

# Spatiotemporal structure of components of lidar returns with different orders of multiple scattering

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Components of lidar returns due to different orders of multiple scattering are analyzed for different model disperse media. Contribution of each component to the spatiotemporal structure of signal is estimated for the case of sensing optically thick scattering media. The estimates are made by Monte Carlo method for coaxial viewing geometries.

## Formulation of the problem

The viewing geometry used in the numerical experiments was described earlier in Ref. 1. Suppose that a point source of monochromatic radiation is located at the point (0,0,0) of Cartesian coordinate system (Fig. 1); it emits radiation within the divergence angle  $\nu_0$  at the wavelength  $\lambda$  along the direction of  $Oy$ -axis.

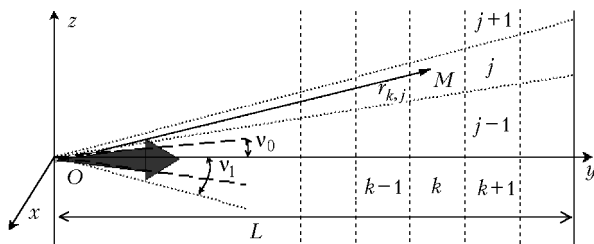


Fig. 1. Viewing geometry used in the numerical experiments.

The detector is collocated with the source (coaxial viewing geometry) and has the field of view (FOV)  $\nu_1$ . The scattering medium is plane-stratified, that consists of the layers whose boundaries are parallel to the  $xOz$  plane, and extends infinitely in the directions  $n \perp Oy$  while having boundaries at  $y = 0$  and  $y = L$  in the  $Oy$ -direction. The optical properties of the medium are characterized by the extinction  $\beta_{\text{ext}}(y)$ , scattering  $\beta_{\text{sct}}(y)$ , and absorption  $\beta_{\text{abs}}(y)$  coefficients and scattering phase function  $g(\mu)$  (here  $\mu$  is the cosine of the scattering angle), corresponding to a polydisperse system of spherical particles of cloud or aerosol type. It is assumed that the source emits  $\delta(t)$  pulse.

In Ref. 1, this problem was formulated as the problem on determining the spatial structure of instantaneous brightness body  $P(\mathbf{r}/t)$  produced by the photons simultaneously incident on the detector at a time  $t$ . Here, we consider the influence of the viewing optical and geometrical parameters on the signal  $P_i(\mathbf{r}/t)$ , produced by the photons due to different

orders,  $i$ , of multiple scattering. The purpose of the study is to determine when, within a preset accuracy, this function can be treated approximately, in terms of only first orders of multiple scattering. In our opinion, this issue is important for problems of correct interpretation of lidar returns produced not only by single-scattered photons. With the results of such studies used as a background, one can construct a theory and methods of solving of inverse problems of laser sensing using, as the initial point, not the lidar equation in the single scattering approximation, but the solutions obtained for higher orders of scattering (see, e.g., Ref. 2).

## Method of study and models of medium

The study is based on direct Monte Carlo method involving local estimates,<sup>3</sup> which in our case can be written as

$$l_{i,j,k} \approx \frac{\omega_{k,j} g(\mu_{k,j}) \exp(-\tau_{k,j})}{2\pi r_{k,j}^2} \Delta_i,$$

where subscript  $k$  indicates the layer number where last collision has taken place,  $j$  is the detector's angular aperture number starting from which the point  $\mathbf{r}_{k,j}$  remains within the detector FOV,  $\Delta_i$  is the indicator of the interval  $(t_i - t_{i-1})$  of photon "lifetime" from photon emission along the trajectory to photon arrival at the point  $\mathbf{r}_{k,j}$  at the detector, and  $\omega_{k,j}$  is the photon statistical weight to account for absorption.

The optical models of the medium used are taken from Ref. 1. In particular, the directional light scattering is described using scattering phase functions for atmospheric urban aerosol and two types of fog, advective and radiative, for the wavelength  $\lambda = 0.86 \mu\text{m}$ .<sup>4</sup> In the numerical experiments, we assumed that the geometrical thickness  $L$  of the medium is fixed at 1000 m. The optical depth of the medium was varied in the range  $2 \leq \tau \leq 8$  by specifying

extinction coefficients in each layer, while keeping the single scattering albedo  $\chi = \beta_{\text{sct}}(y) / \beta_{\text{ext}}(y) = \text{const}$ . Using this model of optical properties of the medium, it is possible, first of all, to explore the dependence of the characteristics under study on the optical depth and asymmetry of the scattering phase function of the medium. The inhomogeneity effect of optical properties of the medium on the signal parameters was studied using uniform, i.e.,  $\beta_{\text{ext}}(y) = \text{const}$ , and "growing" profiles of extinction coefficient (see, e.g., Ref. 5, Fig. 4).

