Refraction of electromagnetic waves in the polluted turbulent atmosphere

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On the basis of three-dimensional spectral fluctuation functions of the number density gradient of a passive conservative admixture in the turbulent atmosphere, refraction phenomena are investigated and a model is constructed. Spectra of the refractive index gradient and fluctuation of the refractive angle for the entire range of inhomogeneity scales are obtained. Variances of the incident angle of radio waves for various levels of air pollution are determined. The horizontal and vertical gradients of the number density of a chemical admixture in the troposphere are found from experimental data. Model calculations were carried out for the following substances: NH_3, NO_2, H_2S, and SO_2. The influence of water vapor is analyzed. This has made it possible to bring the model more closely in line with real conditions. Comparison with a number of experimental works reveals a high degree of correspondence with actual conditions.

Introduction

The problem of scattering of electromagnetic waves by atmospheric turbulence attracted the attention of researchers long ago in connection with the observed phenomenon of propagation of HF and UHF waves. Many authors have studied the problems of formation and evolution of turbulence and its effect on radio communication in detail. Typically, they considered inhomogeneities of temperature, pressure, wind velocity, or concentration of some admixture. But the possibility also exists of assessing the effect of inhomogeneities of one or another parameter of the atmosphere. For example, the refractive angle of an electromagnetic wave depends directly on the gradient of the air refractive index.

Obukhov[1] proposed to study turbulence in terms of structure functions and spectral functions. This method later gained wide acceptance. The structure and spectral functions of the wind velocity, temperature, and passive admixture number density fields were obtained by Tatarskii in 1967.[2] The form of the spectral function is determined by the atmospheric parameters, acting forces, and redistribution of turbulent energy and is derived by means of methods of dimensionality theory. Later papers[3,4] successfully used the treatment of structure and spectral functions to describe inhomogeneities of the free-electron number density in the ionosphere.

Reference 5 considered turbulence in the ionosphere, where the gradient of the electron number density was used as an input parameter. The spectral functions of the gradient of the electron number density presented in Ref. 5 are used in the present paper to construct the spectrum of the gradient of the admixture number density in the troposphere. Molecules of chemical substances present in the atmosphere are considered as a passive conservative admixture.

Spectral functions of the gradient of the number density of a passive conservative admixture

The shape of the spectrum is determined by different factors for each range of the turbulent inhomogeneity scale. The small-scale turbulent fluctuations are represented by a measure of the inhomogeneity of the number density of the admixture g_N, the molecular diffusion coefficient D_{m}, and the wave number p[5]:

$$E(p) = c_0 \frac{g_N}{D_{\mathrm{m}}} p^{-1}, \tag{1}$$

where c_0 is a constant, g_N is a measure of the inhomogeneity of the admixture number density per unit time due to turbulence:

$$g_N = K \left(\frac{\mathrm{d}\bar{N}}{\mathrm{d}z}\right)^2 . \tag{2}$$

Here K is the turbulent exchange coefficient and $\frac{\mathrm{d}\bar{N}}{\mathrm{d}z}$ is the mean vertical gradient of the admixture number density.

The spectrum in the isotropic subrange of the inertial range is given by the formula[5]

$$E(p) = c_1 \frac{g_N}{\varepsilon^{1/3}} p^{1/3}, \tag{3}$$

where ε is the specific dissipation energy.

Processes in the Archimedean (buoyancy) subrange of the inertial range are determined by the action of buoyancy forces, and the spectrum is represented in the form[5]

$$E(p) = c_2 \frac{g_N}{M^{1/5}\, \beta^{2/5}} p^{3/5}, \tag{4}$$

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where M is the rate of smoothing of the temperature inhomogeneities and β is the Archimedean (buoyancy) parameter.

The following formula is obtained for the range of large-scale quasi-2D turbulence[5]:

$$E(p) = c_3 \frac{g_N}{\varepsilon_e^{1/3}} p, \tag{5}$$

where ε_e is the rate of spectral energy transfer.

