Derivation of the system of evolutionary equations for optical bundles

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Received December 30, 2005

The system of evolutionary differential equations describing the propagation of radiation in optical bundles with the allowance for the coefficients of coupling between the fibers is obtained. It is shown that, in optical bundles consisting of single-mode fiber elements, the field amplitude in bundle cross section satisfies the parabolic (diffusion) or Helmholtz equation, in which the diffusion coefficient is determined by the distance between the centers of fiber cores and by the overlap integral of interacting modes. For few-mode and multimode fiber channels, the system of equations can be solved by the method of splitting into physical processes. The problem of the influence of adiabatic (conic) increase of the fiber core radius on the contrast in the spatially unsteady regime is considered. Based on the model of pairwise interaction of the fibers, the parameters of the transfer function in optical bundles are calculated from the analysis of cross talk. The influence of polarization corrections to propagation constants of simple fiber on the parameters of the transfer function is estimated from the results of numerical calculations.

Introduction

The progress in fiber optics became possible, in the first turn, due to the development of high-purity technologies of fabrication of fibers with low optical losses based on fused silica. The promising field of application of fibers, in particular of optical bundles, is presented by sensors of various physical fields, such as the electric and magnetic fields, mechanical displacements, pressure, temperature, etc. A wide variety of materials allows the physical parameters of fibers to be varied in a wide range and thus the needed characteristics of fiber sensors to be obtained.

Now the need in sensors grows fast in connection with the rapid development of automated control systems, implementation of new technological processes, transition to flexible manufacturing systems, growing safety requirements to dangerous industries. Fiber-optics sensors and devices based on fibers, in particular, optical bundles meet these requirements best of all. In the case of a multifiber bundle, each optical fiber has a diameter of 14–30 μm, and the density of image elements ranges from 1000 to 10 000 fibers. Such light guides allow images to be transferred to a distance of 5 to 10 m.[1] Another type of optical bundles is presented by multicore light guides, in which the core diameter is 4 to 12 μm, and the image density is more than 10 000 light-guiding cores. Such bundles allow images to be transferred to a distance of 100 m.[2]

The intense investigations connected with the study of the transfer properties of optical bundles and the optimization of their parameters are carried out nowadays. For example, in Ref. 2 Nakamura and Kitayama study the image skew in an optical bundle and the influence of the bundle bend on the dispersion of modes of different light-guiding channels. It is also shown there that to provide for the transfer rate of 1Gb/s/channel at a length of 100 m, the parameters of all cores making up the bundle should be identical during the bundle production. In Ref. 3, the theory is proposed for the determination of the transfer function of an optical bundle. The approach is based on the assumption of the presence of strong cross talk. The transfer function is determined with the use of the Fourier–Bessel transform. The transfer function of the cross talk of light-guiding channels is studied experimentally in Ref. 4.

The connection between light-guiding channels in optical bundles is determined by the optical cross talk.[5] The cross talk can be described in two ways: 1) to calculate the power exchange between the light-guiding channels, the modes of the complex waveguide are determined directly, and the interaction manifests itself in the interference between modes of the complex waveguide; 2) the modes of each individual light guide are determined separately, and the cross talk is characterized by the coupling coefficient.

In this paper, the system of equations is derived for the amplitudes of the modes of individual light guides in optical bundles. It is shown that, using the method of splitting into physical processes, this system of equations for the amplitudes of identical modes can be reduced to economical splitting schemes and solved numerically. In the limit of large number of single-mode light-guiding channels, it is shown that the process of energy exchange between the light-guiding elements is described by the diffusion equation, in which the "diffusion coefficient" is determined by the value of the cross talk.

0235-6880/06/02–03 223-06 $02.00

1. Derivation of the system of difference-operator evolutionary equations for optical bundles

Consider the area $AEDF$ (see Fig. 1), filled with simple light guides with the diameter of the light-guiding element $d = 2\rho$ (ρ is the core radius) and the shield diameter $D = 3d$. In this case, the light-guiding channels are few-mode light guides. Let the radius of the circle inscribed into the $AEDF$ trapezium is equal to R_0. The position of the light-guiding elements is characterized by the numbers i, and j, where

