

## LASER-MICROWAVE DISCHARGE FOR CONTROL OVER FLIGHTS OF SUPERSONIC BODIES

V.N. Tishchenko

*Institute of Laser Physics,  
Siberian Branch of the Russian Academy of Sciences, Novosibirsk  
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*A new type of discharge, i.e. laser-microwave discharge (LMD) with high velocity of motion in gas ( $\sim 0.4\text{--}3$  km/s) and remote contactless power supply, is considered for the first time. Laser radiation creates a channel in which more powerful microwave radiation is absorbed. The motion of the discharge is realized by displacement of the focal regions of beams. An approximate LMD model is based on experimental data. Relative consumption of energy in LMD is small what indicates the possibility to use it in promising methods of control over flights of supersonic bodies.*

The quasi-continuous optical pulsing discharge in a supersonic argon flow and possibility to use such a discharge for reducing aerodynamic resistance of a small body (by a factor of  $\sim 2$ ) were demonstrated for the first time in Refs. 1–3. To advance in the range of high power, the possibility of combined application of laser and more powerful microwave radiation was studied. In this paper, the scheme of the laser-microwave discharge (LMD) and its empirical model are presented for the first time. We determine some properties of LMD and energy conditions of its existence in the atmosphere. Experimental data on the microwave discharge in decomposing laser plasma are partially presented in Ref. 4.

### LMD FORMATION

The focal region of pulse-periodic laser radiation moves in the direction of the ray axis with velocity  $V$  with respect to the ambient gas. Every laser pulse creates an optical breakdown of gas near the focus. The length  $L$  of laser plasma significantly exceeds the radius. If the ray focus displaces by distance  $L$  during the pause between the pulses  $\sim 1/f$  (i.e.,  $fL/V \cong 1$  holds), separate optical breakdowns form a continuous channel whose length is bounded by cooling ( $f$  is the frequency of laser pulses' sequence).

Pulse-periodic microwave radiation enters the channel through its lateral surface. The beam parameters are chosen so that the microwave discharge is localized in the channel. There exist conditions under which microwave radiation compensates cooling of the medium in the channel. In this case, medium temperature and transmittance in the channel can be set at the level optimal for absorption. This opens the possibility to increase LMD power and length significantly. A field of microwave radiation moves

along the channel. With channel length  $L_c \gg L$  and weak interaction of regions of separate optical breakdowns (mainly by shock waves), distribution of LMD parameters in the channel can be considered on the base of the study of interaction between microwave radiation and decomposing plasma of a separate optical breakdown. Further, replacing  $t$  by  $Z/V$ , we can obtain dynamics of the medium in the channel. The value  $Z = 0$  corresponds to the focal point of the laser beam.

### FORMATION OF A CHANNEL UNDER THE ACTION OF LASER RADIATION

One can distinguish three stages with characteristic times differing more than by an order.

1. Heating and ionization by optical radiation during the time  $t_1 \ll r_1/c_0 \cong 2 \mu\text{s}$  ( $r_1$  is the characteristic radius of optical breakdown  $\sim 0.3$  cm,  $c_0$  is the velocity of sound in a non-disturbed gas).

2. Heat expansion (explosion) of laser plasma at  $t_1 < t \leq t_2 \leq r_2/c_0$ . With  $t \cong t_2$ , the pressure in the cavern reduces to that of non-disturbed gas ( $r_2(t_2)$  is the radius of the cavern boundary).

3.  $t_2 < t < 10t_3 \cong 3$  ms, isobaric expansion of the cavern, gas temperature is  $\sim 4000$  K, density is small, the boundary moves with velocity of several meters per second; by the moment  $t \geq 10t_3$ , medium density and temperature become equal to their values in non-disturbed gas.

4. The stage of long-living small-scale vortical structures  $t_4$ . The typical values of characteristic times are  $t_1 \cong 1 \mu\text{s}$ ,  $t_2 \cong 10 \mu\text{s}$ ,  $t_3 \cong 0.3$  ms,  $t_4 \geq 20$  ms (see below). The great difference in the values of  $t_i$  permits one to separate the processes and obtain approximate relations for the LMD model.

### OPTICAL BREAKDOWN

Optical breakdown of gas begins in the focal region where intensity of laser radiation is maximal and must exceed the threshold value. The plasma front moves toward the radiation. The threshold of breakdown and mechanism of its propagation considerably depends on the radiation wavelength. In this paper, as well as in Ref. 4, we consider the breakdown under the action of CO<sub>2</sub> laser radiation ( $\lambda \cong 10.6 \mu\text{m}$ ) of duration  $\sim 1 \mu\text{s}$ .

