EFFECT OF ORIENTATION OF ELONGATED ICE PARTICLES TO A HORIZONTAL PLANE ON THE PARTICLE LIGHT SCATTERING CHARACTERISTICS

D.N. Romashov and R.F. Rakhimov

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk
Received November 2, 1993

In this paper we analyze the effect of preferred orientation of axisymmetric elongated particles (AEP) to the horizontal plane on the light scattering phase matrix (LSPM). It has been shown that analysis of the relationships between the elements of the LSPM provides for determining the angle and the degree of preferred orientation of the AEP.

1. CALCULATIONAL TECHNIQUE

Investigations of the patterns of light scattering by nonspherical particles are of interest not only for specialists in the theory of radiative transfer through the atmosphere but also for improvement of the methods of optical monitoring of the atmosphere, particularly of polarization sounding of mixed clouds.

Hexagonal column and plate crystals whose size varies from a few to hundreds of micrometers are the most commonly encountered shapes of ice particles in mixed clouds. As is well known,[1] orientation of particles sedimentating in the quiescent atmosphere depends on the particle shape and the Reynolds number $Re=lu/\nu$, where l is the characteristic size of a particle, u is the rate of sedimentation, and ν is the kinematic viscosity of a medium. In particular, sedimentating particles with $Re \le 0.1$ retain their orientation at the beginning of motion. With increasing Re particles start to orient in such a way that their resistance to motion becomes maximum.

For example, according to the data of Ref. 2, the precise orientation is observed for plate and column crystals with $Re \le 100$, and one may expect the noticeable oscillations of free–falling crystals only when they are very large ($Re > 100$). But the atmosphere is far from being the stable system, and it is clear that the intense turbulent mixing may result in oscillations of the axes of crystals even with small Reynolds number. The role of turbulent air fluxes in orientation of particles is poorly known.

This paper is devoted to analysis of the effect of the slope angle of the column particle axes with respect to the horizontal plane on the light scattering phase matrix (LSPM) elements.

It should be noted that in the general case of arbitrary oriented hexagonal ice crystals the light scattering characteristics of an individual particle must be averaged over all possible orientations determined by the rotation of a column about its principal axis. It is clear that for uniform distribution the available peculiarities of light scattering caused by fine geometric structure of a crystal (its faces) are strongly smoothed out. This makes it possible to interpret the crystal as circular cylinders of finite length (CCFL).

The approximate solution of the problem on electromagnetic wave scattering by homogeneous circular cylinder of finite length was obtained in Ref. 1 in terms of amplitude functions. In the theory the scattered field is expressed in the $x'y'z'$ coordinate system (Fig. 1) affixed to a cylinder whose symmetry axis is aligned with the z' axis and the wave vector $\mathbf{k}^{(i)}$ of the incident radiation lies in the $x'z'$ plane at the angle β with respect to the z' axis. The vector $\mathbf{k}^{(s)}$ of the scattered field has arbitrary orientation specified by the polar angles θ' and φ'. Then the expression for the amplitude functions in this coordinate system is written in the form

$$S_i(\varphi', \theta', \beta, r, l) = \frac{k\,l}{p}\, E\left[\frac{k\,l}{2}(\cos q' - \cos b)\right] T_i(\varphi', \beta, r), \qquad (1)$$

where T_i (φ', β, r) are the elements of the amplitude matrix for an infinite cylinder[3]; r is the cylinder radius; $i = 1, 2, 3, 4$; $k = 2\pi/\lambda$ is the wave number; l is the cylinder length; and, $E(x) = \sin(x)/x$ is the Kotel'nikov function. The scattering phase matrix elements F_{ij} are expressed in terms of the amplitude functions S_i in a standard way[4] (i, j = 1, 2, 3, 4).

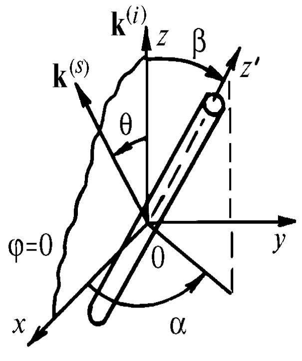

FIG. 1. Geometry of scattering by arbitrarily oriented circular cylinder.