Effect of chemical admixtures

According to the molecular theory of matter, the dielectric constant of a gas depends on the absolute temperature T_a, the pressure p_a, and the number density of polar molecules N. Polar chemical admixtures[6] have a noticeable effect on the propagation of HF and UHF waves. In general, the refractive index of air is represented by the formula[6]

$$(n - 1) \cdot 10^6 = \frac{a_1\, p_0}{T_0} + \frac{a_2\, e}{T_0} + \frac{a_3\, e}{T_0^2}. \tag{6}$$

Here the first term represents the effect of non-polar molecules of the gaseous mixture, and the second and third terms represent the contributions of polar molecules. The coefficients of this formula are equal to[7] $a_1 = 77.6$, $a_2 = 12.96$, and $a_3 = 3.72 \cdot 10^5$ deg/mbar.

Since $a_2 \ll a_3/T_0$ under atmospheric conditions, we can ignore the second term.[6] Taking into account the Maxwell formula ($n = \sqrt{\varepsilon\mu}$, where $\mu = 1$), and making the substitution

$$e = \frac{M_a}{M_s} p_0\, q, \tag{7}$$

where M_a is the mean molecular weight of air ($M_a = 29$); M_s is the molecular weight of the chemical admixture; q is the relative number density $\left(q = \frac{m_{sub}}{m_{air}}\right)$, and setting $A = a_1 \cdot 10^{-6} = 1.552 \cdot 10^{-4}$ and $B = a_3 \cdot 10^{-6} = 0.746$, we obtain a formula for the refractive index of the gaseous mixture[7]:

$$n = \frac{p_0}{T_a}\left(A + \frac{B}{T_a}\frac{M_a}{M_s}\frac{m_{sub}}{m_{air}}\right) + 1 \tag{8}$$

and, hence, the gradient of the refractive index:

$$\frac{dn}{dz} = \frac{B\, p_0}{T_a^2}\frac{M_a}{M_s}\frac{1}{m_{air}}\frac{dm_{sub}}{dz}. \tag{9}$$

It is a generally known fact that the molecules of the overwhelming majority of chemical substances have a constant magnetic moment[8]; this is the reason for the suitability of the above theory for calculating the parameters of radio paths in polluted air.

According to the theory of random fields, a relation exists between the spectral and structure functions for a locally isotropic field which has the form[2]

$$E(\eta) = \frac{1}{4\pi^2\eta^2}\int_0^\infty \frac{\sin \eta r}{\eta r}\frac{d}{dr}\left[r^2 D'(r)\right]dr. \tag{10}$$

By definition of the structure function, we can write it in the form

$$D_\xi(r) = \langle[\xi(\bar{r}_1 + \bar{r}) - \xi(r_1)]^2\rangle. \tag{11}$$

After substituting expression (9) in formula (11) and performing various transformations, we obtain a formula for the gradient of the refractive index:

$$D_{\nabla n}(r) = \left[\frac{B\, p_a}{T_a^2}\frac{M_a}{M_s}\frac{\langle[m_{sub}(\bar{r}_1 - \bar{r}) - m_{sub}(\bar{r}_1)]\rangle}{m_{air}}\right]^2. \tag{12}$$

Substituting the last formula into relation (10), we write the spectral function of the gradient of the refractive index as a function of the spectral function of the gradient of the number density of the chemical admixture:

$$E_{\Delta\theta}(p) = \frac{B^2}{4\pi^2\, p_w^2}\left[\frac{p_a}{T_a^2}\frac{M_a}{M_s}\frac{1}{m_{air}}\right]^2 E_{\nabla m}(p). \tag{13}$$

Substituting the spectral functions of the gradient of the number density of the admixture (1), (3)–(5) into formula (13), one can construct the fluctuation spectrum of the gradient of the refractive index for all inhomogeneity scales.

