

# Numerical model of broken clouds adapted to results of observations

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A 3D stochastic model of broken cloud geometry is constructed; it is adjusted according to satellite or ground-based observations. The model input parameters are an autocorrelation function of the cloud indicator field and a distribution of cloud layer thickness. A numerical algorithm is constructed based on spectral models of homogeneous random fields and on the method of nonlinear transformation of Gaussian functions. The proposed approach to broken clouds modeling is quite simple, universal, and makes it possible to reproduce key characteristics of cloud field geometry, estimated according to field measurements.

## Introduction

Clouds, covering a considerable part of the globe, are a significant factor determining the radiative transfer processes in the atmosphere. The geometry and optical structure of clouds are highly diverse and have a pronounced stochastic character. Study of influence of stochastic structure of broken clouds on characteristics of radiation fields is an urgent problem in the context of research in the field of atmospheric general circulation, theory of climate, meteorology, as well as in solution of many applied problems of atmospheric optics. It is noteworthy that, in addition to experimental studies, mathematical simulation has recently become an efficient tool of investigation.

History of mathematical modeling and numerical algorithms for simulation of stochastic cloud structures numbers several decades (see, e.g., Refs. 1–12). Cloud models evolve in parallel with the development of mathematical theory and calculation facilities, as well as simultaneously with the refinement of technical capabilities for obtaining experimental data. Whereas in first models the clouds were presented as simplest geometrical bodies<sup>1,2,4</sup> (parallelepipeds, spheres, and paraboloids), subsequently modelers had turned to using general methods of numerical simulation of random processes and fields, enabling one to simulate more complicated structures.

One of the methods of numerical simulation of broken clouds was developed in Refs. 13 and 14 using spectral models of Gaussian random fields and nonlinear transformations of Gaussian functions. This method allows one to simulate quite diverse stochastic cloud configurations. The corresponding cloud models have been named Gaussian. They were used to study radiative transfer processes in cumulus clouds by Monte Carlo method.<sup>14–17</sup> One of the disadvantages of the Gaussian models is their certain complexity and indefiniteness of the procedure of adjusting the model parameters.

This paper is devoted to solution of this problem. The model we propose is a modification of the Gaussian model of broken clouds. Therefore, we first will give a brief outline of the Gaussian model, and then present a new model modification and method of estimation of the parameters, which make it possible to adapt the model to real data.

## 1. Gaussian model of broken cloud field

The Gaussian model of broken clouds was first proposed by Mullamaa,<sup>18</sup> who hypothesized that the cumulus clouds can be described using stationary Gaussian process. Based on this hypothesis, a theoretical-experimental model of statistical structure of cumulus clouds was created (see Refs. 19 and 20). Then, in Refs. 13 and 14 this hypothesis was used to construct a numerical model of cloud structure for simulation of solar radiation transfer.

Let us proceed to description of the Gaussian model. We assume that the clouds are bounded by the plane  $z = H_0$  below (cloud base, defined by the water vapor condensation level, varies little in space), while the top boundary  $z = w(x, y)$  is given by the expression (*model A*)

$$w(x, y) = H_0 + \max[\sigma(v(x, y) - d), 0], \quad (1)$$

where  $d \in (-\infty, +\infty)$ ,  $\sigma > 0$ ,  $v(x, y)$  is the homogeneous Gaussian field with zero mean, unit variance, and normalized correlation function  $K(x, y)$ ,  $K(0, 0) = 1$ . Thus, it is assumed that the cloud medium is concentrated on the set  $\{(x, y, z): H_0 < z < w(x, y)\}$ . The value  $w(x, y) = H_0$  means that there is a gap in the cloud field over a point with coordinates  $(x, y)$  in a horizontal plane. The absolute cloud fraction  $n_0$  for this model is defined by the formula

