ITERATIVE METHODS FOR RECONSTRUCTION AND COMPENSATION FOR WAVE FRONT MODES

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Two iteration techniques are proposed for restoration and compensation for wave front modes using the image, which are invariant relative to image source. The results of numerical simulation for the case, when Zernike polynomials are the modes, are presented.

We consider an adaptive optical system in which the pupil function is represented by a finite part of a series over a linearly independent system of functions

$$\Phi(\xi, \eta) = \sum_{k=1}^{n} \zeta_k \, \Phi_k(\xi, \eta) \ , \tag{1}$$

where $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)$ is the unknown mode vector. For instance, such an independent system can consist of response functions of actuators of a flexible adaptive mirror and of piecewise linear functions for a segmented mirror.

It is assumed that the mode compensation proceeds by the adaptive optical system using the control vector $\zeta_u = (\zeta_1^u, \zeta_2^u, \ldots, \zeta_n^u)$:

$$\Phi(\xi, \eta) = \sum_{k=1}^{n} (\zeta_k - \zeta_k^u) \, \Phi_k(\xi, \eta) \ . \tag{2}$$

Let us consider two different formulations of the problem on mode reconstruction and compensation using an image. In the first one, a hardware solution is proposed for an equation in the control vector. For a point source, the equation has the form

$$H(f;\, z,\, \zeta - \zeta_u) = H(f;\, z,\, 0), \tag{3}$$

where $H(f;\, z,\, \zeta)$ is the optical transfer function at the spatial frequency f at a given defocusing z and the unknown mode vector ζ.

If the source of incoherent radiation is arbitrary, two measurements corresponding to the planes $z_1 = 0$ and $z_2 = z$ are required. In this case, we obtain

$$\frac{H(f;\, z,\, \zeta - \zeta_u)}{H(f;\, 0,\, \zeta - \zeta_u)} = \frac{H(f;\, z,\, 0)}{H(f;\, 0,\, 0)} \ . \tag{4}$$

instead of equation (3).

The left-hand sides of equations (3) and (4) are measured in the process of forming the discrepancy vector $\zeta - \zeta_u$ by an optical system. It is supposed that the solution is found by iterative methods for which $\zeta_u \to \zeta$.

Another formulation of the problem of mode reconstruction implies a numerical solution of the equation

$$H(f;\, z,\, \zeta) = H(f;\, z,\, \zeta^*) \tag{5}$$

for to the mode vector ζ. For an arbitrary source, the equation takes the form

$$\frac{H(f;\, z,\, \zeta)}{H(f;\, 0,\, \zeta)} = \frac{H(f;\, z,\, \zeta^*)}{H(f;\, 0,\, \zeta^*)} \ . \tag{6}$$

The solution of equations is supposed to be sought for by iterative methods, but now only one measurement is needed: at the mode vector equal to ζ^*.

Let us examine the solution of the equation (3) in more detail by use of an iterative scheme of the modified Newton method. The equation for discrepancy in such a scheme has the form

$$P(f;\, z,\, \zeta - \zeta_u) = H'(f;\, z,\, 0)(\zeta - \zeta_u) \ , \tag{7}$$

where

$$P(f;\, z,\, \zeta - \zeta_u) = H(f;\, z,\, \zeta - \zeta_u) - H(f;\, z,\, 0).$$

The operation of the iterative scheme is shown in Fig. 1, where $I(z, \zeta)$ is the intensity distribution in the plane displaced from the focal plane by the value of the given defocusing z, $J(f;\, z,\, \zeta)$ is the Fourier transform of the intensity at the spatial frequency f.

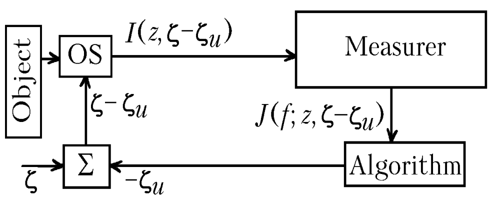

FIG. 1.

The order of calculations by Newton's scheme for the equation (7) is as follows:

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1. Take a zeroth-order approximation for the control vector ζ_u which is always assumed to be zero.

2. Measure the function $H(f; z, \zeta)$ forming the left-hand side of the equation (7).

3. In correspondence with the solution of equation (7), the algorithm forms the control ζ_u transforming the optical system (OS) into a new state for which $\zeta_u \to \zeta$.

