RAYLEIGH VS. SODIUM BEACON FOR A PARTIAL CORRECTION OVER WIDE FIELD OF VIEW

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Ground and low–altitude layers perturbing the incoming wave front are characterized by a very high field–of–view isoplanatism. However, images of a sodium beacon and stars are affected also by high–altitude layers, characterized by a small isoplanatic angle. Correction of an off–axis image of an object using the latter type of reference wave will introduce an additional degradation of the wave front due to the lack of correlation between the high–altitude layers seen at different angles, while the use of small altitude Rayleigh reference source is affected only by the loss of information on the higher layers. A trade–off between the two approaches, versus the covered field of view, and the $C_n^2(z)$ behavior is discussed in this paper.

I. INTRODUCTION

The problem on the isoplanatism arises when creating any adaptive optical system. An attempts to overcome this restriction have been considered in a number of papers (see, e.g., Refs. 1, 2) using so–called multidithering adaptive optics. Such an approach allows the fully corrected image to be obtained within a greater field of view than the area of the isoplanatism.

Behavior of an incident wave front being corrected using the reference source displaced by an angle Θ is often described in the following way:[3]

$$\sigma^2 = (\Theta/\Theta_0)^{5/3}, \tag{1}$$

where σ^2 is the standard deviation of the wave front expressed in squared radians; Θ_0 is the parameter characterizing the atmosphere and defined as

$$\Theta_0 = \left[2.91 \left(\frac{2\pi}{\lambda} \right)^2 \int\limits_0^\infty C_n^2(z)\, z^{5/3}\, dz \right]^{-3/5}, \tag{2}$$

where the designations used are commonly accepted.

Equation (1) does not hold at $\Theta > \Theta_0$, where the resulting values of σ^2 strongly exceed the true ones.

Asymptotic behavior of the true function $\sigma^2(\Theta)$ can be found under conditions that for very large Θ angles the radiation coming from an object and the reference wave front are fully uncorrected so that

$$\sigma^2(\Theta \to \infty) \approx 2\, \sigma^2, \tag{3}$$

where σ is the standard deviation for an infinitely far object. For an arbitrary height h, the standard wave front deviation σ_h is given by for following expression:[4]

$$\sigma_h^2 = 0.134\, (D/r_h)^{5/3}, \tag{4}$$

where D is the telescope diameter, r_h is the Fried's radius for the heights less than h determined using a standard technique as

$$r_h = \left[0.423 \left(\frac{2\pi}{\lambda} \right)^2 \int\limits_0^h C_h^2(z)\, z^{5/3}\, dz \right]^{-3/5}. \tag{5}$$

In Ref. 4 we assumed that no tilts are present in the wave fronts coming from an object and a reference source.

In a similar manner the angle Θ_h can be obtained by substituting h for ∞ in Eq. (2) as well as σ_∞ by substitution of r_∞ for r_h in Eq. (4).

Since C_n^2 is mostly concentrated in the ground layers characterized by Θ_h values which in most cases are far greater than Θ_0 (usually at 1–km height the excess is about one order of magnitude), we study the possibility of using a Rayleigh beacon (Fig. 1) for a moderate improvement of the image contrast in the field–of–view sectors far greater than Θ_0.

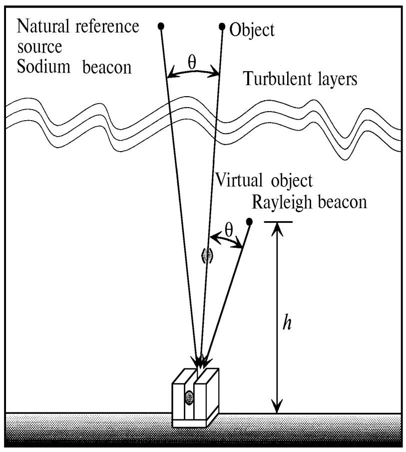

FIG. 1. Conceptual diagram of a comparison between the Rayleigh and sodium (or natural) reference sources.

2. ANALYTICAL APPROACH

As an example, let us consider a telescope with a 3.5–m – diameter mirror located at the Canarias (project of Italian National telescope "Galilei"[5]), the C_n^2 profile for

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which can be found in literature. We used the table values of the profile published by Barletti (Ref. 6) scaled to bring into agreement with recent measurements made with the Nordic Optical telescope.[7] The resulting plots obtained using spline interpolation are shown in Fig. 2.

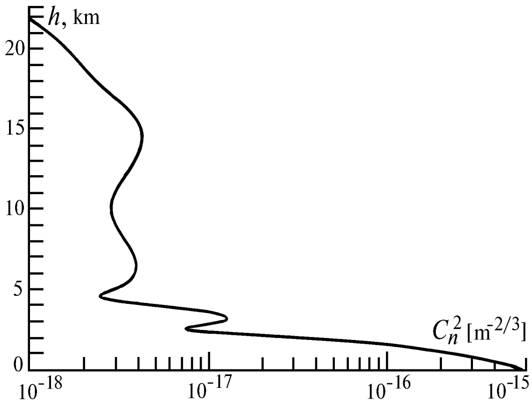

FIG. 2. $C_n^2(z)$ function constructed using the chosen atmospheric model.