To form a continuous channel, it is necessary to chose optimum radiation energy  $Q$  of a separate laser pulse and angle of ray focusing  $\alpha = d/F$  ( $d$  is diameter of the ray at the focusing lens,  $F$  is the distance from the lens to the focus). The minimal value of  $Q$  is bounded by the breakdown threshold  $Q > Q_0$ . The maximal value of  $Q$  and minimal angle are bounded by the condition of continuity of the optical breakdown. At the same time, the contribution of energy per unit of the breakdown length  $Q/L$  must not be small what is necessary for creating a channel with diameter compared with microwave radiation wavelength  $\lambda \cong 3 \text{ cm}$ . According to experiments, the optical breakdown is continuous in a one-mode beam with  $\alpha < 0.05$  if  $Q$  insignificantly exceeds the value  $Q_0 \approx 0.5 - 1 \text{ J}$ . For higher  $Q$ , the breakdown consists of separate plasmoids with characteristic diameter  $\sim 1 \text{ cm}$ . Such a structure is formed as a result of initiating a breakdown at aerosol particles when radiant exitance in the beam is  $\geq 100 \text{ MW/cm}^2$ . For  $\alpha \cong 0.1$ , the breakdown is close to continuous if  $Q_0 \leq Q \leq 7 \text{ J}$ . The maximal value  $\alpha = 0.1$  is bounded because of technical conditions, namely, the necessity to use a focusing lens of large diameter.

The length  $L$  of a continuous breakdown of gas can depend on the mechanism of plasma front propagation in the process of breakdown evolution. The regime of light detonation wave is the most probable in the energy range mentioned above. For this case, we obtained an analytical expression for  $L$  and length density of radiation energy absorbed at unity of breakdown length

$$L = \frac{b_i}{\alpha} \left( \frac{W_L}{P_0} \right)^{1/2} \left\{ 6.3 + \ln P_0 + \ln \left[ b_i \left( \frac{W_L}{P_0} \right)^{1/2} (6.3 + \ln P_0)^{3/4} \right] \right\}^{3/4};$$

$$g(Z) = 1.2 \cdot 10^{-4} W_L^{2/3} (Z\alpha)^{2/3} \left( \frac{A}{\gamma^2 - 1} \right)^{1/3}; \quad (1)$$

$$b_i = \frac{A^{1/4} (\gamma - 1)^{5/4}}{I^{3/4} (\gamma + 1)}.$$

Here  $Z \leq L$ ,  $A$ ,  $I$ ,  $\gamma$  are atomic weight of the gas, ionization potential (K), and specific heat ratio,

respectively. The pressure  $P_0$  of non-disturbed gas expresses its concentration at  $T_0 \cong 300 \text{ K}$ . The power of laser radiation  $W_L$  is constant during the whole pulse, and duration is not less than the time sufficient to upset the regime of propagation of the light detonation wave

$$t_i = 4.5 \cdot 10^{-5} b_i^{5/3} \left( \frac{W_L}{P_0} \right)^{1/2} \times \left( \frac{A}{\gamma^2 - 1} \right)^{1/3} \alpha^{-1} \left\{ 6.3 + \ln P_0 + \ln \left[ b_i \left( \frac{W_L}{P_0} \right)^{1/2} (6.3 + \ln P_0)^{3/4} \right] \right\}^{5/4}.$$

If radiation duration  $t_j$  is less than  $t_i$ , the length of optical breakdown is

$$L = 360 \alpha^{-2/5} \left( \frac{\gamma^2 - 1}{A} \right)^{1/5} \left( \frac{W_L}{P_0} \right)^{1/5} t_j^{3/5} = 360 \alpha^{-2/5} \left( \frac{\gamma^2 - 1}{A} \right)^{1/5} \left( \frac{Q}{P_0} \right)^{1/5} t_j^{2/5}.$$

Note that the value  $\alpha L$  corresponds to the maximal diameter of the zone of optical breakdown.

The experimental data can be approximated by the following expressions.