$$x_i = iD; \quad y_j = jD. \tag{1}$$

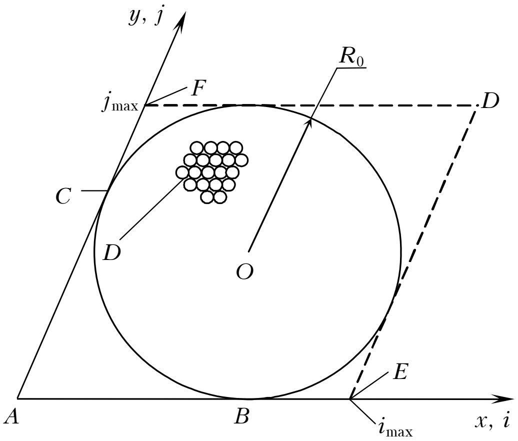

Fig. 1. Arrangement of light-guiding elements with respect to the axes AX and AY.

If the field amplitude at the input of the optical bundle is

$$\Phi(x,y) = \sum_{N,m} \Phi_{N,m}(x,y), \tag{2}$$

then the solution is sought in the form of the sum of linear combinations of eigenfunctions $\psi_{N,m}(x - iD, y - jD)$ of the modes entered into each light-guiding element

$$\Psi(x,y) = \sum_{N,m;i,j} A_{i,j}^{N,m}(z)\Psi_{N,m}(x - iD, y - jD), \tag{3}$$

where, according to Eqs. (2) and (3),

$$A_{i,j}^{N,m}(z = 0) = \iint \Psi^*_{N,m}(x - iD, y - jD)\Phi_{N,m}(x,y)\,dx\,dy. \tag{4}$$

For the coefficients $A_{i,j}^{N,m}$ we obtain, from Eq. (3), the system of evolutionary equations

$$\frac{\partial^2 A_{i,j}^{N,m}}{\partial z^2} + \beta_{N,m}^2 A_{i,j}^{N,m} + \sum_{\tilde N,\tilde m}\Big(\Lambda_{\tilde N,\tilde m}^{N,m}(i-1,j)\,A_{i-1,j}^{\tilde N,\tilde m} + \Lambda_{\tilde N,\tilde m}^{N,m}(i-1,j+1)A_{i-1,j+1}^{\tilde N,\tilde m} + \Lambda_{\tilde N,\tilde m}^{N,m}(i,j+1)\,A_{i,j+1}^{\tilde N,\tilde m} + \Lambda_{\tilde N,\tilde m}^{N,m}(i+1,j)\,A_{i+1,j}^{\tilde N,\tilde m} + \Lambda_{\tilde N,\tilde m}^{N,m}(i+1,j-1)\,A_{i+1,j-1}^{\tilde N,\tilde m} + \Lambda_{\tilde N,\tilde m}^{N,m}(i,j-1)\,A_{i,j-1}^{\tilde N,\tilde m}\Big) = 0, \tag{5}$$

where

$$\Lambda_{\tilde N,\tilde m}^{N,m}(i,j) = \frac{k^2\iint \Psi^*_{N,m}(\mathbf{r})\Psi_{\tilde N,\tilde m}(\mathbf{r}-\mathbf{R}_{ij})(n^2 - n_{ij}^2)\,dx\,dy}{\iint |\Psi_{N,m}|^2\,dx\,dy}; \tag{6}$$

$$\mathbf{R}_{ij} = (iD, jD);$$

$$n^2 - n_{ij}^2 = \begin{cases} n_{\rm co}^2 - n_{\rm cl}^2, & 0 \le |\mathbf{r} - \mathbf{R}_{ij}| \le \rho, \\ 0, & |\mathbf{r} - \mathbf{R}_{ij}| > \rho. \end{cases}$$

The system of equations obtained can be solved with the aid of the method of splitting into physical processes,[6] according to which the coefficients $A_{i,j}^{N,m}$ in Eq. (5) are calculated by the implicit scheme, while the coefficients $A_{i,j}^{\tilde N,\tilde m}$ are taken from the explicit layer. It is seen from Eq. (5) that this system of equations is a system of difference equations with a seven-point stencil.