In order to calculate the total LSPM of an ensemble of randomly oriented cylindrical particles, it is convenient to introduce another xyz coordinate system whose z axis is aligned with $\mathbf{k}^{(i)}$. The cylinder orientation, i. e., the direction of its symmetry axis, is specified by the polar angles (α, β) in the xyz coordinate system. Let us specify the polarization state of the incident light with respect to the plane $\varphi = \varphi_0$.

0235-6880/94/03 149-05 $02.00

Then, according to Ref. 5, the LSPM of an individual arbitrarily oriented cylinder in the xyz coordinate system with respect to the plane $\varphi = \varphi_0$ has the form

$$\mathbf{Z}(\theta,\ \varphi_0,\alpha,\beta,\ r,\ l) = \mathbf{L}(\pi - \gamma)\ \mathbf{F}(\theta',\ \varphi',\ r,\ l)\ \mathbf{L}(-\gamma), \qquad (2)$$

where $L(-\kappa)$ is the matrix of transformation of the Stockes parameters upon clockwise rotation of the reference plane through the angle κ when looking in the wave propagation direction,

$$\mathbf{L}(-\kappa) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2k & -\sin 2k & 0 \\ 0 & \sin 2k & \cos 2k & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad (3)$$

$\mathbf{F}(\theta',\ \varphi',\ r,\ l)$ is the matrix of transformation of the Stockes parameters in the $x'y'z'$ coordinate system .

The angles θ', φ' and γ in Eq. (2) for the given cylinder orientation $(\alpha,\ \beta)$ can be expressed in terms of the angles θ, β, and $\alpha - \varphi_0$:

$$\cos\theta' = \cos\theta\ \cos\beta + \sin\theta\ \sin\beta\ \cos(\alpha - \varphi_0),$$

$$\cos\varphi' = [\cos\theta\ \sin\beta - \sin\theta\ \cos\beta\ \cos(\alpha - \varphi_0)]\ /\ \pm \sin\theta', \qquad (4)$$

$$\cos\gamma = [\cos\beta\ \sin\theta - \sin\beta\ \cos\theta\ \cos(\alpha - \varphi_0)]\ /\ \pm \sin\theta',$$

where the sign plus in two last cases is taken for $0 < \alpha - \varphi_0 < \pi$, and minus is taken for $\pi < \alpha - \varphi_0 < 2\pi$.

Let us consider the LSPM for polydisperse ensemble of cylinders that have uniformly oriented angle α and the angle β symmetric about the z axis. In this case the scattered field is independent of the azimuth angle φ_0. So we can select the xz plane as a scattering plane. The Stokes parameters of the radiation scattered by all randomly oriented particles represent the sum of the Stokes parameters of each particle. So the average LSPM $\overline{\mathbf{F}}(\theta)$ for the case of arbitrarily oriented particles can be derived by integrating the LSPM elements over all possible orientations and size

$$\overline{\mathbf{F}}(\theta) = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\int_{l_1}^{l_2}\int_{r_1}^{r_2} \mathbf{Z}(\theta, 0, \alpha, \beta, r, l)\, g(\beta)\, n(r, l) \sin\beta\, \mathrm{d}\beta\, \mathrm{d}\alpha\, \mathrm{d}r\, \mathrm{d}l, \qquad (5)$$

where $g(\beta)$ and $n(r,l)$ are the distribution functions of cylinders over the angle β and size.