The dependence of the background due to multiply scattered radiation is easier to predict in view of the fact that, the less the single scattering albedo, the weaker the multiple scattering effect is and, hence, the monotonically less is the contribution of multiply scattered photons to lidar return. Therefore, the single scattering albedo will not be changed in the course of statistical experiments.

The shape of the scattering phase function will be characterized by the coefficients

$$\gamma = \frac{\int_0^1 g(\mu) d\mu}{\int_{-1}^0 g(\mu) d\mu}; \quad \xi = \frac{g(1)}{g(-1)}. \quad (1)$$

For the  $g(\mu)$  models chosen here,  $\gamma$  ranges from 3.987 (urban aerosol) to 61.5 (radiative fog), while  $\xi$  from 723 (advective fog) to 27200 (radiative fog).

In the calculations, the angular divergence of the sounding beam was kept constant ( $v_0 = 3'$ ), while the detector's FOV was varied from 0.05 to 5°.

### Results

Before analyzing the results of statistical experiments, we introduce some new notation to simplify the subsequent presentation. The total lidar return recorded at a given time is represented by the sum

$$P(t) = P_1(t) + P_2(t) + P_3(t) + P_4(t) + \dots,$$

where  $P_1(t)$  is the contribution of photons singly scattered in the medium to  $P(t)$ ;  $P_2(t)$  is the double-scattering contribution, etc. Then, the relative contributions of photons with different orders of scattering will be

$$D_1(t) = P_1(t) / P(t), \quad D_2(t) = P_2(t) / P(t), \\ D_3 = P_3(t) / P(t), \dots$$

In turn,  $D_i$  can be represented by the sum

$$D_i = D_{i1} + D_{i2} + D_{i3} + D_{i4} + D_{i5} + D_{i6},$$

where the second subscript indicates the layer number  $j$ , wherefrom the relative contribution from the scattering

order  $i$  comes. The  $D_{1j}$ ,  $D_{2j}$ , and  $D_{3j}$  values are presented in Figs. 2 and 3.

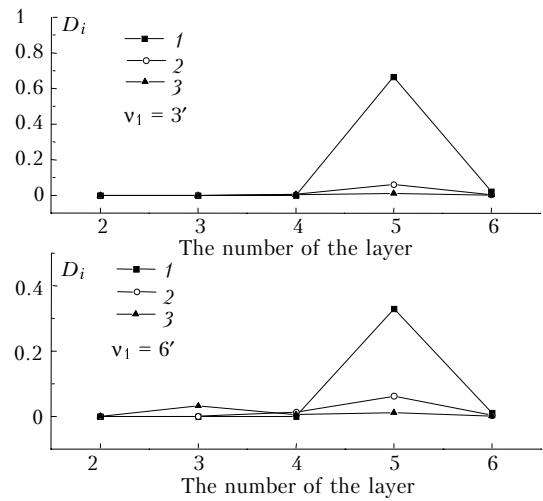


Fig. 2. Dependence of  $D_{ij}$  for different detector FOVs for aerosol model and  $\tau = 8$ .

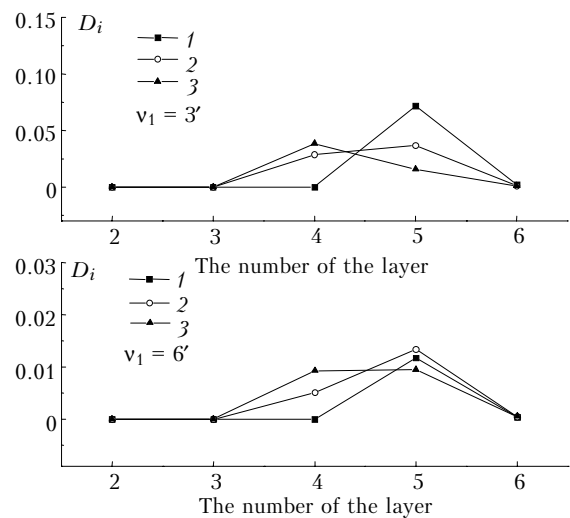


Fig. 3. Dependence of  $D_{ij}$  for different detector FOVs for radiative fog model and  $\tau = 8$ .