The difference-operator system of equations of hyperbolic type (5) can be written in the form of the difference-operator system of equations of parabolic type by separating out the fast oscillating part $A_{i,j}^{N,m} \sim \exp(i\beta z)$ of the solution:

$$2i\beta\frac{\partial A_{i,j}^{N,m}}{\partial z} + \left(\beta_{N,m}^2 - \beta^2\right)A_{i,j}^{N,m} + \sum_{\tilde N,\tilde m}\Big\{\Big(\Lambda_{\tilde N,\tilde m}^{N,m}(i-1,j)\,A_{i-1,j}^{\tilde N,\tilde m} + \Lambda_{\tilde N,\tilde m}^{N,m}(i-1,j+1)\,A_{i-1,j+1}^{\tilde N,\tilde m}\Big) + \Lambda_{\tilde N,\tilde m}^{N,m}(i,j+1)\,A_{i,j+1}^{\tilde N,\tilde m} + \Lambda_{\tilde N,\tilde m}^{N,m}(i+1,j)\,A_{i+1,j}^{\tilde N,\tilde m} + \Lambda_{\tilde N,\tilde m}^{N,m}(i+1,j-1)\,A_{i+1,j-1}^{\tilde N,\tilde m} + \Lambda_{\tilde N,\tilde m}^{N,m}(i,j-1)\,A_{i,j-1}^{\tilde N,\tilde m}\Big\} = 0. \tag{7}$$

This system of equations can be also solved by the method of splitting into physical processes and is a system of difference equations with a seven-point stencil as well.

2. Derivation of the system of evolutionary equations of parabolic and hyperbolic type for bundles with single-mode light-guiding channels

The system of equations allows unique theoretical results to be obtained. Let an optical bundle consist of single-mode light-guiding elements. In this case, there is only the main mode and, correspondingly, one element $\Lambda_{\tilde N,\tilde m}^{n,m}(i,j)$. Let

$$\Lambda_{\tilde N,\tilde m}^{N,m}(i,j) = \Lambda, \quad A_{i,j} = A_{i,j}^{\tilde N,\tilde m}, \quad \beta_{N,m}^2 = \beta_0^2. \tag{8}$$

Then Eq. (5) can be written in the form

$$\frac{\partial^2 A_{i,j}}{\partial z^2} + \beta_0^2 A_{i,j} +$$

$$+ \Lambda\left(A_{i-1,j} + A_{i-1,j+1} + A_{i,j+1} + A_{i+1,j} + A_{i+1,j-1} + A_{i,j-1}\right) = 0. \quad (9)$$

Equation (9) is a difference-operator equation. The expression in parenthesis is an approximation of some operator in the nonorthogonal coordinate system (see Fig. 1). Let us show that the expression in parenthesis can be represented through the difference Laplace operator in a two-dimensional domain.

Let i be plotted along the OX axis and j be plotted along the OY axis. The angle between the axes is equal to $\pi/3$. According to the formulas known from differential geometry,[7] we obtain that the metric tensor g_{nm}, Laplacian Δ, and the determinant of the metric tensor in the curvilinear coordinate system x and y are:

$$g_{nm} = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}, \quad g^{nm} = \frac{4}{3}\begin{pmatrix} 1 & -0.5 \\ -0.5 & 1 \end{pmatrix}, \quad g = 1,$$

$$\Delta = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^n}\left(\sqrt{g}\, g^{nm}\frac{\partial}{\partial x^m}\right) = \frac{4}{3}\left\{\frac{\partial^2}{\partial (x)^2} - \frac{\partial^2}{\partial x\,\partial y} + \frac{\partial^2}{\partial (y)^2}\right\}. \quad (10)$$

To prove that the expression in the second parenthesis is the difference Laplace operator in a two-dimensional domain, we superpose the origin of the Cartesian system of x^1 and x^2 coordinates onto the point $(i,\ j)$ and direct the axis x^1 through the point $(i + 1,\ j + 1)$ and the axis x^2 through the point $(i - 1,\ j + 1)$. With this choice of the Cartesian coordinate system, we obtain

$$A_{i-1,j+1} - 2A_{i,j} + A_{i+1,j-1} = D^2\left\{\frac{\partial^2}{\partial (x^2)^2}A\right\} + \Theta(D^4), \quad (11)$$

where $A = A(x^1,\ x^2)$; $\Theta(D^4) \sim D^4$.