Due to the above–enumerated conditions of orientation of the angles α and β of the particles and taking into account that cylindrical particles have the symmetry plane, the LSPM $\overline{\mathbf{F}}(\theta)$ is the function of only scattering angle θ and consists of six independent elements

$$\overline{\mathbf{F}}(\theta) = \begin{pmatrix} \overline{f}_{11} & \overline{f}_{12} & 0 & 0 \\ \overline{f}_{12} & \overline{f}_{22} & 0 & 0 \\ 0 & 0 & \overline{f}_{33} & -\overline{f}_{43} \\ 0 & 0 & \overline{f}_{43} & \overline{f}_{44} \end{pmatrix}. \qquad (6)$$

We have used this characteristic form of the matrix to check the accuracy of integration of Eq. (5). Based on the fact that the cylinder has the symmetry plane perpendicular to its axis, we may integrate over the angle α in Eq. (5) from 0 to π.

It is convenient to use the normalized LSPM

$$\mathbf{P}(\theta) = \frac{4\pi}{k^2\,\overline{C}_s}\,\overline{\mathbf{F}}(\theta) \qquad (7)$$

for the subsequent analysis of the obtained results, where $\overline{C}_s$ is the ensemble averaged scattering cross section and k is the wave number.

2. CALCULATED RESULTS

The values of the LSPM elements given by Eq. (5) were calculated by the algorithm described in Ref. 6, i.e., by the Monte Carlo method. The convergence of the integral was tested on the basis of numerical estimates and check of the property (6). The distribution of particles over radii of cross section was modeled by the lognormal distribution with mean geometric radius $r_m = 1$ μm (Fig. 2) and 5 μm (Fig. 3) and the standard deviation $\sigma = 0.5$.

The cylinder length was calculated by the formula[1]

$$l = A\ (2\ r)^{\varepsilon}, \qquad (8)$$

where $A = 2.07$ and $\varepsilon = 1.079$. The distribution over the angle α was modeled by the uniform distribution. The distribution over the angle β that characterizes the deviation of the cylinder axes from the horizontal plane was modeled by the uniform distribution in the interval $\pi/2 - \Delta\beta < \beta < \pi/2 + \Delta\beta$. For the two characteristic parameters we have analyzed five different orientations of the angle β with $\Delta\beta = 0$ (curve *1*), 3 (*2*), 10 (*3*), and 20° (*4*) and uniform distribution of β (*5*). All calculations were carried out for the incident radiation wavelength $\lambda = 1.06$ μm and the ice refractive index[7] $n = 1.299 - i{\cdot}10^{-6}$.

Let us analyze the angular dependence of the LSPM elements simultaneously on the degree of orientation of an ensemble of the CCFL and on the characteristic size of particles involved in it. The comparison of the behavior of curves *1* and *5* corresponding to ensembles of horizontally and uniformly oriented particles (HOP and UOP) receives primary attention.

(a) The element P_{11} is the normalized phase function. As can be seen from Figs. 2*a* and 3*a*, the behavior of P_{11} is similar for the same degrees of orientation of both considered characteristic parameters. Thus it is primarily determined by the degree of orientation. Scattering by the UOP is stronger than by the HOP at the angles $\theta < 120°$. Conversely, at $\theta > 120°$, backscattering by the HOP is 50 – 100 times stronger than by the UOP. We note that in the case of the UOP P_{11} has rather flat angular distribution slightly increasing as the scattering angle approaches 180°.

(b) The parameter $p = P_{12}/P_{11}$ that determines the degree of linear polarization of singly scattered unpolarized incident light is characterized by distinctly pronounced peak near the scattering angle $\theta = 150°$ for the HOP with $r_m = 1$ μm (curve *1* in Fig. 2*b*) and near 140° for $r_m = 5$ μm (curve *1* in Fig. 3*b*). The peak is narrower in the last case. The parameter p is largely positive for the HOP irrespective of the characteristic size. For the UOP with $r_m = 1$ μm (curve *5* in Fig. 2*b*) and 5 μm (curve *5* in Fig. 3*b*) p takes both positive and negative values and also has positive peaks near the same angles as for the HOP but their amplitudes are half as high as those for the UOP. The oscillations of p about zero are faster for $r_m = 5$ μm than for $r_m = 1$ μm.

