Method for parameterization of gas absorption of atmospheric radiation giving the *k*-distribution with minimum number of terms

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The method based on line-by-line calculations is described. In a wide spectral region for a preset accuracy, this method provides for efficient estimation of the minimum possible number of terms in the *k*-distribution and then determination of their parameters in order to simulate radiative fluxes and radiative cooling / heating in the atmosphere. Application of this method can significantly increase the accuracy and roughly triple the speed of current radiative blocks of climatic models based on the method of *k*-distribution.

Introduction

The method of *k*-distribution have been widely used in recent years for parameterization of atmospheric radiation.[1,2] In this method, gas spectra in "wide" ($\sim 100\ \text{cm}^{-1}$) intervals are replaced by a sum of several rectangular "quasilines," whose sets of spectral points will be referred to as channels following Ref. 3. Quasilines are obtained by sorting spectral points, joining them into channels, and then assigning the same effective absorption coefficient to all channel points. After that, a quasiline can be treated in transfer equations as monochromatic radiation. Since the rate of parameterization is inversely proportional to the number of channels, it should be minimized without loss in the needed accuracy. The minimization problem was often solved by selecting standard quadratures (usually, Gauss-Legendre ones in the so-called g-space[1–4]). This technique, efficient for homogeneous conditions, turned out to be less efficient for an inhomogeneous atmosphere, and for this problem an additional resource to increase the rate was found. This paper describes the method for efficient estimation of the minimum possible number of channels (terms of the *k*-distribution) in a given spectral range for achieving a preset accuracy and then determination of their parameters just for the inhomogeneous atmosphere. Besides, this method provides for control (more efficient than in the ordinary correlated *k*-distribution method[5]) of the accuracy of approximation of radiative cooling and heating in the upper atmosphere and, which should be emphasized, it takes into account overlapping of bands of different gases.

Implementation of the method involving intense line-by-line calculations proved to be possible, thanks to the efficient algorithm described in Ref. 6.

1. Procedure of determining channels

In the method proposed, as well as in the correlated *k*-distribution method, points of a spectral interval are joined into channels by the criterion of similar behavior of the absorption coefficient. The following procedure is proposed:

(A) For every gas and every level of some atmospheric model (for example, tropical[7]), volume absorption coefficients K_i are calculated at the points i of the wave number grid (as usually for line-by-line calculations of radiative fluxes in the atmosphere).

(B) One of the levels is selected, and its points are selected for the first channel U_1 by the criterion that a point is sought belonging to the channel, if the absorption coefficient at this point at the given level is less than the preset threshold value S_1 ($i \in U_1$, if $K_i < S_1$).

(C) At the set of points of the channel U_1, line-by-line calculation of three parameters is performed, namely: upward and downward fluxes of atmospheric radiation, and radiative cooling / heating for all horizontal levels of the atmospheric model.

(D) One of these three profiles is used to determine (applying a numerical procedure) the profile of effective volume absorption coefficient at each level of the atmospheric model; being substituted into the radiative transfer equation, the profile returns the profile of the initial parameter (a sort of the inverse problem is solved).

(E) The radiative transfer equation with the obtained profile of the effective volume absorption coefficient is solved, and the solutions obtained are compared with the three initial line-by-line solutions (one of these three pairs of solutions coincides automatically).

(F) If the discrepancies do not meet the preset accuracy or are too small, the channel is compressed or extended by the corresponding decrease or increase of the threshold S_1, and the procedure is repeated starting from the stage B, until the threshold providing for the needed accuracy at the maximum width of the channel U_1 is found.

(G) Then the points of the found channel U_1 are excluded from the further consideration, and for the rest points the procedure is repeated starting from the stage B (other atmospheric level can be used), and this yields sequentially all the N channels U_1, U_2, …, U_N.

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Effective absorption increases with the channel number, and the channel width usually decreases.

The method is based on the following simple fact: it is true only for monochromatic radiation that there exists a single profile of the absorption coefficient that allows both the upward and downward radiation to be described accurately and simultaneously. The proposed procedure separates the sets of spectral points, i.e., channels, in which the absorption coefficient behaves similarly (is almost independent of the radiation frequency) for the equations valid for monochromatic radiation (e.g., Bouguer–Lambert–Beer law can be used with the needed accuracy). Figure 1 depicts a part of actual calculation for the 15 μm carbon dioxide band divided into 10 channels.