$$n_0 = 1 - \Phi(d), \quad (2)$$

where  $\Phi$  is the function of the standard normal distribution. The specific feature of model (1) is that for  $d < 0$ ,  $n_0 > 0.5$  the cloud configuration corresponds to the structure of overcast clouds with gaps. Therefore, Kargin and Prigarin<sup>13</sup> proposed as a model of cumulus clouds, in addition to Eq. (1), the *model B*:

$$w(x, y) = H_0 + \max\{\sigma[|v(x, y)| - d], 0\}, \quad d > 0. \quad (3)$$

In this case

$$n_0 = 2[1 - \Phi(d)]. \quad (4)$$

Let  $m_0$  denotes the average cloud amount per unit area. Based on the results presented in Ref. 21, it is not difficult to obtain

$$m_0 = d(2\pi)^{-3/2} (k_{20}k_{02} - k_{11}^2)^{1/2} \exp(-d^2/2), \quad d > 0, \quad (5)$$

for model (1) and

$$m_0 = 2d(2\pi)^{-3/2} (k_{20}k_{02} - k_{11}^2)^{1/2} \exp(-d^2/2) \quad (6)$$

for model (3). Here it is assumed that the correlation function is twice differentiable and that

$$k_{ij} = -\left. \frac{\partial^{i+j} K(x, y)}{\partial x^i \partial y^j} \right|_{x=y=0}.$$

For isotropic fields,  $k_{20} = k_{02}$ ,  $k_{11} = 0$ , and formulas corresponding to Eqs. (5) and (6) take the form

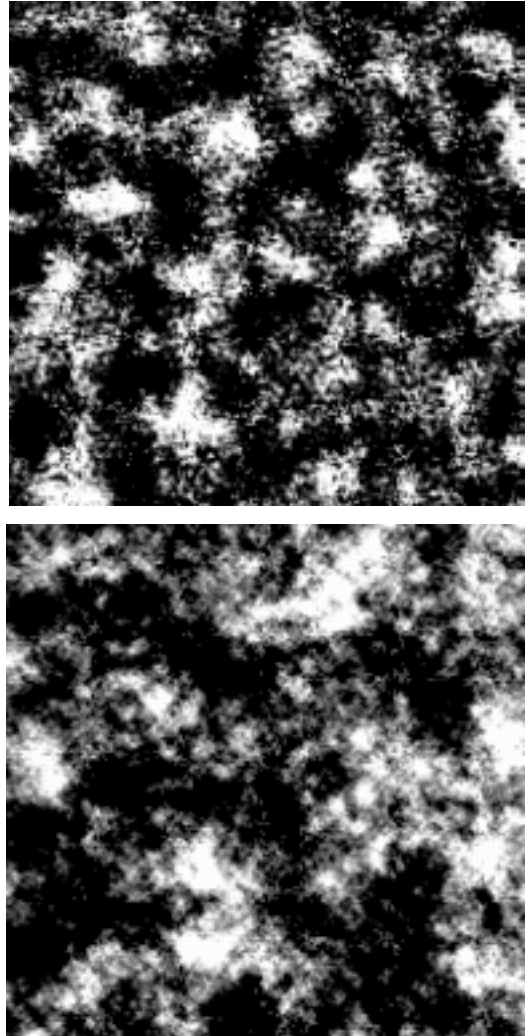
$$m_0 = d(2\pi)^{-3/2} k_{20} \exp(-d^2/2), \quad d > 0, \quad (5')$$

$$m_0 = 2d(2\pi)^{-3/2} k_{20} \exp(-d^2/2). \quad (6')$$

Models *A* and *B* are uniquely determined by  $d$  and  $\sigma$  and by the correlation function  $K(x, y)$ . With an appropriate selection of these parameters, the models can be readily adjusted to correspond to any cloud fraction and mean vertical and horizontal cloud sizes. This allows one to use representations (1) and (3) to simulate different structures of broken clouds.

