STABILITY ANALYSIS OF AN ALGORITHM FOR RECONSRUCTING OPTICAL COMPONENTS PROFILES BASED ON LONGITUDINAL ABERRATIONS

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We study the stability (against longitudinal aberration measurement errors, zone marking errors, and errors in the specification of a priori information) of an algorithm for reconstruction of the surface profile of an optical component, giving the radius of curvature of the wavefront, the radius of curvature of the component surface begin monitored, and the refractive index. We propose relations that enable one to estimate the monitoring error in the component profile using longitudinal aberrations in various ways.

The receiving antenna is a basic component of devices for atmospheric studies, which usually employ either a Cassegrain telescope or a large-aperture refractor. We show here that the optics in these devices can be efficiently monitored using a longitudinal aberration method.

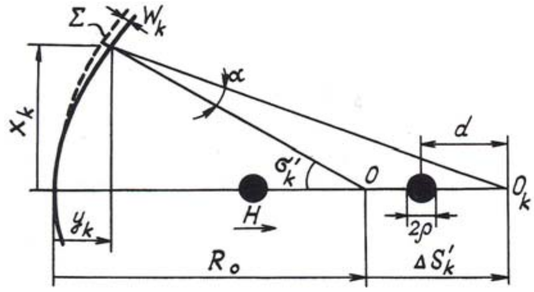

FIG. 1. The arrangement for aspherical wavefront Σ control by longitudinal aberrations $\Delta S'_k$.

The layout for optical control based on longitudinal aberrations is shown in Fig. 1. The wavefront Σ formed by the controlled component, with radius of curvature R_0 at its vertex, is nonspherical; i.e., it has a wave aberration W. For this reason every zone of the wave front at a height K_k from the optical axis has its own radius of curvature and therefore creates its own image of the source O_k, producing a caustic.

In other words the wave aberration W_k in zone κ_k produces a longitudinal spherical aberration $\Delta S'_k$, which is equal to the distance between the paraxial focus (i.e., the focus formed by zones located near the optical axis) and the focus of k-th zone. By placing a fiber F at the focus O_k of each zone, one can observe a shadow pattern that takes the form of the letter Φ superposed on some sort of zone marker that has been applied to the component's exit pupil. The curve on which the points of the wave front cross section are located, i.e., its profile and the corresponding profile of the controlled component can be calculated, provided that the arrays of zone markers and longitudinal aberrations are known. The wave aberration W and deviation of the actual component's profile from the nominal one can also be calculated. This procedure is well known[2], but it is not used in practice for the following reasons:

1. Precision of control is low due to instability of the technique with respect to errors in the measurement of $\Delta S'_k$ of radius of curvature R_0. In analyzing the sources of error, D.D. Maksutov took account only of the capabilities of the equipment then available, but did not carry out a detailed analysis of the method or optimize the control scheme.

2. The necessity of digital integration. This disadvantage was important in the 1930's, when the design procedure was formulated, but has been completely eliminated with the advent of computers.

3. The development of alternative control methods. The procedure discussed in this paper was proposed for quantitative control of spherical mirrors from their centers of curvature. The problem of quantitative control of spherical surfaces was solved with the appearance of Filber's method[3] and with the development of interferometry and automated devices for processing the interferograms[4]. But the method can also be used to control optical components that form aspherical wavefronts, such as aspherical concave mirrors and symmetric lenses[1]. Since the fiber in that case breaks up into distinct elements, it is possible to Improve the precision of $\Delta S'_k$ measurements compared with the case of spherical mirrors. The application of Filber's method and interferometry to the control of aspherical fronts requires the use of additional optical compensating elements to cancel the influence of asphericity. The method's capabilities are then limited, the control procedure becomes complicated, and the reliability of the results is reduced. For these reason, we must discuss the sources of errors in the control method based on

longitudinal aberration measurements. Using Fig. 1 and a simple geometrical relation one can write

$$(R_k - y_k) = x_k \operatorname{ctg} \sigma'_k, \tag{1}$$

where x_k is the height of the k-th zone the wave front, σ'_k is the aperture angle of the k-th zone, $\Delta S'_k$ is the longitudinal aberration of the k-th zone, y_k is the sag, R_k is the radius of curvature of the k-th zone. If there is an error due to wave aberration W_k in the k-th zone it leads to deviation of the normal to the wave front by an angle α, and to the magnification of the longitudinal aberration by the quantity $\delta(\Delta S'_k)$. For relation (1) one obtains

$$dR_k - dy_k = dx_k \operatorname{ctg} \sigma'_k - x_k \alpha / \sin^2 \sigma'_k$$

where $\alpha = d\sigma'_k$.