One-mode radiation,  $P_0 = 1 \text{ atm}$ ,  $\alpha = 0.1$ :

$$L = 2.2 Q^{1/2} (\text{cm}); \quad d_p = 0.6 Q^{0.77}.$$

Multi-mode radiation:

$$L = 1.6 Q^{0.4} / P_0^{0.1};$$

$$d_p = 0.66 Q^{0.45};$$

$$Q_0 = 0.3 / P_0^{0.43} - \text{breakdown threshold.}$$

The value  $d_p$  corresponds to luminescence diameter of a broadening laser plasma at  $t \cong 20 \mu\text{s}$ .

Thus, in forming a channel, one can treat the following laser radiation parameters as optimal:  $Q \cong 5 \text{ J}$ ,  $\alpha \cong 0.1$ ,  $L \cong 5 \text{ cm}$ ,  $Q/L \cong 1 \text{ J/cm}$ . Radiation duration must be  $t_r \cong 1 - 2 \mu\text{s}$ . The optical pulsing discharge moving with velocity  $V$  creates a continuous rarefaction channel if frequency  $f$  and average power of pulse-periodic laser radiation satisfy the conditions

$$f = V/L = 0.45 V / Q^{1/2} (\text{Hz});$$

$$W_1 = 0.45 V Q^{1/2} (\text{W}).$$

Here  $V$  is expressed in  $\text{cm/s}$ , and radiation is one-mode.

### SUPERSONIC BROADENING OF LASER PLASMA

The main characteristics of this stage are as follows: in the beginning,  $t \cong t_1 - T > 10^4$  K; then the supersonic broadening of plasma follows. The broadening ends in equalization of pressure in the cavern and ambient gas medium at  $t = t_2$ . With  $t \cong t_2$ , the shock wave goes off the cavern's boundary. Medium density decreases by 10–20 fold, and temperature, according to measurements,<sup>11</sup> falls to ~8000 K. In this stage, electron concentration is much more than the critical value for microwave radiation. To create an approximate model of a continuous channel and LMD in the whole, it is necessary to know cavern parameters (radius, temperature).

These values can be obtained as functions of parameters of the medium and laser radiation, as an approximation of a cylindrical strong explosion. With  $t \leq t_1$ , the ratio of the maximal diameter to the length of optical breakdown is small,  $2r_1/L = \alpha \leq 0.1$ .

The velocity of the shock wave  $D$  can be determined from the relation for the cylindrical point-source explosion<sup>6</sup>

$$\frac{r}{r^*} = \frac{k_1}{M - (1/M)};$$

$$k_1 = [(\gamma - 1)/\pi]^{1/2} (\gamma + 1)/2\gamma,$$

where  $M = D/c_0$  is the Mach number;  $r^* = (Q/LP_0)^{1/2}$  is the dynamic length.

The dependence of  $M$  on  $r/r^*$  differs from the experimental data obtained in a wide range of pressure ( $P_0 \cong 0.05 - 1$  atm) and radiation energy. This is caused by the fact that the explosion is not point-source. The experimental data well agree with a single curve. For  $r/r^* \geq 0.1$ , the experimental and theoretical values  $M_1$  and  $M$ , respectively, are connected by the relation  $M_1 = 1.6M - 0.6$ . In the range  $r/r^* < 0.1$ , we have  $M > M_1 \cong 5$ , because the shock wave is formed just here. With  $r/r^* \geq 1$ , the velocity of the shock wave decreases to  $M_1 \cong 1.2-1.3$  and, besides, the cylindrical shock wave passes into a spherical one.

By the moment  $t \cong t_2$ , medium pressure in the cavern decreases to pressure of a non-disturbed gas, the velocity of the plasma boundary sharply decreases to a few meters per second, the shock wave goes off the plasma cavern and carries away a considerable part of energy.

The cavern radius  $r_2$  at  $t \cong t_2$  can be determined by use of the theory from Ref. 6

$$r_2 = \frac{1}{2} \left[ \frac{120(\gamma - 1) \alpha^{2(\gamma - 1)} Q}{\pi P_0 L^{3 - 2\gamma}} \right]^{1/2\gamma}.$$