Model parameters are determined as follows. Provided the cloud fraction  $n_0$  is known, using formulas (2) and (4), we can first calculate the “cutting level”  $d$ . Then, based on some additional information on cloud configuration in the horizontal plane, it is necessary to specify the correlation function  $K(x, y)$  (this problem as well as a new method of its solution with the help of real images of cloud fields are discussed below). Finally, the parameter  $\sigma$ , responsible for the extension along the vertical [see formula (7) below], should be adjusted to correspond to the data on vertical cloud extents. For more detailed information on adjustment and numerical implementation of the models, as well as for calculated results on the cloud radiative properties obtained using Gaussian simulation models, one can see Refs. 13–16.

The first problem to be solved in constructing Gaussian model of broken clouds is the choice of correlation function  $K(x, y)$ . It is the correlation function that determines the geometry of modeled cloud field and configuration of individual clouds and cloud gaps (Fig. 1).



**Fig. 1.** Examples of Gaussian model realizations of broken clouds for different correlation functions (cloud fraction is 0.5 in both cases).

In the very first numerical experiments modelers frequently used correlation function for isotropic fields with the simplest structure

$$K(x, y) = \sigma^2 J_0[\rho(x^2 + y^2)^{1/2}],$$

where  $J_0$  is the Bessel function of the first kind (and at the same time,  $k_{20} = \rho^2/2$ ). Here, parameter  $\rho$  is responsible for horizontal cloud sizes (the smaller  $\rho$ , the greater mean horizontal cloud sizes). For its determination one can use formulas (5') and (6'), which relate the mean cloud amount  $m_0$  per unit area, as well as the second derivatives of the correlation function.

The use of such a simplified model for studying the qualitative influence of stochastic cloud structure on radiative characteristics of radiation field may be partially justified. However, the configuration of a cloud field for the model with such a correlation function seems unnatural (Fig. 2).

Therefore, it is reasonable to consider the correlation functions of a general form; in the isotropic case, it can be presented in terms of the integral:

$$K(x, y) = \sigma^2 \int_0^\infty J_0[\rho(x^2 + y^2)^{1/2}] z(\rho) d\rho,$$

where  $z(\rho)$  is the radial spectral density of a Gaussian field, i.e.,  $z(\rho)$  is an arbitrary nonnegative (possibly reduced) function, defined on real half-axis so that  $\int_0^\infty z(\rho) d\rho = 1$ . In section 2 we will describe a method which allows one to choose the correlation function  $K(x, y)$ , based on satellite and ground-based observations.

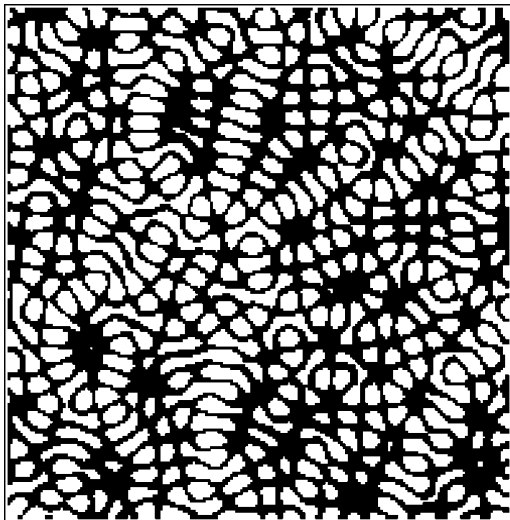


Fig. 2. Configuration of cloud field for Gaussian model with correlation function  $J_0$  (cloud fraction is 0.5).

