INTEGRAL RESOLUTION OF THE TURBULENT ATMOSPHERE––TELESCOPE OPTICAL SYSTEM

I.P. Lukin

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk
Received September 19, 1994

Integral resolution of the turbulent atmosphere–telescope optical system has been studied for various methods of postdetection processing of short–exposure images. The Labeyrie and Knox–Thompson methods of image processing as well as the method of triple correlation of the image intensity have been investigated. The integral resolution has been computed in terms of the optical transfer function of the turbulent atmosphere and telescope for corresponding image processing method. Knowledge of the system integral resolution allows us to estimate the minimum resolvable distance in observations made through the turbulent atmosphere. All investigated methods are shown to give close results in that parameter better than the results obtained in recording of average images. The method of triple correlation of the image intensity gives the maximum resolution, whereas the Knox–Thompson method gives the minimum one.

Integral resolution of an optical system is one of the three basic characteristics of image quality.[1] Integral resolution having a meaning of a frequency bandwidth is usually defined as the integral of the optical transfer function $\tau\tau(\mathbf{f})$ of an optical system over the spatial frequency $\mathbf{f}$

$$\Re = \int\limits_{-\infty}^{\infty}\int \mathrm{d}\,\mathbf{f}\;\tau(\mathbf{f}),$$

where $\mathbf{f} = \mathbf{p}k/F$ is the spatial frequency, $k = 2\pi/\lambda$, λ is the optical radiation wavelength in vacuum, F is the focal length of the optical system, and $\mathbf{p}$ is the spatial scale. Integral resolution R, as a measure of optical quality, determines the minimum resolvable distance

$$\delta\, l \simeq 1/(2\sqrt{\Re}).$$

A telescopic system has maximum resolution in vacuum, when

$$\Re_0 = \frac{k^2}{F^2}\int\limits_{-\infty}^{\infty}\int \mathrm{d}\,\mathbf{p}\;\tau_0(\mathbf{p}),$$

where $\tau_0(\mathbf{p})$ is the normalized optical transfer function of the telescope. For the Gaussian transmission function of the receiving aperture[2]

$$\tau_0(\mathbf{p}) = \exp\left(-\frac{p^2}{4\,R^2}\right),$$

where R is the aperture radius, the integral resolution of the telescope in vacuum and minimum resolvable distance connected with it are equal to

$$\Re_0 = \frac{4\pi\, k^2\, R^2}{F^2}$$

and $\delta\, l \simeq \dfrac{F}{4\sqrt{\pi}\,k\,R}$, respectively.

We note that the radius of circular physical aperture R_f corresponding to the Gaussian one is $R_f = \sqrt{2}\,R$.

The presence of the atmospheric turbulence distorting an image formed by a telescope leads to lower integral resolution of optical system (frequency bandwidth gets narrower) and correspondingly to the increase of the minimum distance resolvable by the optical system. The distorting effect of the atmospheric turbulence manifests itself most strongly in recording of average image,[1,2] when the integral resolution of the turbulent atmosphere–telescope system may be represented as follows:

$$\Re_1 = \frac{k^2}{F^2}\int\limits_{-\infty}^{\infty}\int \mathrm{d}\,\mathbf{p}\;\tau_1(\mathbf{p}) = \frac{k^2}{F^2}\int\limits_{-\infty}^{\infty}\int \mathrm{d}\,\mathbf{p}\,\exp\left\{-\frac{p^2}{4\,R^2} - \left(\frac{p}{\rho_c}\right)^{c}\right\}, \tag{1}$$

where $\tau_1(\mathbf{p})$ is the optical transfer function of the turbulent atmosphere––telescope system in observation of average image[1,2]

$$\gamma = \begin{cases} 2\,, & \text{for } p < l_0\,,\\ 5/3\,, & \text{for } p > l_0\,,\end{cases}$$

$$\rho_c = \begin{cases} \rho_m = (0.225\,C_\varepsilon^2\,k^2\,L\,\kappa_m^{1/3})^{-1/2}\,, & \text{for } p < l_0,\\ \rho_0 = (0.365\,C_\varepsilon^2\,k^2\,L)^{-3/5}\,, & \text{for } p > l_0\,,\end{cases}$$

C_ε^2 is the structure parameter of the air permittivity fluctuations in the turbulent atmosphere, L is the depth of optically active layer of the turbulent atmosphere, $\kappa_m = 5.92/l_0$, and l_0 is the inner scale of the atmospheric turbulence. The coherence radius of a plane optical wave in the turbulent atmosphere is $\rho_c = \rho_m$, for $\rho_c < l_0$ and $\rho_c = \rho_0$, for $\rho_c > l_0$. Hereafter we will consider the case $\rho_c > l_0$ most often encountered in practice.