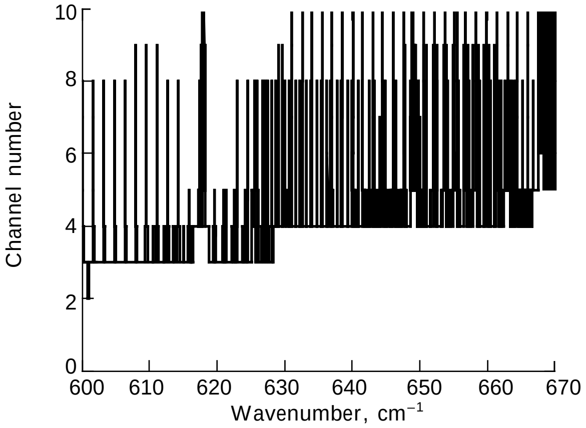

Fig. 1. A part of carbon dioxide 15-μm band divided into 10 channels.

The levels of the atmospheric model, using which the points are divided into channels, and the model itself are selected rather arbitrarily. Note that in the upper atmosphere it is better to use radiative cooling / heating, rather than fluxes; the former significantly increases the accuracy of parameterization in this case. It is important to note that, unlike the ordinary correlated *k*-distribution method (that uses sorting of the spectrum), the proposed method saves the information about what channel every spectral point belongs to. This considerably facilitates solution of the problem of overlapping of bands from different gases. Thus, consideration of intersections of the channels (as sets) obtained for each of the gases actually determines the channels of the mixture.

2. Procedure for obtaining parameters of the *k*-distribution

The procedure described above completely solves the problem of determining the *k*-distribution, but only for the atmospheric model used. If this procedure *with the same channels* is applied to other atmospheric models, it will yield different profiles. For four of the six standard atmospheric models (Tropical (TRP), Mid-Latitude Summer (MLS), Mid-Latitude Winter (MLW), Sub-Arctic Summer (SAS), Sub-Arctic Winter (SAW), and Standard USA (USA)), Fig. 2 shows the vertical profiles of effective absorption cross section, in cm^2, of Í$_2$Î molecules in the interval of 50–250 cm^{-1} for the single channel. Hereinafter the examples are presented only for description of longwave radiation. Variations in the profiles of effective absorption coefficients reflect the dependence of the actual absorption coefficients on temperature and pressure. For this interval, as known, the temperature dependence of water vapor absorption is weak and often can be neglected. Therefore, for the considered case some average profile of the effective absorption coefficient common for all atmospheric conditions can be recommended. It is specified in Table 1 that serves for simple linear pressure interpolation. The quality of this simple single-channel parameterization provides the accuracy in calculation of radiative cooling higher than 0.1 deg / day for all the atmospheric models listed above.

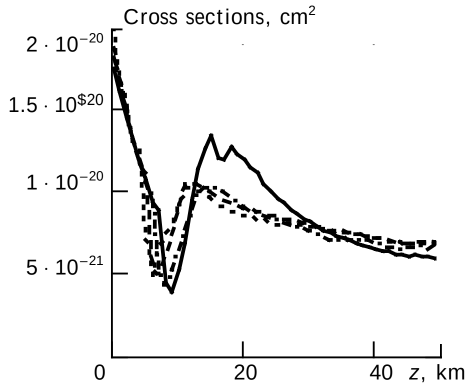

Fig. 2. Spectral interval of 50–250 cm^{-1}. Vertical profiles of the effective absorption cross section of Í$_2$Î molecules (cm^2) obtained for TRP (—), SAW (- - -), MLW (– – –), and USA (– · – · –) atmospheric models.

In the other spectral ranges, the temperature dependence should be necessarily taken into account. It turns out that the following extremely simple method provides for a rather good accuracy in temperature dependence:

(à) effective coefficients are calculated for two most different atmospheric models (usually TRP and SAW);

(b) for every pressure value, these coefficients corresponding to different temperatures are used as node points in simple linear temperature interpolation.