The second problem here is that the model outlined above provides no possibility of performing an independent control of the shape of distribution of the cloud layer geometrical thickness  $w(x, y) - H_0$  (leaving only possibility to scale the distribution using the parameter  $\sigma$ ). This distribution is “automatically” determined by other model parameters and is a truncated Gaussian distribution with the density

$$f_\sigma(h) = \sigma^{-1} \varphi(\sigma^{-1}h + d) / C, \quad h > 0, \quad C = \int_d^\infty \varphi(x) dx, \quad (7)$$

where

$$\varphi(x) = 1/2\pi [\exp(-x^2/2)]$$

is the density of standard normal distribution. Real distributions may considerably differ from the truncated Gaussian distributions. Figure 3 presents histograms constructed according to observations on Nauru island for periods (a) December 20–22, 1998 and (b) July 5 – August 5, 1999. Here, we used the results of Bayesian data analysis, kindly provided by Evans and McFarlane.

In section 3 we will describe a modification of the Gaussian model that allows one to reproduce an arbitrary distribution of the cloud layer thickness (e.g., determined through statistical processing of field measurements).

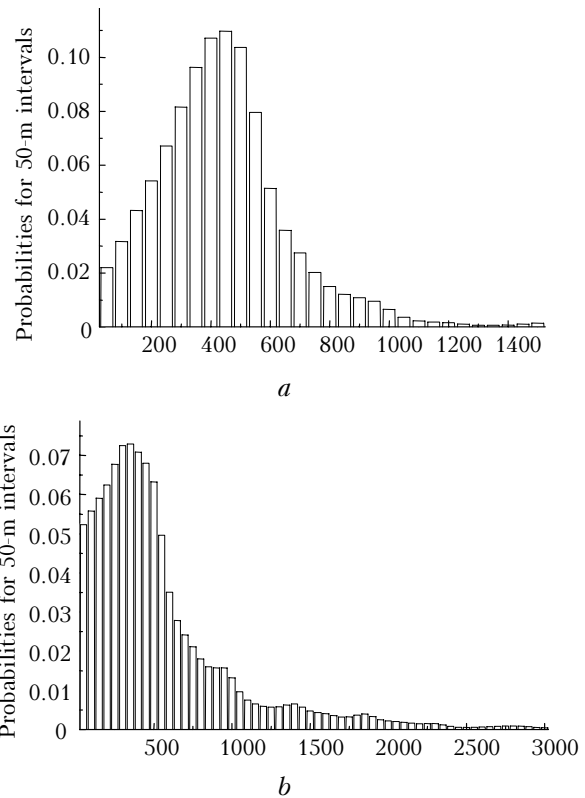


Fig. 3. Examples of histograms of the geometric thickness of a cloud layer, in m.

*Note.* This paper is not concerned with the problem of numerical simulation of homogeneous Gaussian random field  $v(x, y)$ . This problem is quite extensively discussed in Refs. 22 and 23, where, in particular, spectral models and schemes based on moving summation are considered. As our experience shows, spectral models have certain advantages.<sup>22</sup> Actually, we have used the spectral models of Gaussian homogeneous and isotropic fields in the numerical experiments; their results are presented below.

## 2. Calculation of correlation function $K(x, y)$ for Gaussian model of broken clouds according to observations

Let  $I(x, y)$  denotes a cloud indicator field, equal to 0 if there is a gap in the cloud over a point with the coordinates  $(x, y)$  in a horizontal plane, being equal to 1 otherwise. Suppose that ground-based or satellite observations allow us to estimate the mathematical expectation  $m_I$  (which coincides with the cloud fraction  $n_0$ ) and the covariance function  $K_I(x, y)$  of the cloud indicator field  $I(x, y)$ ,  $K_I(x, y) = \mathbf{E} I(x, y) I(0, 0)$ . From formulas (2) and (4) we can determine the parameter  $d$  of the Gaussian model. The covariance functions of the indicator field  $I(x, y)$  and Gaussian field  $v(x, y)$  are related by equalities (for more information about nonlinear one-point transformations of the Gaussian functions, see Refs. 15 and 23):

$$K_I(x, y) = \iint_{\xi>d, \eta>d} \varphi_{K(x,y)}(\xi, \eta) d\xi d\eta \quad (8)$$