From Eq. (1), one can easily derive asymptotic dependences of the telescope integral resolution in recording of an average image

0235-6880/95/03 248-03 $02.00

$$\Re_1 \simeq \begin{cases} \Re_0 (1 - 4\, R^2/\rho_m^2), & \text{for } R \ll l_0, \\ \Re_0 (1 - 2.99 (R/\rho_0)^{5/3}), & \text{for } l_0 \ll R \ll \rho_c, \\ \Re_0 \left(\dfrac{\rho_0^2}{4 R^2}\right) (1 - 0.25\, \rho_m^2/R^2), & \text{for } R \gg \rho_c. \end{cases} \quad (2)$$

As follows from Eq. (2), in observation made through the turbulent atmosphere the limiting resolution of average image (i.e., minimum δl) realized for arbitrary large telescope ($R > \rho_c$) is determined by the coherence radius of a received optical wave

$$\delta\, l_{\min} \simeq \frac{F}{2 \sqrt{\pi}\, k\, \rho_0}\,. \quad (3)$$

If a series of short–exposure images is recorded, they may be processed by one of the known techniques: Labeyrie, Knox–Thompson, or triple correlation of image intensity.[2] In these cases integral resolution of the turbulent atmosphere–telescope system is determined by the following relations:

$$\Re_2 = 2 \frac{k^2}{F^2} \int\!\!\int_{-\infty}^{\infty} \mathrm{d}\, \mathbf{p}\; \tau_2(\mathbf{p}), \quad (4)$$

$$\Re_3 = \frac{k^2}{F^2} \sqrt{\int\!\!\int_{-\infty}^{\infty} \mathrm{d}\, \mathbf{p}_1 \int\!\!\int_{-\infty}^{\infty} \mathrm{d}\, \mathbf{p}_2\; \tau_3(\mathbf{p}_1, \mathbf{p}_2)}\,, \quad (5)$$

$$\Re_4 = \frac{k^2}{F^2} \sqrt{3 \int\!\!\int_{-\infty}^{\infty} \mathrm{d}\, \mathbf{p}_1 \int\!\!\int_{-\infty}^{\infty} \mathrm{d}\, \mathbf{p}_2\; \tau_4(\mathbf{p}_1, \mathbf{p}_2)}\,, \quad (6)$$

where $\tau_2(\mathbf{p})$, $\tau_3(\mathbf{p}_1, \mathbf{p}_2)$, and $\tau_4(\mathbf{p}_1, \mathbf{p}_2)$ are the optical transfer functions of the turbulent atmosphere–telescope system for the Labeyrie and Knox–Thompson methods, and method of triple correlation of the image intensity,[2] repectively. Coefficients in Eqs. (4) and (6) are chosen from the condition of equality of the values $\Re_2$ and $\Re_4$ in the absence of the atmospheric turbulence to the integral resolution of optical system in vacuum $\Re_0$. The extraction of roots in Eqs. (5) and (6) is needed for dimensionality of integral resolution of the Knox–Thompson method and method of triple correlation of the image intensity to coincide with that of the turbulent atmosphere–telescope system when using other image processing techniques.

Substituting the optical transfer functions of the turbulent atmosphere–telescope system calculated in Ref. 2. into Eqs. (4), (5), and (6) we derive the following asymptotic dependence describing the integral resolution of a telescopic system in the turbulent atmosphere for the above–considered techniques of processing of short–exposure images:

$$\Re_2 \simeq \begin{cases} \Re_0\, [1 - 0.33 (\kappa_m\, R)^2\, (R/\rho_m)^2]\,, & \text{for } R \ll l_0, \\ \Re_0\, [1 - 0.73 (R/\rho_0)^{5/3}\,, & \text{for } l_0 \ll R \ll \rho_c, \\ \Re_0 \left[0.44 \left(\dfrac{\rho_0}{R}\right)^2\right] \left[1 + 0.61 \left(\dfrac{\rho_0}{R}\right)^{1/3}\right], & \text{for } R \gg \rho_c \end{cases}$$