The coordinates x and y (see Fig. 1) and the coordinates of the Cartesian coordinate system are related as follows:

$$x^1 = \frac{\sqrt{3}}{2}x + \frac{\sqrt{3}}{2}y + \text{const}, \quad x^2 = -\frac{1}{2}x + \frac{1}{2}y + \text{const}. \quad (12)$$

It follows from Eq. (12) that

$$\frac{\partial^2}{\partial (x)^2} = \left(\frac{\sqrt{3}}{2}\frac{\partial}{\partial x^1} - \frac{1}{2}\frac{\partial}{\partial x^2}\right)^2,$$

$$\frac{\partial^2}{\partial (y)^2} = \left(\frac{\sqrt{3}}{2}\frac{\partial}{\partial x^1} + \frac{1}{2}\frac{\partial}{\partial x^2}\right)^2. \quad (13)$$

From Eq. (11) we finally obtain that

$$A_{i+1,j} + A_{i-1,j} + A_{i,j+1} + A_{i,j-1} - 4A_{i,j} =$$

$$= D^2\left\{\frac{\partial^2}{\partial (x)^2} + \frac{\partial^2}{\partial (y)^2}\right\}A(x^1,x^2) =$$

$$= \left\{\frac{3}{2}\frac{\partial^2}{\partial (x^1)^2} + \frac{1}{2}\frac{\partial^2}{\partial (x^2)^2}\right\}A(x^1,x^2). \quad (14)$$

Using Eqs. (11) and (14), for the expression in parenthesis in Eq. (9) we can write:

$$A_{i-1,j} + A_{i-1,j+1} + A_{i,j+1} + A_{i+1,j} + A_{i+1,j-1} + A_{i,j-1} =$$

$$= 6A_{i,j} + \frac{3}{2}D^2\left\{\frac{\partial^2}{\partial (x^1)^2} + \frac{\partial^2}{\partial (x^2)^2}\right\}A(x^1,x^2). \quad (15)$$

With the allowance made for Eq. (15), the Eq. (9) takes the form

$$\frac{\partial^2 A(x,y)}{\partial \tilde{z}^2} + \Delta_\perp A + k_0^2 A = 0, \quad (16)$$

where

$$\Delta_\perp = \frac{\partial^2}{\partial (x^1)^2} + \frac{\partial^2}{\partial (x^2)^2}$$

is the two-dimensional Laplacian;

$$k_0^2 = \xi^2\left(\beta_0^2 + 6\Lambda\right); \quad z = \xi\tilde{z};$$

$$\xi = \left(\frac{2}{3\Lambda}\right)^{1/2}\frac{1}{D}; \quad k_0 = \xi\beta. \quad (17)$$

Thus, the field amplitude in an optical bundle consisting of single-mode light-guiding elements satisfies the Helmholtz equation (16), that is, the steady-state wave equation.

Correspondingly, with the aid of Eq. (7) for the optical bundle consisting of single-mode light-guiding elements, the evolutionary equation for the field amplitude can be written in the form of the parabolic (diffusion) equation:

$$2i\beta\frac{\partial A(x,y)}{\partial z} + \frac{3}{2}\Lambda D^2\Delta_\perp A(x,y) = 0. \quad (18)$$

Thus, the field amplitude in the optical bundle consisting of single-mode fibers can be found from the solution of the diffusion equation (18).

3. Methods for solution of evolutionary equations

As was noted above, the system of equations for the coefficients $A_{i,j}^{N,m}$ in the case of an optical bundle with few- or multimode light-guiding channels can be solved numerically in the general case, for example, by the method of splitting into physical processes. The situation changes for single-mode light-guiding channels. In this case, there is a wide class of analytically soluble problems. For the Helmholtz equation (16), it is possible to use the method of Green's function, well known in mathematical physics and widely used in optics. In this method, the solution at a given point can be expressed through surface integrals of the initial wave front.[8] In the method of Green's function itself, there is a wide class of problems, solved analytically in accordance with the theory of light diffraction, for example, the light diffraction on the boundary of an opaque half-plane. In the case of the parabolic equation (18), there is also a wide class of analytically solved problems, in particular, those solved by the method of Green's function.

With modern powerful computers, Eq. (18) can be solved numerically based on well developed and studied efficient difference schemes. The high stability of these schemes allows the corresponding programs to be realized quite easily, without the aid from highly skilled specialists. In addition, the diffusion equation describes the transformation of spherical wave fronts in an optical lens. To solve Eq. (18), it is possible to use the Fourier transform for the transverse coordinates and, using then the inverse Fourier transform, to obtain the analytical solution.

4. Method of increasing the image contrast and the translational length of an optical bundle

The coupling between light-guiding channels in optical bundles is determined by the optical cross talk.[5] The cross talk can be described in two ways. In the one case, to calculate the power exchange between light-guiding channels, modes of a complex waveguide are determined directly, and the interaction manifests itself in the interference of individual modes of a complex light guide. In the other case, the modes of each simple light guide are determined separately, and the cross talk is characterized by the coupling coefficient.

