OPTIMIZATION OF ACTUATOR POSITIONS IN FLEXIBLE SINGLE-PLATE MIRRORS

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An approximate optimization of actuator positions in flexible single-plate mirrors within the context of the problem pertaining to compensation for the phase distortions of a light wave transmitted through a layer of the turbulent atmospheric. Numerical calculations are carried out for a correction system based on lower-order Zernike polynomial compensation.

Wavefront correctors with flexible plane mirrors are capable of considerable improvement of optical system performance in a variety of cases[1]. Their salient feature is the accurate restoration of deterministic or random light-wave aberrations using the least number of control channels. The problem of creating high-quality correctors with plate mirrors is closely related to the problem of finding an optimum layout of mirror actuators determined from the criterion of the least mean-square error of the phase-distortion approximation.

The present work seeks to provide an approximate optimization of flexible mirror actuator positions within the context of the problem of the modal compensation for light-wave phase distortions induced by the turbulent atmosphere. Let us consider a correction of the phase distortions $\varphi(r)$. We choose a grid of possible positions for m actuators, fill all the grid nodes with actuators, and then remove those" making the least contribution to the residual error of the approximation of the phase $\varphi(r)$. Hopefully, it will result in an adequate approximation to the optimal placement of the least number of actuators. We write the correction error as

$$\langle\Delta\rangle = \left\langle \frac{1}{S} \int_\Omega \left[\varphi(r) - \sum_{i=1}^{m} P_i R_i(r)\right]^2 d^2r \right\rangle \tag{1}$$

where Ω is the spatial domain of correction, S is the correction area, P_1 is the actuator control actions chosen from the correction error minimization condition, and $R_1(r)$ is the mirror response to the control actions. The angular brackets denote ensemble-averaging over all possible realizations.

We will now derive the correction error $\langle\Delta'\rangle$ that occurs after l actuators have been removed. Assume for definiteness that the first l actuators are withdrawn: $P'_t = 0$, $i = 1, \ldots, l$. Here and in what follows the prime denotes the corresponding transformed quantities. The correction error Δ' is then expressed in terms of Δ by the Lagrangian multiplier rule: $\Delta' = \Delta + \sum \lambda_i P'_i$. Minimizing Δ' with respect to λ_i ($i = 1, \ldots, l$) and P_j ($j = 1, \ldots, m$) readily yields

$$P'_i = \sum_{j=l+1}^{m} c'_{i+j}\, b_j, \qquad j = l+1, \ldots, m,$$
$$c'_{ij} = c'_{ij} - \sum_{\alpha=1}^{l} \sum_{\beta=1}^{l} h_{\alpha\beta}\, c_{i\alpha}\, c_{\beta j}, \quad i, j = 1, \ldots, m, \tag{2}$$

where c_{ij} and $h_{\alpha\beta}$, are elements of the inverse matrices of

$$a_{ij} = \frac{1}{S}(R_i, R_j) \text{ and } c_{\alpha\beta} \ (\alpha,\ \beta = 1, \ldots, l);$$

respectively, and

$$b_i = \frac{1}{S}(\varphi, R_i),$$

where (f, g) denotes the scalar product of the functions f and g over Ω.

It follows from Eqs. (1) and (2) that

$$P_i' = P_i - \sum_{\alpha=1}^{l} \sum_{\beta=1}^{l} h_{\alpha\beta}\, c_{i\alpha}\, P_\beta, \quad i = l+1, \ldots, m;$$
$$\Delta' = \Delta + \sum_{\alpha=1}^{l} \sum_{b=1}^{l} h_{\alpha\beta}\, P_\alpha\, P_\beta, \quad P_i = \sum_{j=1}^{m} c_{ij}\, b_j. \tag{3}$$

Light-wave phase distortions induced by an atmospheric turbulent layer are random and are characterized by the structure function $D_\varphi(r - \rho)$ (Ref. 2). Let us average Eq. (3) over an ensemble of realizations. Since the mean displacement of the mirror as a unit has no effect on the light-wave phase correction, the response functions can be conveniently replaced by

$$\tilde{R}_i = R_i = \frac{1}{S}(R_i,\ 1).$$

Upon averaging Eq. (3) by a standard technique[2], we obtain

$$\langle\Delta'\rangle = \langle\Delta\rangle - \frac{1}{2S_1^2}\sum_{i,j=1}^{m}\sum_{\alpha,\beta=1}^{l} h_{\alpha\beta}\, c_{\alpha i}\, c_{\beta j} \times \int_{\Omega}\int_{\Omega} \tilde{R}_i(r)\, \tilde{R}_j(\rho)\, D_{\varphi}(r-\rho)\, d^2r\, d^2\rho. \quad (4)$$

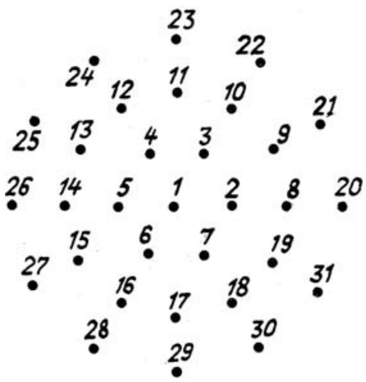

Fig. 1. Disposition of actuators in plane flexible mirrors.