for model A and

$$K_I(x, y) = 2 \iint_{\xi>d, \eta>d} \varphi_{K(x,y)}(\xi, \eta) d\xi d\eta + 2 \iint_{\xi>d, \eta<d} \varphi_{K(x,y)}(\xi, \eta) d\xi d\eta \quad (9)$$

for model B. Here  $\varphi_p$  is the density of two-dimensional Gaussian vector with zero mean, unit variances of components, and correlation coefficient  $\rho$  between the components:

$$\varphi_p(\xi, \eta) = \left[ 2\pi\sqrt{1-\rho^2} \exp\left(\frac{\xi^2 + \eta^2 - 2\rho\xi\eta}{2(1-\rho^2)}\right) \right]^{-1}$$

Thus, the correlation function  $K(x, y)$  for the Gaussian model can be calculated from the covariance function  $K_I(x, y)$  of the indicator field and by inverting formulas (8) and (9). For calculations it is reasonable to use representations in terms of Owen functions (see Ref. 24, as well as Ref. 25, where the method based on transformations of Gaussian fields was used for modeling binary random fields). These are the equalities

$$K_I(x, y) = \Phi(-d) - 2T(d, a(x, y)) \quad (10)$$

for model A and

$$K_I(x, y) = 4[\Phi(-d) - T(d, a(x, y)) + T(d, 1/a(x, y))] \quad (11)$$

for model B, where

$$a(x, y) = \sqrt{\frac{1 - K(x, y)}{1 + K(x, y)}}$$

and

$$T(d, a) = \frac{1}{2\pi} \int_0^a \exp(-d^2(1+u^2)/2) \frac{du}{1+u^2}$$

are Owen functions.

The above-mentioned method of construction of Gaussian models of broken clouds was tested by use of experimental data of satellite and ground-based observations. Figure 4 presents an image (indicator function) of a cloud field with the resolution of 1 km, obtained from a satellite using multichannel radiometer (see Ref. 26), together with the results of simulation of the cloud field, based on the Gaussian model.

Real cloud field was used to estimate cloud fraction and covariance function of the indicator field  $K_I(x, y)$ , and then formula (10) was numerically inverted to determine the correlation function  $K(x, y)$  for the Gaussian field  $v(x, y)$  and a realization of the model A was constructed according to formula (1). Note that, in estimating the covariance function of the indicator field and in constructing the Gaussian model, we used assumed random fields isotropic.

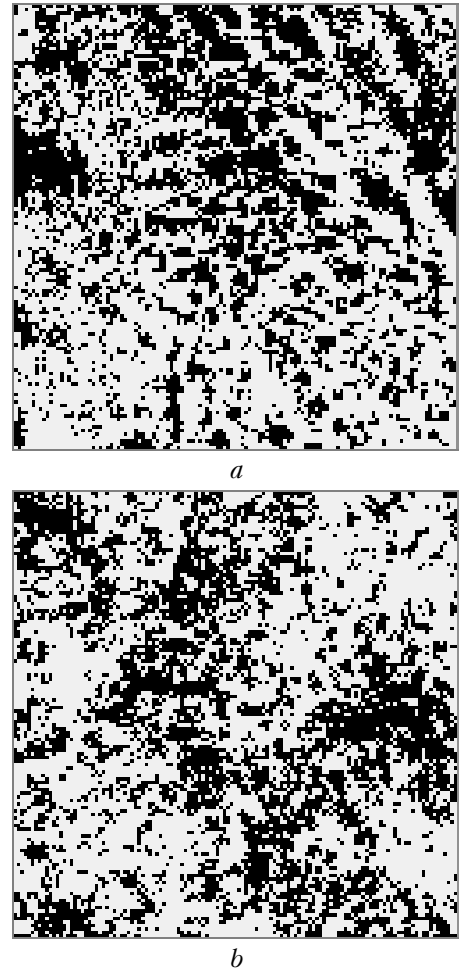


Fig. 4. Image (indicator function in horizontal plane) of a cloud field 200 × 200 km in size with the resolution of 1 km, obtained from satellite (a), and a realization of the mathematical simulation model. Cloud fraction is 0.6 (b).