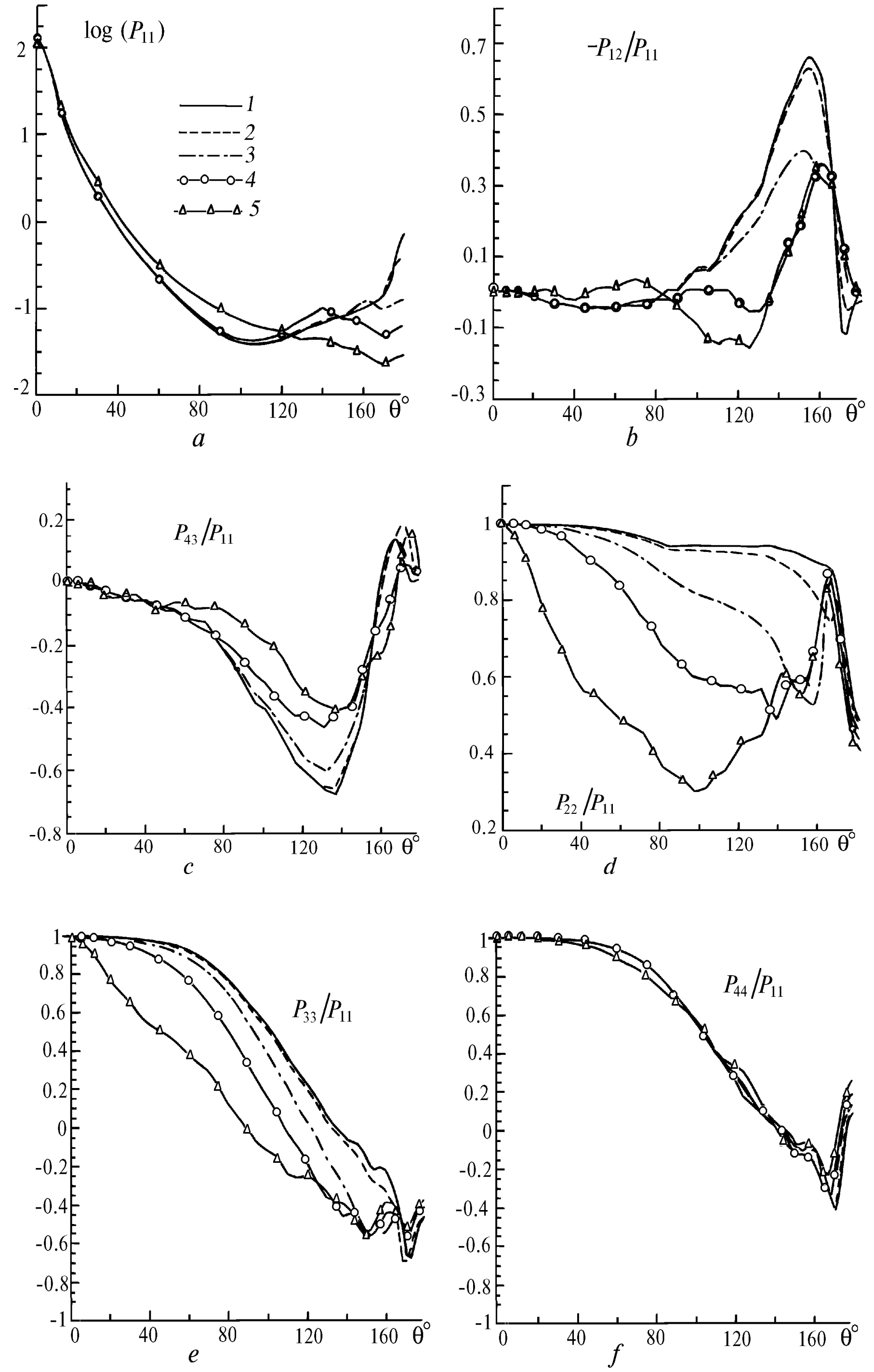

FIG. 2. Angular dependence of the normalized elements of the scattering phase matrix for an ensemble of circular cylinders with $r_m = 1$ μm *and different degree of orientation of their axes to the horizontal plane.*