4. Let the zeroth-order approximation be equal to ζ_u.

5. Go to the item 2 of the algorithm until the control ζ_u is stabilized at the required accuracy.

Reference 1 presents the results of solution of equations (3) and (4) by Newton method for a segmented mirror when piecewise linear functions are taken as an independent function system for equation (1). In the present paper, we demonstrate the feasibility and convergence of the Newton method for equations (3) and (4) in the case of a flexible mirror what essentially extends the applicability of such a formulation of the problem.

Let Zernike polynomials corresponding to defocusing, spherical aberration, general wave front slope, coma, and astigmatism be taken as an independent function system. In polar coordinates they have the form

$$\Phi_1(\rho) = 2\rho^2 - 1 , \quad \Phi_2(\rho) = 6\rho^4 - 6\rho^2 + 1 ,$$
$$\Phi_3(\rho, \theta) = \rho \cos(\theta) , \quad \Phi_4(\rho, \theta) = \rho \sin(\theta) ,$$
$$\Phi_5(\rho, \theta) = (3\rho^3 - 2\rho) \cos(\theta) , \qquad (8)$$
$$\Phi_6(\rho, \theta) = (3\rho^3 - 2\rho) \sin(\theta) ,$$
$$\Phi_7(\rho, \theta) = \rho^2 \cos(2\theta) , \quad \Phi_8(\rho, \theta) = \rho^2 \sin(2\theta) .$$

Consider now the function $H(f; z, \zeta)$ at a finite number of frequencies $f_k = (r, \psi_k)$, $(k = 1, \ldots, n)$, in polar coordinates and introduce a frequency vector $\mathbf{f} = (f_1, \ldots, f_n)$. Then one can consider a system of equations

$$P(r, \psi_k; z, \zeta - \zeta_u) = H'(r, \psi_k; z, 0)(\zeta - \zeta_u) . \qquad (7a)$$

instead of the equation (7).

The optical transfer function at the frequencies $f(\xi, \eta)$ has the form[2]

$$H(f; z, \zeta) = \int\int_{-\infty}^{\infty} G(\xi + \xi', \eta + \eta')\, G_0(\xi + \xi', \eta + \eta') \times$$
$$\times\, G^*(\xi', \eta')\, G_0^*(\xi', \eta')\, d\xi' d\eta', \qquad (9)$$

to a constant factor. Here $G_0(\xi, \eta) = \exp(-iz(\xi^2 + \eta^2)/2)$ is the pupil function containing aberrations corresponding to the given defocusing z, $G(\xi, \eta) = \exp(-i2\pi\Phi(\xi, \eta))$ is the pupil function with an unknown aberration function (1) (i is the imaginary unit).

The iterative scheme for the system of equations (7a) is feasible if a nonsingular matrix is formed from the derivative vector $H'(r, \psi; z, 0)$ by choosing frequencies $f_k = (r, \psi_k)$. Using Eq. (9), one can obtain the expression for the derivative vector in polar coordinates

$$H'(r, \psi; z, 0) = \sum_{j=1,2} \varphi_j(r)\, \zeta_j + \sum_{j=3,5} \varphi_j(r)\, \zeta_j \cos(\psi) +$$
$$+ \sum_{j=3,5} \varphi_j(r)\, \zeta_{j+1} \sin(\psi) + \varphi_7(r)\, \zeta_7 \cos(2\psi) +$$
$$+ \varphi_7(r)\, \zeta_8 \sin(2\psi) , \qquad (10)$$

where

$$\varphi_1(r) = 16\pi^2 (r/b)\, [J_0(b) - 2J_1(b)/b] ;$$
$$\varphi_2(r) = 48\pi^2 (r/b)\, [J_0(b) - 6J_1(b)/b + 16J_2(b)/b^2] ;$$
$$\varphi_3(r) = 4\pi^2\, r\, J_1(b)/b ; \qquad (11)$$
$$\varphi_5(r) = 4\pi^2\, r\, [4J_1(b)/b - 12J_2(b)/b^2 - J_3(b)/b] ;$$
$$\varphi_7(r) = -8\pi^2\, r\, b\, J_2(b)/b^2 .$$

Here $J_s(b)$ are Bessel functions of the sth order, and the parameter $b = rz$. Let us take $\psi_k = (2/3)\pi k$ and make up the following expressions

$$M0 = \sum_{k=0}^{2} P(r, \psi_k; z, \zeta - \zeta_u) ,$$
$$M1 = \sum_{k=0}^{2} P(r, \psi_k; z, \zeta - \zeta_u) \exp(i\psi_k) ,$$
$$M2 = \sum_{k=0}^{2} P(r, \psi_k; z, \zeta - \zeta_u) \exp(i2\psi_k) . \qquad (12)$$