Another example presented below is based on the data kindly provided by McFarlane et al.<sup>27</sup> The received data array was obtained using Bayesian analysis of ground-based observations performed on Nauru island as part of Atmospheric Radiation Measurement (ARM) research program for the period from July 5 to August 5, 1999 (Fig. 5).

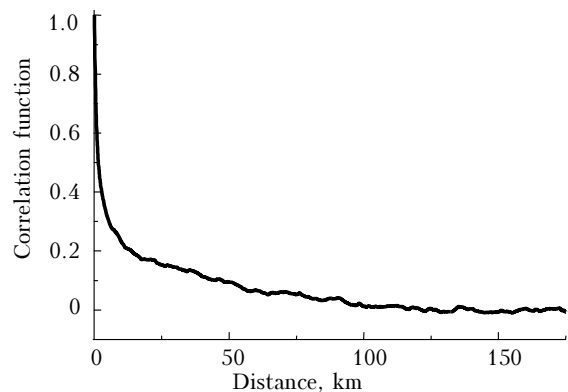
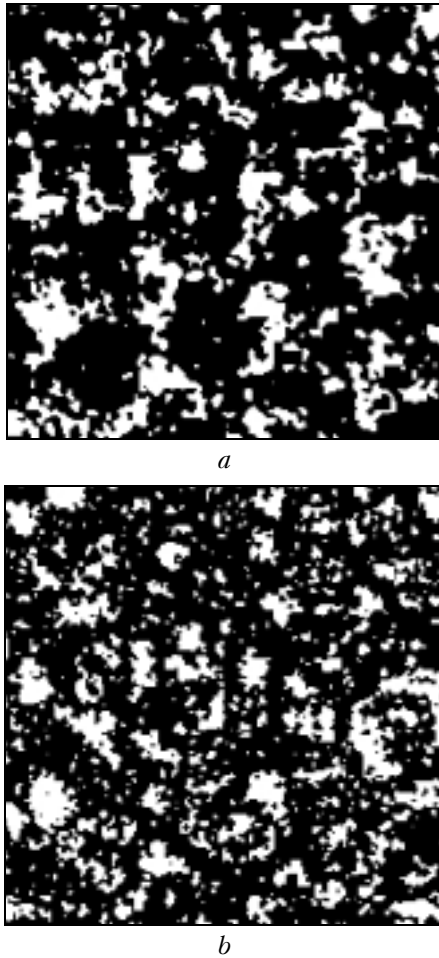


Fig. 5. Estimate of normalized correlation function of cloud indicator field.



**Fig. 6.** Gaussian model realizations of broken clouds (200 × 200 km), constructed using “real” correlations of the indicator field: model *A* (*a*), and model *B* (*b*).

Based on available time series and information about wind velocity, we estimated cloud fraction and covariance function of the indicator field. And again, as in interpretation of satellite observations, we used an assumption of statistical homogeneity and isotropy of the cloud field. Realizations of the Gaussian models *A* and *B* are presented in Fig. 6. Cloud fraction (0.25) and correlations of the indicator field were estimated according to data of observations on Nauru island between July 5 and August 5, 1999.

An additional analysis (comparison of distributions for the lengths of the cloud-free and cloud-covered segments) showed that the model *B* better agrees with the results of observations than the model *A*. Here we note that, in fact, model *B* has more clouds per unit area than model *A* [see Eqs. (5) and (6)].