In this paper, for optical bundles of various design the value of K, being an analog of contrast, is determined using the second method from the model of the pairwise interaction of simple light guides (light-guiding elements). Based on the results obtained, a method is proposed to increase the image contrast and the translational length of an optical bundle due to the conical thickening of the light-guide core in the domain of the spatially unsteady regime. Let us consider the problems formulated.

In quantum mechanics, there is the theory of adiabatic change of parameters of a quantum system (Ref. 7, p. 223), according to which in the limit of perturbation varying arbitrarily slowly in time the probability that a system transits from one state to another tends to zero. This means that if a certain spectrum of states (for light guides, this means a certain number of modes with a certain probability) is generated at the initial time, then the spectrum composition does not change upon the adiabatic perturbation. Since the process of radiation propagation in complex light guides is described by a scalar wave equation, being essentially the Schrödinger equation, in which the longitudinal coordinate z plays the role of time, the theorem of adiabatic perturbation can be applied to light guides.

In this paper, it is proposed to use the smooth increase of the core radius ρ of a fiber in an optical bundle as a function of the longitudinal coordinate z as the adiabatic perturbation. According to the theorem of adiabatic perturbation, if the cross talk is neglected, the fraction of the mode power does not change. The fraction of the power transferred at the expense of the cross talk is determined both by the length of the region of the mode interaction and by the integral of overlap of the interacting modes. If the length of the region of adiabatic increase of the core radius is chosen small, then the cross talk can be neglected, because the mode interaction due to the cross talk decreases sharply as the core radius increases, as will be shown below with the aid of numerical calculations. This means that if the core radius is not changed after its adiabatic increase, then no power is transferred due to the cross talk in the further translation of a signal. Let us study this issue in a more detail.

The contrast in optical bundles is affected both by the power transferred due to the cross talk, and by the amplitude of the generated mode of a simple light guide. Therefore, we first estimate the amplitude of different modes in a simple light guide.

Let the Gaussian amplitude profile be entered into the light guide

$$\Psi = \Psi_0 \exp\left(-\tilde{r}^2/(2a^2)\right);$$
$$a = \rho\sqrt{N/100};\ N = 35\,000; \qquad (19)$$
$$\rho = 3\ \mu\text{m};\ \tilde{r}_{\max} \approx \rho\sqrt{N}.$$

The amplitude of the generated mode $m \neq 0$ is determined by the value of the mth derivative of Ψ with respect to the transverse coordinates. The derivative of the amplitude of the input field (19) in the light guide with respect to the radius is maximum at $\tilde{r} = a$. The amplitude of the generated mode $m \neq 0$ is proportional to $\Psi(\tilde{r} = a)$ and $\sim(\rho/a)^m \ll 1$. Correspondingly, the fraction of power of the mode $\Psi_{N,m}$ is proportional to $\sim(\rho/a)^{2m} = (100/N)^m$. At $N = 35\,000$ for $m = 1$ we obtain $(\rho/a)^{2m} \approx 3 \cdot 10^{-3}$ (at $m > 1$ this number is even smaller). Therefore, in the further consideration for the rather smooth function of the input signal Ψ we take into account only the modes $m = 0$, and neglect the modes $m \geq 1$.

The interaction between the light-guiding channels in a complex light guide is characterized by the parameter K, being an analog of contrast:

$$K = (W_1 - W_2)/W_1, \qquad (20)$$

where $W_1(z)$ and $W_2(z)$ for the modes with $m = 0$ are determined as

$$W_2(z) = W_0 \sin^2\{\Lambda z/(2\beta_{N=1,m=0})\},$$
$$W_1(z) = W_0 - W_2(z), \qquad (21)$$

where

$$W_0 = \iint_{core} \left|\Psi_{N=1,m=0}(r,\varphi,z)\right|^2 r\mathrm{d}r\mathrm{d}\varphi. \qquad (22)$$