Ignoring small values and using the obvious equality $dR_k = \delta(\Delta S'_k)$ one can write

$$\delta(\Delta S'_k) = -x_k \alpha / \sin^2 \sigma'_k \tag{2}$$

The error in determination of the longitudinal aberration results in the step-wise error in the profile of the component. In order to affirm this thesis we have calculated the profile of a parabolic mirror. Longitudinal aberration was assumed to vanish in all zones with the exception of the eleventh, where an error was assumed to be present. Figure 2 shows the numerical results obtained with the "Profil" software package. The latter is based on spline interpolation, and is intended for use in reconstructing wavefront profiles and the profiles of controlled components using longitudinal aberrations. A detailed description appears in Ref. 1.

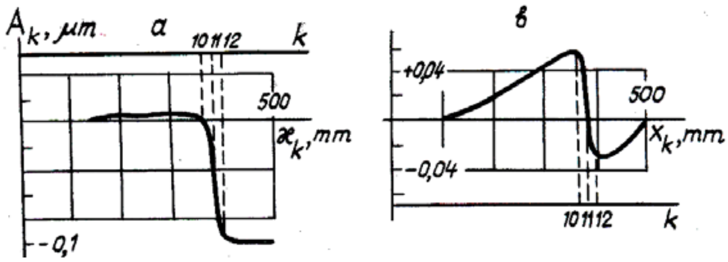

FIG. 2. Deviation of the parabola surface profile ($D/R_0 = 1000/2000$) in the case when the error $\delta(\Delta S'_{11}) = 65$ μm in longitudinal aberrations of the 11-th zone occurs; a) is the deviation of the profile from parabola at the 11-th zone; b) is the deviation of the profile from the closest parabola.

From an analysis of the curves in Fig. 2, one can write down the obvious relation

$$\alpha = -\frac{c A_k}{x_{k+1} - x_{k-1}}, \tag{3}$$

where $c = 1$ if the deviation of the wave front is calculated, $c = 2$ if the deviation of the profile of a mirror is calculated, $c = (1 - n)$ if the deviation of the profile of a lens surface is calculated, and

$$A_k = c\, W_k .$$

The combination of expression (2) and (3) yields

$$A_k = -\frac{\delta(\Delta S'_k) \sin^2 \sigma'_k}{x_k c} (x_{k+1} - x_{k-1}) \tag{4}$$

TABLE 1

The results of calculations of amplitude A_k of error in the surface of parabola $D/R_0 = 1000/2000$ obtained using the "Profil" package: A_k^p and (from Eq. (4)) A_k^t.

x_k, mm / $\delta(\Delta S'_k)$ μm	111.8		273.9		370.8		447.2	
	$-A_k^p$	$-A_k^t$	$-A_k^p$	$-A_k^t$	$-A_k^p$	$-A_k^t$	$-A_k^p$	$-A_k^t$
+5	0.011	0.01	0.08	0.076	0.008	0.0075	0.008	0.0074
+20	0.046	0.044	0.032	0.03	0.032	0.03	0.032	0.029
+30	0.05	0.052	0.047	0.046	0.046	0.044	0.046	0.045
+35	0.08	0.077	0.057	0.054	0.057	0.053	0.056	0.052
+50	0.1	0.11	0.09	0.086	0.08	0.076	0.078	0.074
+65	0.15	0.14	0.1	0.1	0.1	0.098	0.1	0.096

In order to verify Eq. (4) we made model calculations using the "Profil" package for a perfect parabola with D/R_0 = 1000/2000. The numerical results obtained using the package, A_k^p and (from Eq. (4)) A_k^t are given in Table 1. A comparison of and A_r^p shows the high accuracy of Eq. (4). Therefore, it can be used to analyze the stability of the "Profil" algorithm.