For dependence  $L = 2.2 \sqrt{Q}/P_0^{0.1}$  we will obtain

$$r_2 = 1.1 [3.6 b(\gamma - 1)]^{1/2\gamma} \alpha^{1 - (1/\gamma)} \times$$

$$\times \frac{Q^{1/2[1 - (1/2\gamma)]}}{P_0^{0.35/\gamma}}.$$

Here  $b \cong 0.9$  is the factor taking into account the losses by plasma radiation. The boundary of the cavern reaches the radius  $r_2$  at the moment  $t_2 \cong t_1 + r_2/(2c_0)$ . The value  $r_2$  is by ~20% greater than that measured in Refs. 2 and 4 where laser radiation energy was ~0.1 and 15 J, respectively. For  $Q \cong 5$  J what is optimum for LMD, the radius is  $\cong 0.9$  cm, and duration of the supersonic broadening stage is ~15–20  $\mu$ s. Within the time interval  $t = t_1 \div t_2$ , particles' temperature and concentration in the cavern decrease to the values

$$T_2 = \frac{200}{(\gamma - 1)^{1 - (1/\gamma)}} \left\{ \frac{3}{P_0^{0.7} Q^{0.5} \alpha^2} \right\}^{1/\gamma};$$

$$N_2 = N_0 T_0/T_2.$$

We obtain  $T_2 \cong 8000$  K and  $N_2 \cong 0.04 N_0 \cong \cong 10^{18}$  cm<sup>-3</sup> for the condition of LMD appearance. This value of  $T_2$  is close to that measured in Ref. 11 where a weak dependence of  $T_2$  on  $Q$  was observed.

The shock wave goes off the boundary of the cavern with  $t \cong t_2$  and carries away a part of energy

$$E_s/Q = 1 - \left[ \frac{\pi \alpha^2 P_0 L^3}{118b(\gamma - 1)Q} \right]^{1 - (1/\gamma)}.$$

For  $\alpha = 0.1$ ,  $\gamma \cong 1.18$ ,  $b = 0.8$ ,  $L \cong 2.2\sqrt{Q}/P_0^{0.1}$ , we obtain

$$E_s/Q = 1 - 0.5Q^{0.075}P_0^{0.1}.$$

It follows that, in the case  $Q \cong 5$  J,  $P_0 = 1$  atm, the shock wave carries away ~38% of energy of laser radiation absorbed in the plasma of optical breakdown of gas. The experimental value of  $E_s/Q$  is 20–30%.

In the LMD, shock waves of separate breakdowns create a common shock wave of conical form. The vertex of the cone is in the focal point of the laser beam. The angle  $\beta$  of the  $Z$  axis of optical radiation with the element of the cone can be determined by the relation  $\tan \beta = c_0/V$  ( $V$  is the velocity of the focus with respect to gas).

### THE ISOBARIC STAGE OF LASER PLASMA DECOMPOSITION

During the time interval  $t_2 < t \leq 10 t_3 \cong 3$  ms, temperature in the cavern decreases from  $T_2 \cong 8000$  K to the temperature of ambient gas  $T_0 \cong 300$  K, and medium density increases. The main mechanism of cooling is turbulent heat transfer. The small-scale structure of density is observed both at  $t \cong t_1 \cong 2$   $\mu$ s and after large time  $> 3$  ms when the averaged values of temperature and density of the medium in the cavern are equalized to the ambient gas.<sup>4</sup>

Turbulence of a pulsing optical discharge burning ahead of a supersonic body can influence upon its aerodynamic characteristics, i.e., bound the length of the rarefaction channel and make the windstream turbulent. This effect was not considered in the existing papers on control over bodies' flow.<sup>7-10</sup>

The isobaric stage of decomposition is the most complicated for exact theoretical description. When the medium is cooled from 8000 to 1000 K, complicated physical-chemical and gas dynamic effects take place. However, to estimate the discharge energy, one can use empirical relations based on measuring the efficient time  $t_3$  for cooling of the medium of a separate breakdown. In Ref. 11,  $t_3$  and efficient thermal conductivity  $\varphi$  were determined by measurements of cavern temperature  $T(t)$ . In Ref. 4,  $t_3$  and  $\varphi$  were obtained by shadow measurements of the characteristic radius of the cavern  $R_c(t)$  and by use of the relation for  $R_c$ :

$$R_c(t) = r_2 [1 + (t - t_2)/t_3]^{1/2} r_2 \psi^{1/2},$$

which well agrees with experimental data. In the experiment,<sup>4</sup> the characteristic time  $t_3$  was obtained by measured  $r_2$ ,  $t_2$ , and  $R_c(t)$ . According to Ref. 11, the time corresponds to temperature decrease by  $\sim 2$  fold of the level  $T_2$ . In the air,  $t_3 \cong 1.74 \cdot 10^{-4} \times Q^{0.25}$  s, and in argon  $t_3 \cong 1.63 \cdot 10^{-4} (Q/L)^{0.5}$  (pressure is close to atmospheric). Efficient thermal conductivity is connected with  $t_3$  by the relation