It is known that the refractive angle of a radio wave propagating through an elementary layer ΔL depends on the gradient of the refractive index as[5]

$$\Delta\theta = \frac{1}{2}\Delta L \cos\alpha_0\, \nabla n. \tag{14}$$

Finally, after transformations analogous to formulas (10)–(13) we write the spectral function of the refractive angle fluctuations as

$$E_{\Delta\theta}(p) = \left[\frac{B\, \Delta L \cos\alpha_0}{4\pi\, p_w}\frac{p_a}{T_a^2}\frac{M_a}{M_s}\frac{1}{m_{air}}\right]^2 E_{\nabla m}(p). \tag{15}$$

Integrating over all turbulence scales, we obtain the variance of the radio-wave refractive angle within the limits of an elementary part of the propagation path. After integrating over the whole beam trajectory, we obtain the resulting variance of the refractive angle for a path of length L.

Values of parameters entering into the formulas

Let us determine the numerical values of the parameters entering into the obtained formulas. The coefficients $c_0 = 1.3$, $c_1 = 0.41$, $c_2 = 0.25$, $c_3\, \varepsilon_e^{-1/3} = 45$ were determined by matching the spectral functions at the boundary of the transition from one inhomogeneity scale to another.[5] The value of the specific dissipation energy for altitudes of the lower troposphere has the value[2] $\varepsilon = 0.037\ \mathrm{m^2 \cdot s^{-3}}$, and the buoyancy parameter is equal to $\beta = 0.035\ \mathrm{m \cdot s^{-2} \cdot deg^{-1}}$. Then we obtain the value $K = 2900\ \mathrm{m \cdot s^{-1}}$ from the formula for the turbulent exchange coefficient $K = L_0 \dfrac{d\bar{u}}{dz}$. The parameters of the standard atmosphere are $T_0 = 288$ K; $p_0 = 101325$ mbar; $\dfrac{d\bar{T}}{dz} = 5.5\ \mathrm{deg \cdot km^{-1}}$ (Ref. 9).

A measure of the temperature inhomogeneity is given by

$$M = K\left(\frac{d\bar{T}}{dz}\right)^2. \qquad (16)$$

Upon substituting-in the above values for K and $\dfrac{d\bar{T}}{dz}$ we obtain $M = 0.088\ \mathrm{m \cdot deg^2 \cdot s^{-1}}$.

The boundary of the transition from the viscous range to the inertial one is the inner turbulence scale l_0. The value of l_0 in the lower troposphere is on the order of 1 mm.[2,3]

The transition from the inertial range to the Archimedean (buoyancy) subrange is caused by the intensifying effect of buoyancy forces. The boundary value of the inhomogeneity scale is[2]

$$L_K = \varepsilon^{5/4}\, \beta^{-3/2}\, M^{-3/4}. \qquad (17)$$

After substituting-in the numerical values we obtain $L_K = 15$ m.

The boundary of the transition to the large-scale inhomogeneity is the outer inhomogeneity scale L_0. The following expression is given in Ref. 2 for L_0:

$$L_0 = \sqrt{\varepsilon\left(\frac{d\bar{u}}{dz}\right)^{-3}}. \qquad (18)$$

Since the mean gradient of the wind velocity in the troposphere has the value $\dfrac{d\bar{u}}{dz} = 3.6\ \mathrm{m \cdot s^{-1} \cdot km^{-1}}$ (Ref. 9), formula (18) yields $L_0 = 900$ m. The mean value of the diffusion coefficient is $D_q = 1.55 \cdot 10^{-5}\ \mathrm{m \cdot s^{-1}}$ (Ref. 10).

The effect of water vapor

Not only do the molecules of a foreign admixture have a constant magnetic dipole moment, so do water molecules. Thus, to increase the accuracy of the calculations, it is necessary to take into account the effect of water vapor, which is continuously present in the atmosphere. All of the above mathematical apparatus is also applicable for the water vapor present in the atmosphere. The spectra of the refractive angle fluctuations for water vapor are calculated in the same way. Since the variance is an additive value, summing up the variances of the refractive angle of the radio waves for the polluting admixture and for water vapor, we obtain the total effect for the real turbulent atmosphere.