In Eq. (21), the parameter Λ characterizes the cross talk and is calculated as follows:

$$\Lambda = \sqrt{\Lambda_{12}\Lambda_{21}},\quad \Lambda_{21} = \frac{k^2 \iint \Psi_2^*\Psi_1(n^2 - n_2^2)\,dx\,dy}{\iint |\Psi_2|^2\,dx\,dy},$$

$$\Lambda_{12} = \frac{k^2 \iint \Psi_1^*\Psi_2(n^2 - n_1^2)\,dx\,dy}{\iint |\Psi_1|^2\,dx\,dy}, \qquad (23)$$

where in Λ_{21}

$$\Psi_1(\mathbf{r}) = \Psi^{\mathrm{I}}_{N,m}(\mathbf{r}),\quad \Psi_2(\mathbf{r}) = \Psi^{\mathrm{II}}_{N,m}(\mathbf{r}-\mathbf{R}),$$

and in Λ_{12}

$$\Psi_1(\mathbf{r}) = \Psi^{\mathrm{III}}_{N,m}(\mathbf{r}),\quad \Psi_2(\mathbf{r}) = \Psi^{\mathrm{IV}}_{N,m}(\mathbf{r}-\mathbf{R}).$$

In Eq. (23), the following designations are introduced:

$$n^2 - n_2^2 = \begin{cases} n_{co}^2 - n_{cl}^2, & 0 \le r \le \rho, \\ 0, & r > \rho, \end{cases}$$

$$n^2 - n_1^2 = \begin{cases} n_{co}^2 - n_{cl}^2, & 0 \le |\mathbf{r}-\mathbf{R}| \le \rho, \\ 0, & |\mathbf{r}-\mathbf{R}| > \rho. \end{cases} \qquad (24)$$

It can be seen that if the wave functions in Eq. (23) are normalized to unity, $\Lambda_{12} = \Lambda_{21}^*$ according to Eq. (23). In this case, we have

$$\Lambda = |\Lambda_{21}| = |\Lambda_{12}|. \qquad (25)$$

The Table presents the values of $K_{N,N}$ for the pairwise interaction of the modes N and $m = 0$, calculated according to Eq. (20) (the subscript N of $K_{N,N}$ corresponds to the radial quantum number of the interacting modes); the period of the complete transfer L of the mode power from one core to another in a complex two-fiber light guide; Λ from Eq. (23) characterizes the cross talk value. For the calculations summarized in the Table

$$\Psi^{\mathrm{I}}_{N,m}(\mathbf{r}) = \Psi^{\mathrm{III}}_{N,m}(\mathbf{r}) = \Psi_{N,m=0}(\mathbf{r}),$$

$$\Psi^{\mathrm{II}}_{N,m}(\mathbf{r}-\mathbf{R}) = \Psi^{\mathrm{IV}}_{N,m}(\mathbf{r}-\mathbf{R}) = \Psi_{N,m=0}(\mathbf{r}-\mathbf{R}).$$

Now assume that the light-guiding elements in an optical bundle with the core radius ρ = 2 μm transit adiabatically into the light-guiding elements with the radius ρ = 3 μm. In this case, according to the adiabatic theorem known from quantum mechanics,[7] the wave function of the main mode $\Psi_{N=1,m=0}$ transit adiabatically into the wave function of the identical mode, but with a different core. According to the Table, in this case the contrast between the modes entered (denoted as $K_{N=1,N=1}$ in the Table) is nearly equal to unity, because the increase of the optical volume leads to the decrease of the cross talk. This improves the characteristics of the optical bundle, but an alternative effect, deteriorating its characteristics, arises.

Because of the increase of the optical volume due to the increase of the core radius, a new launched mode $\Psi_{N=2,m=0}$ arises (see the Table), as a result of which, first, the interaction becomes possible between the main mode of some core with the new mode $\Psi_{N=2,m=0}$ in the neighboring core and, second, strong interaction takes place between the amplitudes of the modes $\Psi_{N=2,m=0}$ (in the Table, the contrast between these launched modes is designated as $K_{N=2,N=2}$ and is equal to 0.68 at ρ = 3 μm). Since the mode $N = 2$, $m = 0$ is absent in the light guide with ρ = 2 μm (this mode is leaking and, therefore, it quickly leaves the core), this mode can appear at ρ = 3 μm only as a result of the interaction with the main mode due to the cross talk. However the numerical calculations show that $\Lambda = 1.58 \cdot 10^{-8}$ μm^{-2} at such an interaction. Since $\beta^2_{N=1,m=0} - \beta^2_{N=2,m=0} \gg \Lambda$, the maximum contrast K_{max} from Eq. (20) can be written in the form

$$K_{max} \approx 4\Lambda^2 / \left(\beta^2_{N=1,m=0} - \beta^2_{N=2,m=0}\right) \approx 5.6 \cdot 10^{-16}.$$