$$\varphi = r_2^2 / 4t_3.$$

Dynamics of the cylinder's cooling can be obtained from the simplified model (by analogy with Ref. 11) by solving the heat conduction equation with  $\varphi$  obtained by the above-mentioned method. The solution can be represented in the form

$$T(r, t) = T(t_2) \psi^{-1} \exp(-1.75 (r/R_c)^2).$$

Concentration of particles in the cavern can be obtained from the condition of equality of pressure and equals  $N = N_0 T_0 / T$ . Electron's concentration can be determined from the Sach equation taking into account  $T$  and  $N$

$$n_e = 6.3 \cdot 10^9 (N_0 T_0 G_+ / G_a)^{1/2} (T_2 / \psi)^{1/4} \times \\ \times \exp[-0.44 (r/R_c)^2 - I/2T].$$

These expressions determine radius of the cavern, temperature, concentration of molecules (atoms) and electrons in it. Substituting  $Z/V$  for  $t$ , one can obtain distribution of the mentioned parameters along the channel created in the pulse-periodic optical breakdown of the medium. The length  $L_c$  of the continuous channel is defined by cooling time  $t_3$  and velocity of LMD motion with respect to gas:  $L_c = V(t_2 + 10t_3)$ , and the plasma part has the length  $L_p = V(t_2 + t_3)$ . With  $0 \leq Z \leq L_p$ , temperature is more than 4000 K and electron concentration is  $\geq 10^{10} \text{ cm}^{-3}$ . The dependence

of  $t_1$  and  $t_2$  on laser radiation energy is presented above.

If we need a channel of length  $L_c$ , the repetition frequency of laser radiation pulses and mean power must satisfy the conditions

$$f = 3.8 \cdot 10^{-7} V^{3.2} / L_c^{2.2} \text{ (Hz)};$$

$$\tilde{W} = 4.2 \cdot 10^4 L_c^{1.8} / V^{0.8} \text{ (W)}.$$

Here  $V$  is expressed in cm/s,  $L_c$  is in cm.

Thus, laser radiation can create a channel with low concentration of particles  $N/N_0 \leq 0.1$  and high temperature  $\sim 5000$  K. The length of the channel can be several meters, and its diameter can be less than 10 cm.

### MICROWAVE DISCHARGE IN LASER PLASMA

A microwave discharge is localized in the channel under the following conditions: a self-maintaining, low-threshold discharge (the velocity of its front is  $\sim 10^4 - 10^5 \text{ cm/s}$ ) does not propagate toward the beam, i.e. pulse duration must be  $\leq 5 \mu\text{s}$ . Pulse intensity is lower than the threshold of gas breakdown exterior to the cavern but sufficient for medium threshold in the channel with  $L_p < Z < L_c$ . The frequency of pulses' sequence  $f_2 \leq 30 \text{ kHz}$  is bounded by the condition of channel's transparency for passing laser radiation. In the plasma part of the channel  $Z \leq L_p$ , both non-self-maintained and self-maintained microwave discharge of diffusive type<sup>4</sup> is possible. For  $L_p < Z < L_c$ , plasma concentration is much less than the critical one for microwaves of the centimeter range  $\lambda \cong 2 - 5 \text{ cm}$ . Here, only a self-maintained discharge of diffusive or spark-like type is possible (depending on field intensity in the channel). Pulse-periodic microwave radiation is focused by an axicon throughout the channel length  $L_c$ . In this case, mean power of the microwave discharge in the channel equals

$$W' \cong 3.7 \cdot 10^{-3} \eta V J \tau f_2 Q^{1/2} \text{ (W)}.$$

Here  $J$ ,  $\tau$ ,  $f_2$  are intensity on the channel surface, duration, and frequency of pulses' sequence, respectively;  $\eta \cong 0.7$  is absorption efficiency. Radiation energy of a separate pulse is  $Q' \cong 3.7 \cdot 10^{-3} V J \tau Q^{1/2} \text{ J}$ . Energy efficiency of a laser-microwave discharge can be characterized by the ratio of  $W'$  to mean power of laser radiation necessary to create a channel  $W'/W_L \cong 8.2 \cdot 10^{-3} \eta J \tau f_2$ . Under atmospheric pressure of gas,  $J = 1 - 2 \cdot 10^4 \text{ W/cm}^2$ . The value  $W'$  is bounded by channel's extent. To lift the restriction, it is necessary to stabilize channel temperature for  $Z \geq L_1$  at the level  $T \cong 4500 \text{ K}$  optimal for absorption (depending on  $\lambda$  and frequency of collisions of electrons with gas molecules). This is possible if energy supply of microwave radiation compensates turbulent cooling of the channel. Mean power absorbed at unity of the channel length must be  $W'' \cong 4 \cdot 10^3 Q^{1/4} \text{ W/cm}$ , the extent (along  $Z$ ) of energy conductor of the microwave beam is