Calculations and results

A model was constructed based on the above dependences. Real values of the specific horizontal gradient of chemical admixtures in the lower layers of the atmosphere were taken from experimental data obtained by a network of measurement stations over a period of three years. According to Ref. 11, the vertical gradient is one order of magnitude greater than the horizontal one. Maximal values of the vertical gradient lie in the range $(1 \cdot 10^{-5} - 5 \cdot 10^{-4})\ \mu\mathrm{g/m^4}$.

The shape of the spectral function of the refraction index gradient fluctuations over the entire range of turbulence scales $10^{-5} - 1500$ m, calculated by formula (13) for radio waves of length 25 cm, is shown in Fig. 1. Maximal fluctuations are observed near the inner inhomogeneity scale and have a value on the order of $3.0 \cdot 10^{-18}$ 1/m, depending on the substance.

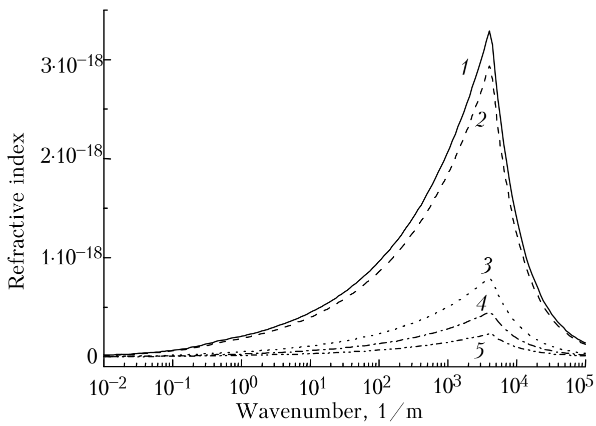

Fig. 1. Spectral function of the refractive index gradient fluctuations. NH_3 (*1*); H_2O (*2*); H_2S (*3*); NO_2 (*4*); SO_2 (*5*).

Figure 2 plots the spectral function of fluctuations of the refractive angle calculated according to formula (15). It has a similar shape and its maximum value is on the order of $4.5 \cdot 10^{-9}$ deg.

Integration of formula (15) over wavenumbers and path lengths gives the resulting variance of the radio-wave incident angle. The results of calculation of the variance of the incident angle for different substances and different values of the admixture concentration gradient are presented in Table 1. Here and below the following parameters were adopted for the model calculations: $L = 10$ km, wavelength 25 cm.

For practical purposes, it is often useful to solve the inverse problem, i.e., to determine the value of the admixture concentration gradient for fairly large variances of the refraction angles (we assume it is on the order of several degrees). Results of these calculations are shown in Table 2.

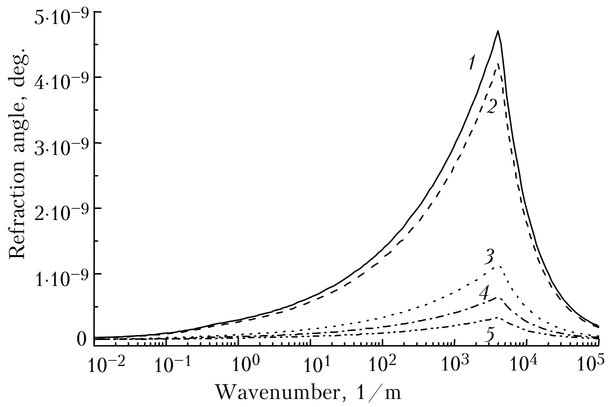

Fig. 2. Spectral function of fluctuations of the specific refraction angle: NH_3 (*1*); H_2O (*2*); H_2S (*3*); NO_2 (*4*); SO_2 (*2*).

As was noted above, we can account for the effect of atmospheric water vapor. Mean values of the water concentration gradient in the troposphere[9] are as follows: 0.37 mg/m^4 in winter at 50°N, 1.8 mg/ m^4 in summer at 50°N, and annual mean value 3.5 mg/ m^4.

