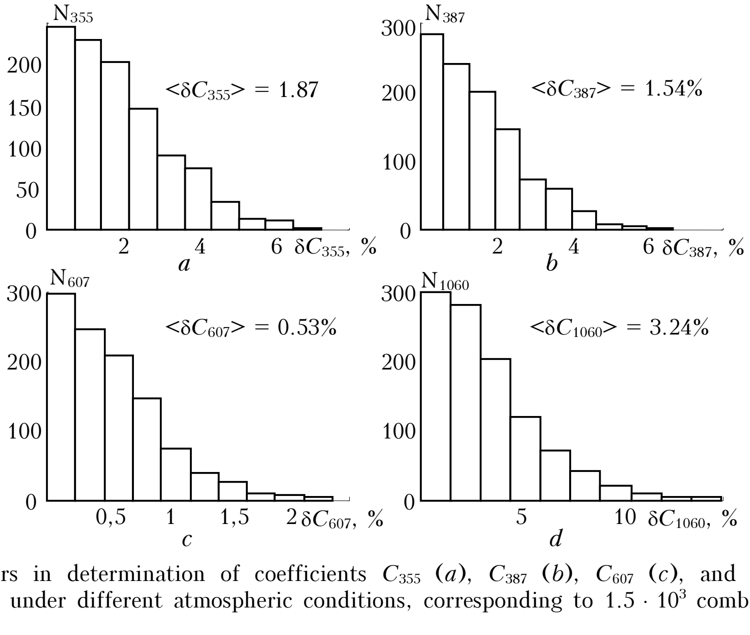

Fig. 1. Histograms of errors in determination of coefficients C_{355} (*a*), C_{387} (*b*), C_{607} (*c*), and C_{1060} (*d*) by Eq. (7) at λ_0 = 532 nm, which appear under different atmospheric conditions, corresponding to $1.5 \cdot 10^3$ combinations of microphysical model[15] parameters.

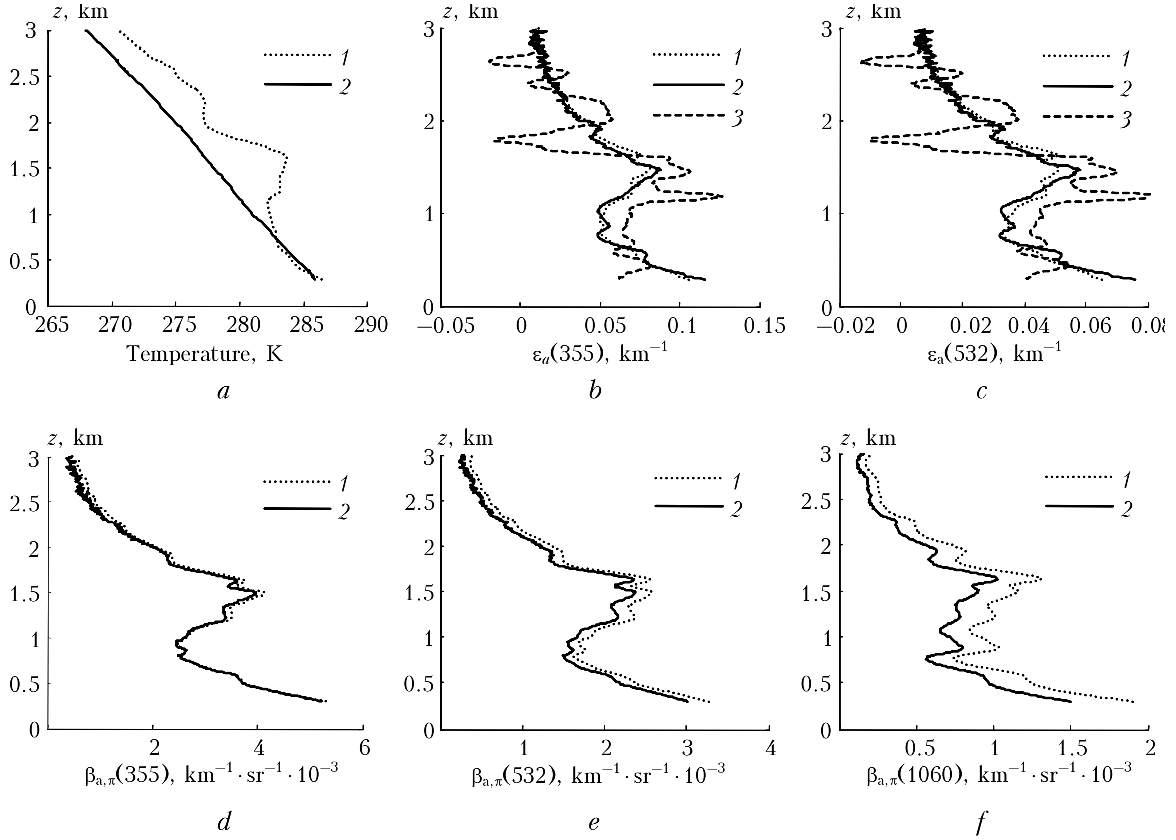

Fig. 2. Results of numerical experiment on multiwave sensing of the atmosphere: *a* is the set temperature profile (curve *1*) and the temperature profile corresponding to the standard model of gas atmosphere[13] (curve *2*); *b* and *c* are the results of retrieving $\varepsilon_a(\lambda_i, z)$ at $\lambda = 355$ and 532 nm, respectively (*1*, *2*, and *3* are the set profiles, retrieved by Eq. (4) and by the method from Ref. 4); *d*, *e*, *f* are the results of retrieving $\beta_{a,\pi}(\lambda_i, z)$ at $\lambda = 355$, 532, and 1060 nm, respectively (*1* and *2* are the set and retrieved $\beta_{a,\pi}(\lambda_i, z)$).