Table 1. Node points for linear pressure *Ð* (mbar) interpolation of cross sections *S* (cm^2) of H_2O molecules in the interval of 50–250 cm^{-1}

Ln(*P*)	–8	–6	–4	–3	–2	–1	0	3
S	3E–19	3.8E–19	4.4E–20	1.35E–20	7.65E–21	6.6E–21	6.6E–21	8.2E–21
Ln(*P*)	4	5	5.8	6.0	6.4	6.6	7.0	–
S	9E–21	1.15E–20	5.7E–21	5.7E–21	1.2E–20	1.4E–20	2E–20	–

Note. 3Å–19 means 3 · 10^{-19} etc.

Table 2. Number of channels obtained by the correlated *k*-distribution method and the proposed method

Reference	1	3	4	8	This paper
Interval, cm^{-1}		50–250	10–250	0–340	40–250
Composition	–	H_2O	H_2O	H_2O	H_2O
Number of channels		7	16	6	1
Interval, cm^{-1}	10–550	50–550	10–500	0–540	40–550
Composition	H_2O	H_2O	H_2O	H_2O	H_2O
Number of channels	10	17	32	12	4
Interval, cm^{-1}	550–990	550–990	500–980	540–980	550–990
Composition	H_2O, CO_2	H_2O, CO_2	H_2O, CO_2	H_2O, CO_2	H_2O, CO_2
Number of channels	29	11	112	24	10

As was already noted in the previous section, analysis of the channels obtained for different gases allows the channels for the gas mixture to be found quite simply. All these facts open already now the possibility of developing efficient parameterizations of radiation, for example, for climate models. Their quality can be judged from Fig. 3, which shows radiative cooling (deg / day) due to absorption at the Ñ Î$_2$ and Í$_2$Î lines in the interval of 550–990 cm^{-1}. As can be seen from Fig. 3, ten channels provide for the accuracy higher than 0.1 deg / day in the troposphere and ~0.5 deg / day in the upper atmosphere (it is quite comparable with the accuracy of known parameterizations, for example, in Refs. 1–3 that are used now in radiative blocks of climatic models).

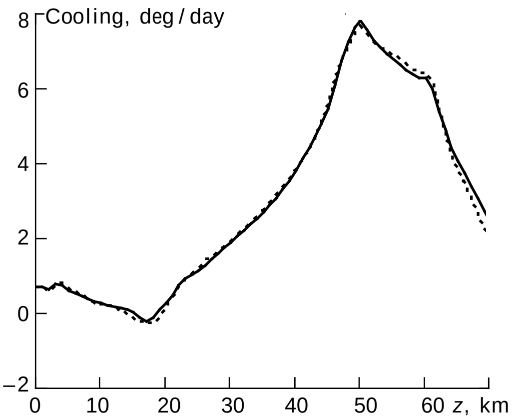

Fig. 3. Radiative cooling obtained by the line-by-line (—) and the proposed (· · · ·) methods (10 channels). TRP model, absorption in Í$_2$Î and Ñ Î$_2$ lines, interval of 550–990 cm^{-1}.

It should be emphasized that this accuracy is not limited for this method. Now it is mostly restricted by the above simple method used to take into account the temperature dependence of the absorption. The accuracy can be increased several times without increasing the number of channels, that is, without loss in the rate, by merely improving the method of taking the temperature dependence into account (for example, by using more than two atmospheric profiles for interpolations). The procedure of dividing an interval into channels itself seems to be quite efficient, as follows from Table 2, which compares the numbers of channels obtained by different methods.

Generally speaking, the efficiency of the parameterizations represented in Table 2 should be compared with a certain care, since they have different accuracy. Nevertheless, it can be expected that the proposed technique may decrease the number of the needed channels by almost three times.

Conclusion

The results obtained are indicative of the possibility of developing parameterizations of longwave radiation for climatic investigations based on the *k*-distributions including ~15 terms for calculations up to 40 km and ~20 terms up to 70 km in the entire spectral range, and the accuracy of these parameterizations for radiative cooling will be higher than 0.1 deg / day. Also it can be expected that the efficiency of parameterizations in the shortwave region will significantly increase as well. As a result, it will be possible to almost triple the speed of radiative blocks of climatic models and to improve their accuracy, especially, for the upper atmosphere. The proposed technique may prove useful in other applications as well, for example, in development of mathematical support for experiments with the spectral resolution ~ 1–100 cm^{-1}.

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