Control based on longitudinal aberrations usually makes use of either square-root spacing or uniformly spaced zones. The height κ_k of a zone is then given by

$$x_k = \begin{cases} D/2\sqrt{\dfrac{k}{M}} & \text{for square-root marking} \\ D/2\ \dfrac{k}{M} & \text{for uniform marking,} \end{cases}$$

where M is the total number of zones. Introducing the notation

$$\chi_k^I = \begin{cases} \dfrac{(k+1)^{1/2}-(k-1)^{1/2}}{M}\,k^{1/2} & \text{for square-root marking, } I = 1 \\ 2k/M^2 & \text{for the uniform marking, } I = 2 \end{cases}$$

and using the relation $\sin\sigma'_k \approx x_k/R_0$, one obtains

$$A_k^I = -\frac{\delta(\Delta S'_k)}{c}\,(D/2R_0)^2\chi_k^I \qquad (5)$$

The dependence of χ_k^I on the total number zones for different marking schemes is shown in Fig. 3.

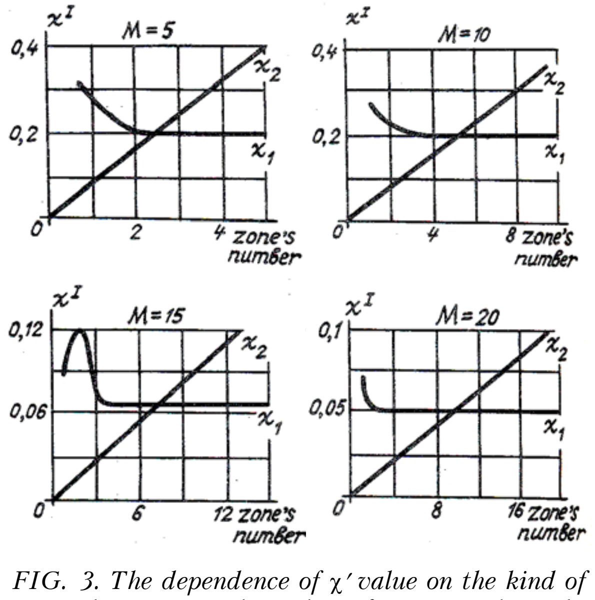

FIG. 3. The dependence of χ′ value on the kind of zones division, total number of zones, and on the particular number of a zone k where an error occurs.

Fig. 3 enables one to draw the following conclusions:

– at the edge of a component, square-root marking scheme is more stable than uniform marking, but the opposite is true at the middle of the component; the situation changes to the opposite one,

– the lowest stability of the square-root marking scheme is in the central zones, and it is at the edge for uniform marking.

ERRORS IN THE MEASUREMENT OF LONGITUDINAL ABERRATION

It might be expected that errors in $\Delta S'_k$ measurements in a few zones would lead to an algebraic sum of amplitudes of the profile's errors; this is confirmed by calculations based on the use of "Profil" package.

Let η_a^I be the admissible error in a longitudinal aberration measurement with I = 1 for square-root marking and I = 2 for uniform marking, where the maximum wavefront error is 3 times smaller than the Rayleigh criterion, i.e. $3W_0 = \lambda/4 = 0.14$ μm. Therefore from (5) we have

$$\eta_a^1 = \frac{0.14}{3}\,\frac{R_0^2 M^{1/2}}{D/2}\ (\mu\text{m}),$$

$$\eta_a^2 = \frac{0.14}{3}\,\frac{R_0^2}{}\,M\ (\mu\text{m}). \qquad (6)$$

The relationship (6) determines the maximum measurement error for longitudinal aberration.

A fiber of diameter 2ρ is placed at the focus O_k of the k-th zone with a residual error η, the value of which is determined by the length of a segment d. It has been empirically established that $|\eta| \approx 0.1$ d. Using relation (6) and geometrical considerations, the requirements imposed on the fiber size in a shadow device are

$$\rho^1 = 10\,\frac{R_0}{D/2}\,M^{1/2}\,\frac{0.14}{3}\ (\mu\text{m}) \quad \text{for square-root marking,}$$

$$\rho^2 = 10\,\frac{R_0}{D/2}\,M\,\frac{0.14}{6}\ (\mu\text{m}) \quad \text{for the uniform marking} \qquad (7)$$

The results of estimation using (7) are presented in Table 2.

Thus, (7) can be used to choose the fiber to be employed in making diffraction measurements of longitudinal aberration. On the other hand, if one has a shadow device with a calibrated fiber of diameter 2ρ, it is possible to determine the achievable control quality. For this, one can use (7):

$$A_a^1 = 0.1\,\frac{3\rho}{c\,M^{1/2}}\,\frac{D}{2R_0},$$

$$A_a^2 = 0.1\,\frac{6\rho}{c\,M}\,\frac{D}{2R_0}; \qquad (8)$$

TABLE 2

The dependence of fiber radius p[mm] for a shadow device on the total number D/R_0 of zones on a component, as well as on the kind of zone marking on the component, which provides for diffraction quality control.