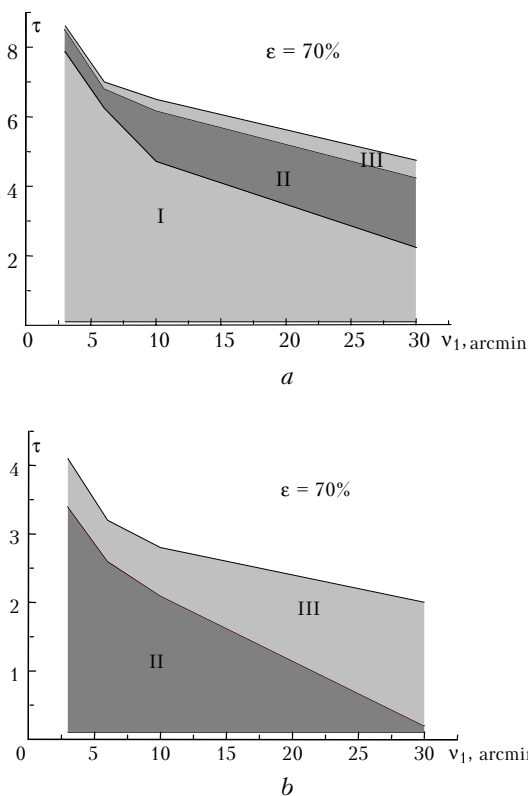
The time interval of the data presented here is chosen such that singly scattered photons from the fifth and sixth layers fall within the detector's FOV.

From Figs. 2 and 3 one can see how sensitive are  $D_i$  components to variations in the detector's FOV. The notation used in Figs. 2 and 3 corresponds to scattering orders. As the detector's aperture increases, the relative contribution of single-scattered photons decreases, fairly gradually for aerosol model (see Fig. 2), and faster for fogs (see Fig. 3), provided that the detector's FOV is quite close to beam divergence of the source (3–10'). The  $D_1(t)$  value is greater for aerosol model than for fog models even at  $v = 3'$ , despite the greater probability of the backscatter in fogs. This is because the scattering phase function of fog (especially radiative fog) has a strong forward directed peak, in

whose aerosol models, as formulated in our experiment, large contribution comes from photons scattered in the layers of the scattering medium closest to the radiation source.<sup>5</sup>

From results simulated it follows that, for small detector's FOV (3–10'), with growing optical depth of the medium,  $D_1(t)$  decreases slower in aerosol than in the radiative fog, primarily because of the growth of the contribution produced by the photons coming from layers 2–4, which component has higher energy for the fog models considered here.<sup>5</sup>

The component of the lidar return  $P_2(t)$ , produced by double-scattered photons at single scattering albedo 0.999, is not a dominating factor in the cases considered here, and its contribution to the total signal does not exceed 20%. The contribution of the third-order component  $P_3(t)$  is less than that of the double-scattered one.



**Fig. 4.** The applicability domains of the first-, second-, and third-order scattering approximations used to describe  $P(\mathbf{r}/t)$  for aerosol model (a) and radiative fog model (b).

Figure 4 shows the applicability limits of the single-, double-, and third-scattering approximations (denoted by I, II, and III in the figure) used to

describe  $P(\mathbf{r}/t)$  with prescribed accuracy  $\varepsilon = 70\%$ . The accuracy  $\varepsilon$  is defined by the inequality:

$$\frac{\sum_{i=1}^R P_i(\mathbf{r}/t)}{P(\mathbf{r}/t)} \cdot 100\% \geq \varepsilon, \quad (2)$$

where  $R$  is the number of scattering orders taken into account.

## Conclusions

Under conditions of formation of the lidar returns, considered here, we formulate the following conclusions.

1. For media characterized by the scattering phase functions with the parameters  $\gamma < 20$  and  $\xi < 5000$  as defined by expression (1), the spatiotemporal structure of lidar returns can be described with the accuracy  $\varepsilon > 70\%$  [as defined by expression (2)] in the single-scattering approximation for optical depths as large as  $\tau < 8$ , provided that the detector's FOV does not exceed the angular divergence of the sounding beam, i.e.,  $v_0 < 3'$ . For media with stronger forward peaked scattering phase function ( $\gamma > 20$  and  $\xi > 5000$ ), at the same detector's FOV, the same accuracy  $\varepsilon$  of reproducing the return  $P(\mathbf{r}, t)$  can be achieved using double-scattering approximation only at  $\tau < 4$ .

2. When low-order approximations are used to describe the return  $P(\mathbf{r}, t)$ , their accuracy degrades with the increasing angular aperture of the detector, especially for media with elongated scattering phase functions (all other conditions being the same). For instance, for scattering media like a radiative fog, even with  $1 < \tau < 2$ , the lidar return can be described accurate to  $\varepsilon > 70\%$  only in triple-scattering approximation, provided  $20 < v_1 < 30'$ .

3. In the statistical experiments, no serious inhomogeneity effect of the scattering medium on the accuracy of description of the  $P_i(\mathbf{r}, t)$  components was found.

## References

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