### 3. Gaussian model modification to reproduce distribution of geometrical thickness of cloud layer

As was already noted in section 1, the distribution density of the cloud layer thickness for Gaussian

model is defined by formulas (7), which may turn out to poorly agree with real distribution determined from experimental data. We will now describe a modification of Gaussian model, allowing one to reproduce an arbitrary distribution of cloud layer thickness. The suggested modification is a variant of the method of inverse distribution function, widely used in statistical modeling.<sup>15,23</sup> The probability density of cloud layer thickness, determined according to observations, will be denoted by  $g(h)$ ,  $h > 0$ , while the corresponding distribution function by  $G$ :

$$G(h) = \int_0^h g(x)dx, \quad h > 0.$$

We will now consider the distribution density  $f_1(h)$  from Eq. (7) for  $\sigma = 1$  and the corresponding distribution function  $F$ :

$$f_1(h) = \varphi(h + d)/C, \quad h > 0,$$

$$C = \int_d^\infty \varphi(x)dx, \quad F(h) = \int_0^h f_1(x)dx.$$

In analysis below, instead of models (1) and (3) we will consider the modified models:

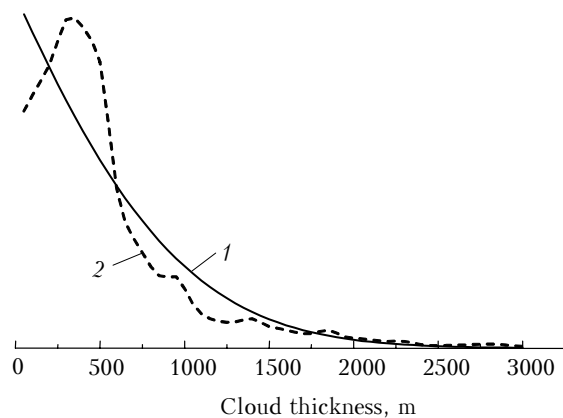
$$w(x, y) = H_0 + G^{-1}F[\max\{v(x, y) - d, 0\}],$$

$$d \in (-\infty, +\infty), \quad (1')$$

$$w(x, y) = H_0 + G^{-1}F[\max\{|v(x, y)| - d, 0\}], \quad d > 0. \quad (3')$$

For these models, the distribution density of cloud layer thickness exactly coincides with density  $g$ , while all the other relations (2), (4)–(6), (8), and (9) (for cloud fraction, mean cloud amount, and covariance function of indicator field) remain unchanged.

Figure 7 shows the distribution density of a cloud layer thickness for the Gaussian model *B*, constructed using data of observations on Nauru island for period July 5 – August 5, 1999.



**Fig. 7.** Shape of the distribution density of geometrical thickness of the cloud layer for Gaussian model *B* with cloud fraction 0.25 (curve 1) and for the corresponding modified model adapted to results of observations (curve 2).

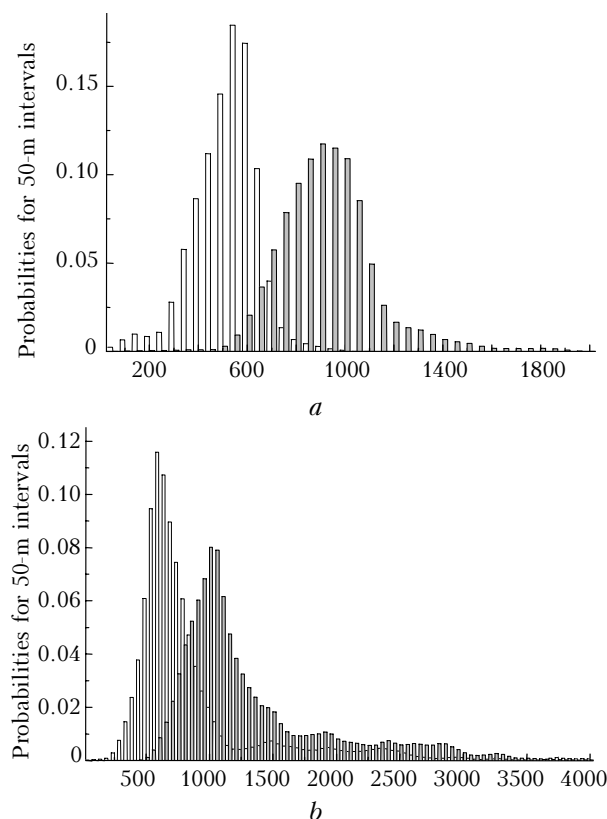
For observed cloud fraction of 0.25 the corresponding value of  $d$  is 1.15. It is worthwhile to note that the model-reproduced distribution bears little resemblance to real distribution of the cloud layer thickness determined according to measurement results (curve 2 in Fig. 7 and lower diagram in Fig. 3). The modified model (3') allows one to reproduce exactly the distribution of a cloud layer thickness.