M / D/R_0	uniform marking				square-root marking			
	5	10	15	20	5	10	15	20
1/1	2	3	3.6	4	2	4.5	7	9.5
1/2	4	5.6	7	8.3	4.5	9	14	19
1/3	6	9	11.8	12	7	14	21	28
1/4	8	12	14	16.5	9	18	28	38
1/6	12	17	21.5	24.8	14	28	42	56
1/10	21	29	36	41	23	45	69	93
1/15	31	44	54	62	35	69	104	138
1/20	41	59	72	83	45	90	140	190

ERRORS IN ZONE MARKING

Let the marking produce the error δx_k in the k-th zone height x_k. As a result, the actual height of the k-th zone is $x_k^a = x_k + \delta x_k$. For simplicity, let us make an estimate for a parabolic wavefront, for which

$$y_k = x_k^2/2R_0$$

For a parabola,

$$\Delta S'_k = y_k = x_k^2/2R_0.$$

The error δx_k is then equal to the measurement error of longitudinal aberration, namely

$$\delta x_k = (R_0/x_k)\ \delta(\Delta S_k')^{-1}.$$

Using the relation (5), one finds that for high-quality control (bearing in mind that $3W = \lambda/4$) it is necessary that the zone be marked with an error of no more than

$$\delta x^I = \frac{0.14 \times 10^{-3}}{3}\left[\frac{R_0}{2D}\right]^3\left[\chi^I_{max}\right]^{-1}. \quad (9)$$

The results of estimation based on (9) are presented in Table 3.

TABLE 3

The dependence of the residual error of zone marking δx_k(mm) on D/R_0, the total number of o zones M on the component, and the kind of zone marking which provides for diffractive control quality.

M / D/R_0	uniform marking				root marking			
	5	10	15	20	5	10	15	20
1/1	0,0015	0,0026	0,003	0,005	0,001	0,002	0,003	0,0034
1/2	0,01	0,02	0,03	0,04	0,009	0,005	0,023	0,027
1/3	0,042	0,07	0,084	0,14	0,03	0,05	0,077	0,09
1/4	0,099	0,17	0,2	0,34	0,07	0,12	0,18	0,25
1/6	0,3	0,6	0,7	1,2	0,25	0,4	0,6	0,73
1/10	1,6	2,7	3,1	5,3	1,2	1,9	2,9	3,37

For a ruler marked off with a certain residual error δx, one can determine from the relation (9) the quality of surface it provides:

$$A^I_p = \frac{3\delta x}{c}\chi^I_{max}\left(\frac{D}{2R_0}\right)^3 \quad (10)$$

MEASUREMENT ERRORS OF THE RADIUS OF CURVATURE

The profile control errors due to measurement errors of the radius of curvature of a wavefront are closely related to the choice of reference curve. For example, if ΔR is the error in the measurement of R_0 measurements then the deviation of the sag at the edge of the entrance pupil of the component is

$$A = \left[\frac{D}{2R_0}\right]^2 \Delta R.$$

The quantity A gives the maximum deviation of the mirror's profile from an ideal mirror with the radius R_0. For a parabolic mirror with $D/R_0 = 250/1000$, we have $A = \Delta R/64$. It is clear then that the to provide the requisite diffraction quality, the radius of curvature must be measured with an error of no more than

$$\Delta R_0 \leq 64 \cdot 0{,}07 = 4{,}5\ \mu\text{m}.$$

It is clear that such requirements are not realistic. In this situation it is more creative to calculate the radius of curvature of the controlled wavefront. If we have available an array of points with known coordinates and equation of the curve, we can calculate R_0 provided that other parameters take their theoretical values.

The algorithm for calculating R_0 can be either the least squares method or it can be based on an a priori specification of the zone through which the reference curve must pass. Both approaches are equivalent and either can be used.