Table 1. Standard deviation of the radio-wave incident angle for different admixtures for several values of the concentration gradient (arcmin)

Gradient, mg/m	Chemical substance				
	NH_3	H_2O	H_2S	NO_2	SO_2
0.005	$1.60 \cdot 10^{-3}$	$1.51 \cdot 10^{-3}$	$7.92 \cdot 10^{-4}$	$5.93 \cdot 10^{-4}$	$4.26 \cdot 10^{-4}$
0.05	$1.60 \cdot 10^{-2}$	$1.51 \cdot 10^{-2}$	$7.92 \cdot 10^{-3}$	$5.93 \cdot 10^{-3}$	$4.26 \cdot 10^{-3}$
0.5	$1.60 \cdot 10^{-1}$	$1.51 \cdot 10^{-1}$	$7.92 \cdot 10^{-2}$	$5.93 \cdot 10^{-2}$	$4.26 \cdot 10^{-2}$

Table 2. Concentration gradient of different admixtures giving rise to the prescribed standard deviation of the radio-wave incident angle (mg/m^4)

Deviation angle, deg	Chemical substance				
	NH_3	H_2O	H_2S	NO_2	SO_2
0.1	7.96	7.99	11.10	12.81	15.50
1.0	14.10	14.43	19.68	22.70	27.47
2.0	16.26	17.06	23.81	27.47	31.69

Fluctuations of the refractive angle for two substances as functions of the admixture concentration gradient for three values of the humidity gradient are plotted in Fig. 3.

To examine the adequacy of the proposed theory, a comparison of the results of our calculations with the experimental data was carried out. Results of measurements of refraction in the troposphere were presented in Ref. 12. The variance of the refractive angle was measured in the radio-wave and optical ranges. The standard deviation of the incident angles for the radio-wave range along a 10-km-long path has values on the order of 1.2 arcmin in spring and 3.2 arcmin in summer.

The model calculations in the absence of the pollutant admixture give 0.9 arcmin for spring and 1.9 arcmin for summer. The underestimated value of the calculated variance of the refraction angles is explained by the fact that the proposed theory does not take into account inhomogeneities of the temperature and pressure fields, whose effects are approximately of the same order.

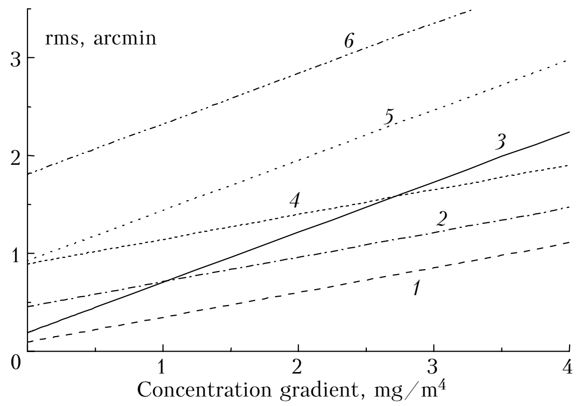

Fig. 3. Standard deviation of the refractive angle taking into account the actual humidity gradient: *1*, *3*, *5* – NH_3 (1.80; 0.37; 3.52); *2*, *4*, *6* – H_2S (0.37; 3.52; 1.80).

Measurements of the refraction angles in the centimeter wavelength range in the coastal zone are described in Ref. 13. The measured values of the standard deviation lie in the range from 8 to 12 arcmin. The model gives the value 11.2 arcmin in the centimeter wavelength range.

Comparison reveals a high degree of correspondence of the proposed model and actual measurements carried out under typical conditions.

Conclusion

The formulas obtained for the 3D spectral functions of the refractive index gradient and the refractive angle of an electromagnetic wave for different inhomogeneity scales are valid for any passive, conservative, polar admixture, including water vapor. The vertical gradient of the admixture distribution in the troposphere is based on experimental data. These formulas are applied to calculate the variance of the angles of refraction of an electromagnetic wave by turbulence inhomogeneities. Such chemical substances as NH_3, NO_2, H_2S, SO_2, and water vapor were used in sample calculations. The model shows a high degree of correspondence to the picture actually observed.

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