### Conclusion

In this paper we have proposed two comparatively simple methods, enabling one to adapt the numerical Gaussian models of broken clouds to satellite and ground-based observations of cloud fields. These methods are capable of reproducing real covariance of the cloud indicator field and distribution of the cloud layer thickness.

Let us highlight certain problems whose solution, in our opinion, is critical for further development of the parametric cloud models by numerical simulations.

1. In the models, considered above, the cloud base is assumed to be constant. This is quite rough assumption (Fig. 8), and variation of the cloud base may have a considerable influence on the characteristics of the radiative field. Therefore it is desirable that the model takes adequately into account the random distribution of the cloud base.



**Fig. 8.** Examples of histograms for distance (m) to bottom (light bars) and top (shaded bars) boundary of cumulus clouds. The histograms are constructed based on observations on Nauru island for periods (a) December 20–22, 1998 and (b) July 5 – August 5, 1999. We used the results of Bayesian data analysis provided by Evans and McFarlane.

2. The table presents correlation coefficients between different characteristics of the broken cloud field. The correlation coefficients were calculated using two data arrays obtained by processing the data of ground-based observations carried out on Nauru island as a part of the ARM program for periods December 20–22, 1998 and July 5 – August 5, 1999.

To calculate the correlation coefficients, we used the results of Bayesian data analysis provided by Evans and McFarlane.

**Correlation coefficients between different characteristics of a broken cloud field according to data of observations on Nauru island for period (1) December 20–22, 1998 and (2) July 5 – August 5, 1999**

Data array	(1)	(2)
Distance to cloud base/ geometrical cloud layer thickness	-0.42	-0.02
Geometrical cloud layer thickness/ cloud extinction coefficient	0.39	-0.04
Distance to cloud base/ cloud extinction coefficient	0.01	0.16

From the table, in particular, it is seen that the geometrical thickness of the cloud layer may depend substantially on the distance to the cloud base, while the extinction coefficient in cloud medium may correlate with geometrical thickness (left column of the table) and with the distance to cloud base (right column). Possibility to take into account not only random cloud geometry but also the inhomogeneity of the cloud optical properties and to adjust the simulation model in accordance with a specified correlation coefficients could have been an invaluable tool for complex study of the influence of stochastic cloud structure on characteristics of the radiative fields.

Studies aimed at developing such models seem to be highly promising. In this regard we only mention Refs. 28 and 29, in which the 3D field of cloud liquid water path is simulated based on the same principles used in the Gaussian models considered in this paper: preliminary modeling of a Gaussian field on the basis of discrete Fourier transform (discrete analog of spectral models) and subsequent nonlinear transformation of the Gaussian field. In particular, in Ref. 29 such an approach is used to simulate 3D fields of liquid water path and effective radius of water droplets in the cloud, based on two-dimensional radar measurements.

Methods suggested in Refs. 28 and 29 are more laborious and they require more complicated procedures of estimation of the parameters than those used by Gaussian models; at the same time, they allow one to construct cloud indicator field correlating with the optical characteristics of the cloud medium.

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