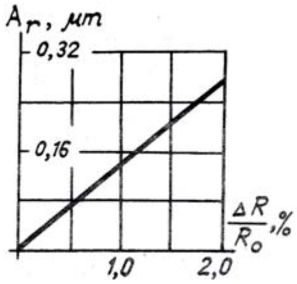

FIG. 4. The dependence of amplitude of an error in the profile of parabolic mirror ($D/R_0 = 250/1000$) on the error of curvature radius measurement $\Delta R/R_0$.

In Fig. 4 the results of a calculations of the parabolic mirror profile with $D/R_0 = 250/1000$, obtained with the "Profil" package, are presented.

One can see from Fig. 4 that diffraction quality control (with $3W = \lambda/4$) can be obtained if the measurement error in the paraxial radius of curvature of the wavefront satisfies the condition

$$\Delta R/R_0 = c\ 0.1\ (\%). \quad (11)$$

The experience of working with the "Profil" program allows us to conclude that the behavior in Fig. 4 takes place not only in this specific case, but is also valid for assessing the amplitude A_k of the profile using the empirical relationship

$$A_R = \frac{23.2}{c}\ \frac{\Delta R}{R}\ (\mu\text{m}). \quad (12)$$

THE INFLUENCE OF A PRIORI PARAMETER MEASUREMENTS

The accuracy of lens control using measurements of longitudinal aberrations by transillumination is influenced by errors in the measurement of the first surface radius R_1, the distance to the light source D_s, and the refractive index n. Ultimately, all these factors determine the measurement errors in the paraxial radius of wavefront curvature.

As a result, one should take ΔR in (12) in the form

$$\Delta R = \Delta R^a + \Delta R^f \quad (13)$$

where ΔR^a is the residual error of the paraxial focus, determination caused by measurement errors in a priori constants, and ΔR^f is the residual error of the paraxial focus measurements.

Using Newton's formula and the expression for the focal length of a lens[7] one can write

$$\Delta R^g = \Delta f' + \Delta x' \quad (14)$$

$$\Delta x' = \begin{cases} -f'\dfrac{\Delta D_n}{D_n^2}; & \\ \dfrac{f'^2}{f_x'^2}\,\Delta x_k', & \text{for } D_n = \infty, \end{cases} \quad (15)$$

$$\Delta f' = \begin{cases} f'^2\left[\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)\Delta n + (n-1)\dfrac{\Delta R_1}{R_1^2}\right] & \\ f'^2\left[\dfrac{\Delta n}{R_2} + (n-1)\dfrac{N\lambda}{(D/2)^2}\right], & \text{for } R_1 = \infty, \end{cases} \quad (16)$$

where ΔD_s is the residual error on measurements of the distance D_s between the lens and the light source, and f_k^{I} is the focal length of the collimator used to simulate a source at infinity; Δx_k is the residual focus error of the collimator at infinity; R_1, R_2 are the radii of the first and second surfaces, respectively; ΔR_1 is the residual error in measurements of the first surface radius; Δn is the residual error of the refractive index measurements; N is the deviation of the first surface from a flat surface, expressed in terms of interference fringes; λ is the wavelength of light.

THE INFLUENCE OF THE LENS MATERIAL INHOMOGENEITY

Inhomogeneities in the optical glass can result in additional phase shifts in the corresponding zone of a lens. The wave aberration due to this phase shift can be determined using the simple relationship

$$W = \delta n\ d,$$

where δn is the maximum deviation of the refractive index from the mean, and d is the thickness of the lens blank. For an admissible wave aberration $W = 0.14$ μm and $d = 30$ mm,

$$\delta n = 4.7 \cdot 10^{-6}, \quad (17)$$

which is a much more stringent requirement on homogeneity than for class A (Ref. 8).

One can draw the following conclusions based on the results of the above investigation.

High quality-control accuracy based on measurements of longitudinal aberrations can be achieved only for the case of mirrors and wavefronts, provided that accuracy of zone marking and measurements of $\Delta S'$ and R_0 is reached (see expressions (8), (10), (11)).

High-accuracy control of the lens surfaces by the longitudinal aberration method is possible if the required accuracy of zone marking and measurements of $\Delta S'$, R_0, R_1 and D_s is provided and the glass of the appropriate class is used (see expressions (8), (10), (15)). The inhomogeneity of the glass can be compensated for, thus exhibiting no noticeable effect on the final results.

It is impossible to ensure high-accuracy control of reflecting surfaces by transillumination (as, for example, in the case of convex mirror with polished rear surface) because of very rigorous constraints on the inhomogeneity of the refractive index.

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