for the Labeyrie technique,

$$\Re_3 \simeq \begin{cases} \Re_0\, [1 - 4\, (R/\rho_m)^2]\,, & \text{for } R \ll l_0, \\ \Re_0 [1 - 2.99\, (R/\rho_0)^{5/3}]\,, & \text{for } l_0 \ll R \ll \rho_c, \\ \Re_0 \left[0.35 \left(\dfrac{\rho_0}{R}\right)^2\right] \left[1 + o \left(\dfrac{\rho_0}{R}\right)^{1/3}\right], & \text{for } R \gg \rho_c \end{cases}$$

for the Knox–Thompson technique, and

$$\Re_4 \simeq \begin{cases} \Re_0\, [1 - 0.33\, (\kappa_m\, R)^2\, (R/\rho_m)^2]\,, & \text{for } R \ll l_0, \\ \Re_0 [1 - 0.63\, (R/\rho_0)^{5/3}]\,, & \text{for } l_0 \ll R \ll \rho_c, \\ \Re_0 \left[0.56 \left(\dfrac{\rho_0}{R}\right)^2\right] \left[1 + 0.70 \left(\dfrac{\rho_0}{R}\right)^{1/3}\right], & \text{for } R \gg \rho_c \end{cases}$$

for the technique of triple correlation of the image intensity.

The Labeyrie method and method of triple correlation of the image intensity are seen to yield a maximum amount of increase in the telescope integral resolution for extremely small apertures ($R < l_0$) in the turbulent atmosphere in comparison with the recording of average image and the Knox–Thompson technique. The last offers an advantage in the integral resolution over measurements of average image intensity only for large apertures ($R \gg \rho_c$). The Labeyrie method and the method of triple correlation of image intensity allow the maximum gain in resolution to be achieved for apertures being both smaller and larger than the coherence radius of a received optical wave.

From the viewpoint of attaining the limiting resolution in the turbulent atmosphere for large receiving apertures ($R \gg \rho_c$), the gain of methods of short–exposure image processing over the recording of averaged image is

$$\lim_{R\to\infty} \frac{\Re_2}{\Re_1} \simeq \frac{4 \int_0^{\infty} \mathrm{d}\, p\, \exp\,(-\, 2\, p^{5/6})}{\int_0^{\infty} \mathrm{d}\, p\, \exp\,(-\, p^{5/6})}\, \Re_4 \simeq 1.74$$

(the Labeyrie technique),

$$\lim_{R\to\infty} \frac{\Re_3}{\Re_1} \simeq \sqrt{2} = 1.41$$

(the Knox–Thompson technique), and

$$\lim_{R\to\infty} \frac{\Re_4}{\Re_1} \simeq$$

$$\simeq \frac{\sqrt{18 \int\!\!\int_{-\infty}^{\infty} \mathrm{d}\mathbf{p}_1 \int\!\!\int_{-\infty}^{\infty} \mathrm{d}\mathbf{p}_2\, \exp(-p_1^{5/3} - p_2^{5/3} - |\mathbf{p}_1 + \mathbf{p}_2|^{5/3})}}{\int\!\!\int_{-\infty}^{\infty} \mathrm{d}\mathbf{p}\, \exp(-p^{5/3})} \simeq 2.26$$

(the technique of triple correlation of the image intensity).

Thus it turns out that the minimum resolvable distance of the turbulent atmosphere–telescope optical system for extremely large telescope in processing of a series of short–exposure images decreases respectively by factors of 1.32 for the Labeyrie technique, 1.19 for the Knox–Thompson one, and 1.5 for the technique of the triple correlation of the image intensity in comparison with recording of the average image intensity (see Eq. (3)).

Consequently, from the standpoint of integral resolution, imaging quality can be improved over that in recording of average image to the greatest extent by the method of triple correlation of image intensity, to a less extent by the Labeyrie method, and to the least extent by the Knox–Thompson method.

REFERENCES

1. D.l. Fried, J. Opt. Soc. Amer. **56**, No. 10, 1372 (1966).
2. I.P. Lukin, Atmos. Oceanic Opt. **8**, No. 3, 235–240 (1995).