The set profile of $\varepsilon_a(355, z)$ is shown in Fig. 2*b*. Profiles of $\varepsilon_a(\lambda_i, z)$ at other wavelengths were set as $\varepsilon_a(\lambda_i, z) = C_{\lambda i}\varepsilon_a(355, z)$, where $C_{\lambda i}$ was equal to $(355/\lambda_i)^k$ in modeling (under supposition that $\varepsilon_a(\lambda_i) \sim \lambda^{-k}$) with imposing of spatial variations (a slow drift and random variations) on $k = 1$ within 25%. The spatial variations, imposed on $C_{\lambda i}$, correspond to numerous atmospheric situations[8] and allow one to trace the resistance of the method to both errors in determining $C_{\lambda i}$ and $C_{\lambda i}$ variations along the path.

Signals of the elastic and Raman scattering for further use were calculated analytically by Eqs. (1) and (2) with superposing of high-frequency noise components on them. Then signals were corrected with respect to molecular extinction, corresponding to the standard gas atmosphere.[13] Profiles of $\varepsilon_a(\lambda_i, z)$ and $\beta_{a,\pi}(\lambda_i, z)$ were retrieved by Eqs. (4) and (5) at $C_{\lambda i}$ values equal to $(355/\lambda_i)^k$, $k = 1$. When differentiating Eq. (4), signals $S_{387}(z)$ and $S_{607}(z)$ were beforehand five-point smoothed. Reference values of $\beta_{a,\pi}(\lambda_i, z_k)$ in Eq. (5) were set in the terminal point of the sensing path with a 25% error.

Results of retrieving $\varepsilon_a(\lambda_0, z)$ and $\beta_{a,\pi}(\lambda_0, z)$ are presented in Figs. 2*b* and *c*. The path-mean errors $\delta\beta_{a,\pi}(\lambda_0, z)$ were calculated by the equation

$$\langle \delta\beta_{a,\pi}(\lambda_0)\rangle = \sqrt{\frac{1}{l}\sum_{m=1}^{l}\left[\tilde{\beta}_{a,\pi}(\lambda_0,z_m)-\beta_{a,\pi}(\lambda_0,z_m)\right]^2} \Big/ \frac{1}{l}\sum_{m=1}^{l}\beta_{a,\pi}(\lambda_0,z_m), \quad (8)$$

where l is the number of registration channels on the sensing path; $\beta_{a,\pi}(\lambda_0,z_m)$ and $\tilde{\beta}_{a,\pi}(\lambda_0,z_m)$ are the actual and retrieved values of the aerosol backscattering coefficient in the point z_m at λ_i, equal to 5.18, 11.8, and 23.7% for $\lambda_0 = 355$, 532, and 1060 nm, respectively. In this case, the path maximum errors $\delta\beta_{a,\pi}(\lambda_0,z_m)$ do not exceed the error of reference value of $\beta_{a,\pi}(\lambda_0, z_k)$, equal to 25%.

Profiles of $\varepsilon_a(\lambda_0, z)$ at the given spatial fluctuations of temperature and $C_{\lambda i}$ are retrieved with mean errors (calculated by Eq. (8) with $\beta_{a,\pi}$ replaced by ε_a), equal to 8.13 and 9.23% for $\lambda_0 = 355$ and 532 nm, respectively, which is satisfactorily for estimation of both $\varepsilon_a(\lambda_0,z)$ on the sensing path and some integral aerosol microphysical parameters (for example, the number and volume concentrations).[17]

Profile of $\varepsilon_a(1060, z)$ can be estimated as $\varepsilon_a(1060, z) = C_{1060}\varepsilon_a(\lambda_0, z)$, where C_{1060} is determined from Eq. (7) with coefficients from Tables 1 and 2.

Evidently, the errors $\delta\varepsilon_a(1060, z)$ are comparable with magnitude of C_{1060} spatial variations.

Note that the errors $\langle\delta\varepsilon_a(\lambda_0)\rangle$ varied between 6 and 15% depending on the model k (at its identical random spatial scatter ring), and $\langle\delta\beta_{a,\pi}(\lambda_0)\rangle$ in all cases did not exceed the set reference values of $\beta_{a,\pi}(\lambda_0, z_k)$. Naturally, the lesser errors in retrieval of $\varepsilon_a(\lambda_0,z)$ and $\beta_{a,\pi}(\lambda_0, z)$ correspond to lesser variations of $C_{\lambda i}$.

For comparison, figures 2*b* and *c* show $\varepsilon_a(\lambda_0, z)$, retrieved by the well-known method,[4] which consists in differentiating Eq. (2) at *a priory* given $N_R(z)$, corresponding to the standard gas atmosphere.[13] It is seen that even at insignificant deviation of the atmospheric temperature profile from the model one, the method from Ref. 4 leads to greater errors in determining $\varepsilon_a(\lambda_0, z)$ and even to physically absurd (negative) values of $\varepsilon_a(\lambda_0, z)$.

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