Then the equations for the discrepancies have the form

$$\mathrm{Re}(M)/3 = \sum_{j=1,2} \varphi_j(r)\, \zeta_j ;$$
$$\mathrm{Im}(M1 + M2)/3 = \sum_{j=3,5} \varphi_j(r)\, \zeta_j ,$$
$$\mathrm{Re}(M1 - M2)/3 = \sum_{j=3,5} \varphi_j(r)\, \zeta_{j+1} ; \qquad (13)$$
$$\mathrm{Re}(M1 + M2)/3 = \varphi_7(r)\, \zeta_7 ,$$
$$\mathrm{Im}(M1 - M2)/3 = \varphi_7(r)\, \zeta_8 ,$$

and the system of equations (7a) can be divided into three systems of the second order and two equations by use of six measurements of the function $P(r, \psi_k; z, \zeta - \zeta_u)$ aΦ $f_k = (r_1, \psi_k)$, $f_k = (r_2, \psi_k)$, $k = 0, 1, 2$.

TABLE I.

Number	1	2	3	4	5	6
$\zeta_1 - \zeta_1^u$	0.1500	0.2000	0.1705	0.1205	0.0705	0.0216
$\zeta_2 - \zeta_2^u$	0.0500	0.0000	−0.0500	−0.0539	−0.0117	−0.0218
$\zeta_3 - \zeta_3^u$	0.1375	0.0964	0.0464	0.0194	0.0078	0.0042
$\zeta_4 - \zeta_4^u$	0.0500	0.0187	0.0048	−0.0004	0.0006	−0.0001
$\zeta_5 - \zeta_5^u$	0.0500	0.0021	0.0142	0.0148	0.0108	0.0060
$\zeta_6 - \zeta_6^u$	0.0500	0.0173	0.0081	0.0057	0.0021	0.0015
$\zeta_7 - \zeta_7^u$	0.0500	0.0000	−0.0234	−0.0124	−0.0089	−0.0030
$\zeta_8 - \zeta_8^u$	0.0500	0.0000	−0.0225	−0.0121	−0.0089	−0.0025
$\|\zeta - \zeta_u\|$	0.2375	0.2235	0.1873	0.1355	0.0738	0.0319

TABLE II.

Number	1	2	3	4	5	6
$\zeta_1 - \zeta_1^u$	0.1500	0.2000	0.1500	0.1000	0.0500	0.0168
$\zeta_2 - \zeta_2^u$	0.0500	0.0000	−0.0500	−0.0074	0.0069	−0.0095
$\zeta_3 - \zeta_3^u$	0.1335	0.0987	0.0487	0.0173	0.0078	0.0056
$\zeta_4 - \zeta_4^u$	0.0500	0.0203	−0.0039	0.0012	−0.0055	0.0015
$\zeta_5 - \zeta_5^u$	0.0500	0.0000	0.0155	0.0171	0.0107	0.0036
$\zeta_6 - \zeta_6^u$	0.0500	0.0118	0.0083	0.0015	0.0067	−0.0011
$\zeta_7 - \zeta_7^u$	0.0500	0.0000	−0.0051	−0.0207	−0.0290	−0.0349
$\zeta_8 - \zeta_8^u$	0.0500	0.0000	−0.0315	−0.0186	−0.0068	0.0092
$\|\zeta - \zeta_u\|$	0.2352	0.2224	0.1694	0.1069	0.0607	0.0415

The solution of the system (13) was performed at the relative frequencies $r_1 = 0.05$ и $r_2 = 0.1$, whose value has the sense of a relative displacement in the pupil plane by the vector $f = -(r_1\cos\psi_k,\ r_2 \sin\psi_k)$ in the formula for optical transfer function (9). The initial values of the mode vector coordinates in Eq.(1) correspond to larger aberrations. The aberrations reach the value 0.8λ (λ is the wavelength) at the edges of a round pupil for $\zeta_k = 0.1$. The values of the discrepancy vector $\zeta - \zeta_u$ at first six iterations under restriction on the module of the control value $|\zeta_k^u| = 0.05$ are presented in Table I. Such a restriction is natural for the control system and, besides, excludes overshoots at the first iteration. In addition to the discrepancies, the table presents a generalized result, namely, the discrepancy vector norm at each iteration.

The result of the algorithm operation with the measurement error equal to 3% of the maximum measurement value is presented in Table II. The table shows that the algorithm has good convergence already at the sixth step.

REFERENCES

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