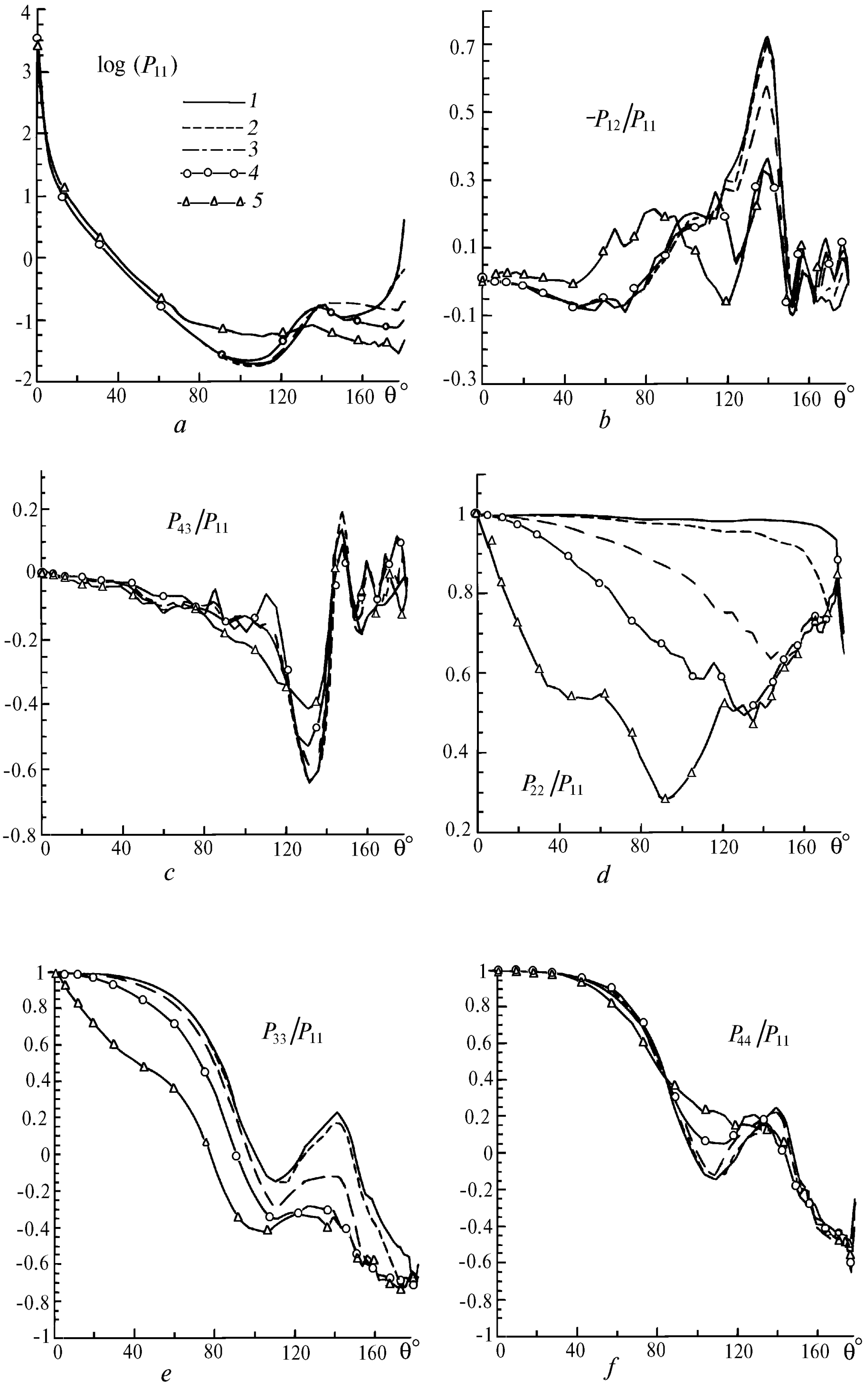

FIG. 3. The same as in Fig. 2 but for r_m = 5μm.