This means that the mode $N = 2$, $m = 0$ virtually is not generated as a result of the interaction with the main mode due to the cross talk. This, in its turn, means that at the adiabatic thickening of the light-guiding channels ρ = 2 μm → ρ = 3 μm the characteristics of the optical bundle do not deteriorate. It is seen from the calculations presented that, after the thickening of the light guide with ρ = 3 μm, the character of the radiation propagation is nearly single-mode (the mode $N = 2$, $m = 0$ is absent).

Table

ρ, μm	5	4.5	4	3.2	3.1	3	2
$K_{N=1,N=1}$	1	1	1	1	1	1	0.97
Λ, μm^{-2}	$1.33 \cdot 10^{-17}$	$6.33 \cdot 10^{-16}$	$3.19 \cdot 10^{-14}$	$1.95 \cdot 10^{-11}$	$4.43 \cdot 10^{-11}$	$1.01 \cdot 10^{-10}$	$5.03 \cdot 10^{-7}$
L, m	$3.4 \cdot 10^{12}$	$7.2 \cdot 10^{10}$	$1.4 \cdot 10^{9}$	$2.3 \cdot 10^{6}$	$1.03 \cdot 10^{6}$	$4.5 \cdot 10^{5}$	90.4
$K_{N=2,N=2}$	1	1	1	0.997	0.97	0.68	
Λ, μm^{-2}	$8.37 \cdot 10^{-15}$	$6.6 \cdot 10^{-13}$	$6.27 \cdot 10^{-11}$	$1.71 \cdot 10^{-7}$	$4.98 \cdot 10^{-7}$	$1.49 \cdot 10^{-6}$	
L, m	$5.4 \cdot 10^{15}$	$6.9 \cdot 10^{7}$	$7.3 \cdot 10^{5}$	$2.7 \cdot 10^{2}$	91.5	30.5	
$K_{N=3,N=3}$	1	0.85					
Λ, μm^{-2}	$1.27 \cdot 10^{-9}$	$1.07 \cdot 10^{-6}$					
L, m	$3.6 \cdot 10^{4}$	42.7					

N o t e s . $NA = 10^0$; $m = 0$; $n_{co} = 1.46$, $n_{cl} = 1.44953$; $R = 6\rho$; $z = 10^7$ μm; λ = 0.63 μm.

It should be noted that, according to the Table, similar situation takes place at the adiabatic transition $\rho = 2$ μm → $\rho = 3.1$ μm, but in this case $K_{N=2,N=2} = 0.97$. In the case of $\rho = 2$ μm → $\rho = 3.2$ μm, $K_{N=2,N=2} = 1$, but the launched mode $N = 2$, $m = 1$ appears (this mode also does not deteriorate the characteristics of the optical bundle). The conditions, analogous to the problem considered above, take place at the adiabatic transition $\rho = 4.5$ μm → $\rho = 5$ μm for the mode $N = 3$, $m = 0$.

Conclusions

For the field amplitude and the bundle with arbitrary light-guiding channels, the system of difference evolutionary equations has been derived, in which the core of the light-guiding channels play the role of the scheme nodes. It can be easily shown that the initial system of difference equations can be readily reduced to the system of differential equations (Helmholtz or parabolic-type equations), which can be solved using other grids, at the investigator's discretion.

It is shown that the field amplitude in the optical bundle, consisting of single-mode light-guiding elements, meets either the Helmholtz equation (16) or the diffusion equation (18).

The Helmholtz equation (16) allows the well-developed method of the Green's function to be used. The diffusion equation (18) is very convenient for estimating the contrast. Indeed, first, from the Green's function, for Eq. (18) it is possible to determine the rms increase a^2 of the initial signal ($a^2 \sim z$). Second, for Eq. (18), using the direct and inverse Fourier transforms, it is possible to obtain a rather wide class of analytical solutions. Third, Eq. (18) can be solved numerically based on economical schemes.[6]

It is predicted and confirmed by the numerical calculations that the adiabatic (conical) increase of the core radius of the light-guiding elements in the region of the spatially unsteady regime improves the contrast.

Acknowledgments

This work was supported, in part, by the Russian Foundation for Basic Research (Grant No. 04–02–96039).

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