$L_b = 2.5 \cdot 10^{-4} W' / Q^{1/4}$  (cm) ( $W'$  is the required LMD power). Experimental results on microwave discharge in decomposing laser plasma in air and argon are presented in Ref. 4.

In conclusion, let us estimate the conditions under which LMD can be efficiently used for control over supersonic bodies. Suppose that a body of diameter  $d_a$  moves in the atmosphere with velocity  $V$  and small angle of attack. Theoretical papers demonstrate, that to lower aerodynamic resistance of a body, it is necessary to create a channel with decreased gas density or a powerful heat flow at a certain distance from the body. The optimal length  $L_c$  can be connected with the diameter  $d_a$  by the relation  $d_a - L_c = k_a d_a$  where  $k_a \cong 2 - 3$ . The method is believed to be efficient if radiation energy is small as compared with kinetic energy of the gas flow passing through a section equal to the cross section of the body  $\pi d_a^2 / 4$ . If we use only laser radiation, the velocity  $V$  must satisfy the condition

$$V \geq \frac{1500 k_a^{0.47}}{(\psi_0 \rho_0)^{0.26} d_a^{0.053}},$$

where  $\rho_0$  is gas density in the flow,  $\psi_0$  is the ratio of the powers of laser radiation and flow. The value  $\psi$  must be small  $\sim 0.001$ . Therefore, Mach's number must be rather large:  $M \geq 3$ . For  $d_a \geq 1.5$  m, the requirements to laser radiation are difficult to be realized. In this case, laser-microwave discharge is necessary, and the velocity of the body must be

$$V \geq 5000 \left( \frac{k_a}{\psi_2 d_a \rho_0} \right)^{1/3} \cong 10^5 \text{ cm/s}.$$

Here  $\psi_2 \cong 10^{-3}$  and equals the ratio of LMD power to the power of the flow.

Thus, combined use of laser and microwave radiation makes it possible to create a new type of a high power discharge which can move in the atmosphere with supersonic velocity and form extended ( $\sim$  several meters) channels of turbulent medium with lower density and high temperature. Since relative expenditure of energy on creating LMD is small, it can be used in promising methods of control over flights of supersonic bodies.

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#### REFERENCES

1. P.K. Tret'yakov, G.N. Grachev, F.I. Ivanchenko, et al., Dokl. Ross. Akad. Nauk **336**, No. 4, 466-467 (1994).
2. G.N. Grachev, A.G. Ponomarenko, V.N. Tishchenko, et al., Laser Physics **6**, No. 2, 376-379 (1996).
3. P.K. Tret'yakov, A.F. Garanin, G.N. Grachev, et al., Dokl. Ross. Akad. Nauk **351**, No. 3, 339-340 (1996).
4. V.N. Tishchenko, V.M. Antonov, A.V. Melekhov, et al., Pis'ma Zh. Tekh. Fiz. **22**, No. 24, 30-34 (1996).
5. V.P. Korobeinikov, in: *Proceedings of V.A. Steklov Mathematical Institute* (Nauka, Moscow, 1973), p. 277.
6. K.P. Stanyukovich, *Physics of Explosion* (Nauka, Moscow, 1975), 704 pp.
7. P.Yu. Georgievskii and V.A. Levin, Pis'ma Zh. Tekh. Fiz. **14**, No. 8, 681-687 (1988).
8. I.V. Nemchinov, V.I. Artem'ev, V.I. Bergelson, et al., Shock Waves, No. 4, 35-40 (1994).
9. V.Yu. Borzov, V.M. Mikhailov, I.V. Rybka, et al., Inzh. Fiz. Zh. **66**, No. 5, 515-520 (1994).
10. L.N. Myrabo and Yu.P. Raizer, AAIA Paper 94-2451.
11. S.N. Kabanov, L.I. Maslova, T.I. Tarkhova, et al., Zh. Tekh. Fiz. **60**, No. 6, 37-41 (1990).