(c) Angular behavior of the normalized element P_{43}/P_{11} of the LSPM for the HOP with r_m = 1 μm and 5 μm (Figs. 2*c* and 3*c*) is characterized by the presence of clearly pronounced negative minimum near θ = 130°. The parameters P_{43}/P_{11} for the UOP with r_m = 1 μm and 5 μm differ from zero to a lesser degree and have less pronounced negative minimums than those for the HOP.

Noteworthy are the very fast oscillations of this parameter at the scattering angles 170° < θ < 180° for r_m = 5 μm.

(d) The parameter $\Delta = 1 - P_{22}/P_{11}$ represents the depolarization ratio for the total intensity and, since $\Delta = 0$ for uniform spheres, characterizes the degree of particle nonsphericity. Angular behavior of the element

P_{22}/P_{11} is most sensitive to the orientation degree of an ensemble of particles and weakly depends on the characteristic particle size except the scattering angles close to 180°. The values of this element are close to 1 at $\theta < 170°$ for the HOP with $r_m = 1$ μm (curve *1* in Fig. 2*d*), whereas for the UOP (curve *5* in Fig. 2*d*) they essentially differ from *1* in the same interval. The values of P_{22}/P_{11} practically coincide and essentially differ from 1 for both orientation types in the interval $170° < \theta < 180°$. Similar reasoning for both degree of orientation with $r_m = 5$ μm shows that the interval of coincidence is narrower than for $r_m = 1$ μm, and the values of the elements differ from 1 to a lesser degree. Thus we can conclude that the depolarization of the radiation scattered at the angle $\theta = 180°$ is independent of the degree of orientation of an ensemble of particles and decreases as the characteristic particle size increases.

(e) Angular behavior of the element P_{33}/P_{11} (Figs. 2*e* and 3*e*) is also very sensitive to the degree of orientation of an ensemble of particles. The values of this element are close to 1 for the HOP with $r_m = 1$ μm and 5 μm, as in the case of spherical particles, whereas they differ markedly from 1 for the UOP. As for P_{22}/P_{11}, there is an interval near 180° in which the values of the element P_{33}/P_{11} coincide for both orientations, but this interval is much narrower for $r_m = 5$ μm than for $r_m = 1$ μm .

(f) Angular behavior of the element P_{44}/P_{11} (Figs. 2*f* and 3*f*) reveals weak dependence on the degree of orientation of an ensemble of particles.

Thus even when the incident radiation is perpendicular to the horizontal plane, there is essential difference between the optical characteristics of scattering by horizontally and uniformly oriented cylindrical particles. This difference becomes especially pronounced near the scattering angles $\theta = 90°$ and 140°. So the polarized incident radiation is depolarized much stronger in the case of scattering by the UOP than by the HOP.

In its turn, scattering by the HOP, first, results in the sharp increase of the values of the phase function at $\theta = 180°$, second, the unpolarized incident radiation is polarized.

This paper has been completed under the financial backing of the Russian Fund of Fundamental Researches (Project Code 93 – 05 – 9376).

REFERENCES

1. O.A. Volkovitskii, L.N. Pavlov, and A.G. Petrushin, *Optical Properties of Crystal Clouds* (Giodrometeoizdat, Leningrad, 1984), 200 pp.
2. J. Zikmunda and G. Vali, J. Atmos. Sci. **29**, No. 7, 1334–1347 (1972).
3. G.F. Bohren and D.R. Huffman, *Absorption and Scattering of Light by Small Particles* (J. Willey & Sons, New York and London, 1983).
4. H.C. Van de Hulst, *Light Scattering by Small Particles* (Willey, New York, 1957).
5. S. Asano and M. Sato, Appl. Opt. **19**, No. 6, 962–974 (1980).
6. R.F. Rakhimov and D.N. Romashov, Atmos. Opt. **4,** No. 7, 496–500 (1991).
7. S.G. Warren, J. Appl. Opt., **23**, No. 8, 1206–1225 (1984).