Equations (2)–(4) provide a relatively simple and time-saving iteration scheme for deriving the relevant quantities after the withdrawal of the next l actuators in turn. If the actuators to be removed are chosen so that the difference $\langle\Delta'-\Delta\rangle$ is minimized, a discrete approximate optimization algorithm will result.

The numerical computations were carried out for a correction system based on lower-order Zernike polynomial compensation. Wang and Markey[2] showed these polynomials to be convenient functions for describing the phase of a light wave transmitted through a layer of the turbulent atmosphere. The flexible mirror was chosen to be a round plate subjected to one of the following boundary conditions: edge restraint, simple edge support, or free edge. Such mirrors will be labeled by 1, 2,and 3, respectively. Their response functions are given in Ref. 3. Since mirror displacements and tilts can be obtained without any surface deformation, they were taken into account for in the expressions for the response functions R_j. Optimization of the actuator positions was considered for the first mirror. Algorithm (3) for $l = 1$ and $l = 2$ was used and the relevant functions $\varphi(r)$ were chosen to represent $Z_i(r)$. Since the edge restraint did not allow us to derive the form of Z_i over the entire mirror surface, the radius R_a of the correction domain Ω was assumed to be 1.5 times smaller than that of the mirror. Figure 1 shows a resolvable coordinate grid of actuator positions. The radii of the actuator circles were 0.4 R_a, 0.8 R_a,and 1,2 R_a.

Algorithms for $l = 1$ and $l = 2$ gave virtually the same results (see Table 1).

TABLE 1.

Polynomial	Numbers of Remaining Actuators	Δ_1
Z_4	8, 10, 12, 14, 16, 18	0.0026
Z_6	20 - 31, 8, 11, 14, 17	0.0012
Z_6	20, 23, 26, 29, 8, 11, 14, 17	0.015
Z_8	8 - 10, 12 - 16, 18, 19	0.0003
	8, 10, 12, 14, 16, 18, 20	
Z_{10}	22, 24. 26, 28, 30	0.0035
Z_{10}	20, 22, 24, 26, 28, 30	0.014
Z_{11}	1 - 31	0.057
Z_{11}	8 - 19, 2, 4, 6	0.084

Note that over the region Ω the first ten. Zernike polynomials satisfy the bend equation for a plate which "sees" no external load applied to Ω (see Ref. 3), i.e., $\nabla^2\nabla^2 Z_i = 0$. Therefore, it is interesting to explore the possibility of deriving the forms of Z_i by means of actuators placed outside Ω. The mandatory boundary conditions for Z_i at $r = R_a$ can be established with the help of actuators distributed on two concentric circles of radii R_1, $R_2 > R_a$. The calculations were performed for the aforementioned three mirrors with radii of $1.5R_a$, $1.5R_a$, and $1.2R_a$.The Poisson coefficient was 0.3. Twenty-four actuators were employed, lying equally distributed (twelve each) on two concentric circles of respective radii R_1 and R_2 equal to 1.1 R_a, 1.3 R_a; 1.1 R_a, 1.3 R_a; 0.9 R_a, 1.1 R_a. Errors of approximation of Δ were: Δ_4:$< 10^{-4}$, $< 10^{-4}$, $< 10^{-4}$, $\Delta_{5,6}$: 2×10^{-4}, 2×10^{-4}, $< 10^{-4}$, $\Delta_{7,8}$: 7×10^{-4}, 3×10^{-4}, 5×10^{-4}, $\Delta_{9,10}$: 1.3×10^{-3}, 3×10^{-4}, 3×10^{-4} respectively. Mirrors 1 and 2 with 1.3 R_a-radii utilizing twelve actuators located on a circle of radius 1.1 R_a were also examined. The resulting approximation errors were as follows: Δ_4: 5×10^{-4}, $< 10^{-4}$, $\Delta_{5,6}$: 0.066, 0.049; $\Delta_{7,8}$: 0.01, 0.009, $\Delta_{9,10}$: 0.065, 0.045.

In conclusion it should be pointed out that the field of application of the proposed optimization procedure is by no means restricted to plate mirrors alone. This technique can be successfully used at the theoretical-evaluation and the drawing-board and engineering stages of the design of wavefront a corrector with a priori known responses to control actions.

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