The corresponding value of Fried's radius is about 10 cm assuming that $\lambda = 500$ nm (this wavelength will be used below). Such r_0 value provides the resolution on the order of 1°, i.e., the value which can be considered as conservative one from the analysis of the Canarias statistics.[8]

Corresponding values of $\sigma^2(\Theta)$ were derived by the following methods:

1) For $- h = \infty$ we calculated the values of r_0 and Θ_0 using Eqs. (5) and (2) and evaluated their asymptotic values from Eq. (3). The two curves of $\sigma^2(\Theta)$ given by Eqs. (1) and (3) are sewn using spline interpolation, which is valid conditionally and only in the inflection area. As will be shown below, the plot area of greatest interest is located far enough from the considered segment of the resulting curve.

2) For $- h < \infty$, the corresponding values of r_h and Θ_h are calculated using Eqs. (5) and (2) modified in accordance with the procedure described above. To sew the curves given by Eqs. (1) and (3), we use the same method as in the above case. It corresponds to behavior of hypothetic object wave front (see Fig. 1) when correcting with the use of a Rayleigh beacon as a reference source. In so doing the constant should be added which corresponds to the degradation of parameters at height from h to infinity. It can be shown that this additive term is approximately equal to the difference $\sigma^2 - \sigma_h^2$. Asymptotic behavior is calculated taking into account the contribution from two wave fronts entering into problem, i.e., $\sigma^2(\Theta \to \infty) \approx \sigma^2 - \sigma_h^2$ in the above assumption on the absence of correlation between them.

In this approximation the problem on focus isoplanatism is neglected. It can be avoided when using a reference source consisting of several beacons, each being used for correction of a separate area of the whole telescope aperture. The technology used falls in the same class as in multidithering adaptive optics (although the light scattering effect is far less for a Rayleigh beacon placed farther from the object, as concerning the degradation of the object image).

The following points should be emphasized:

– if the beacon grid is created by a successive launchings of beacons to different points then the duration of this process must be shorter than the atmospheric time constant;

– wave–front sensor must record the radiation from all beacons to measure the phase gradient between them;

– relative position of beacons in a layer must be known as accurate as possible.

For the case of a sodium beacon, the number n_{Na} of sources needed is defined, by the order of magnitude as

$$n_{\mathrm{Na}} \approx (\text{Field of view} \,/\, \Theta_0)^2, \tag{6}$$

whereas in the case of Rayleigh beacons the number of sources is approximately calculated by the following formula:

$$n_{\mathrm{Ra}} \approx (D/r_h)^2. \tag{7}$$

Because $\Theta_0 \approx 0.6\, r_0/L$, where L is the characteristic height of disturbing layers (it is reasonable to suggest that $L \approx 10$ km), by setting $r_h \approx r_0$ we obtain

$$n_{\mathrm{Na}}/n_{\mathrm{Ra}} \approx 23.7\,(\text{Field of view}\,(\text{in min. of arc}))^2 \,/\, (D(\mathrm{m}))^2\,. \tag{8}$$

For $D = 3.5$ m the ratio (8) is close to unity in the field of view about 2′. The results are presented in Fig. 3.

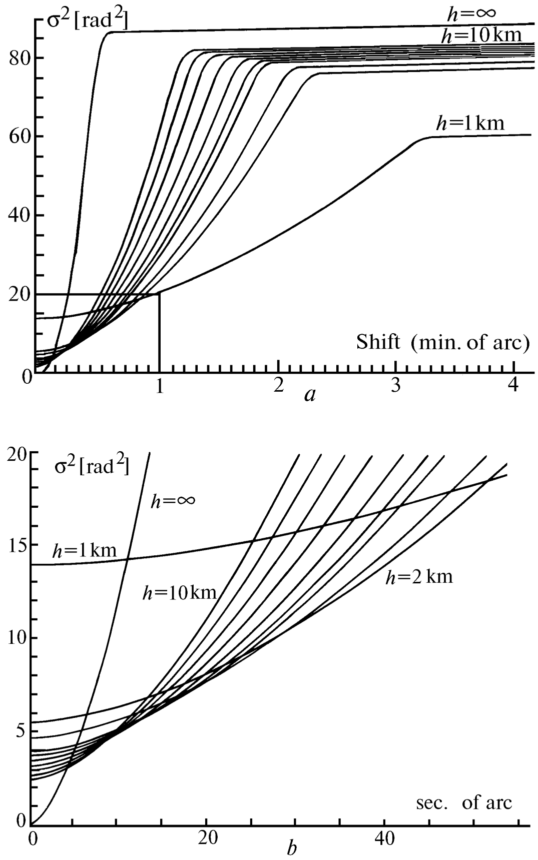

FIG. 3. Wave front standart deviation for the source image corrected using radiation from beacons located at different altitudes h. The rectangle separated at the lower left is drawn magnified in Fig. 3b.

Thus, a number of the key features should be noted. A Rayleigh beacon located at an altitude about 5 km is capable to hold $\sigma^2 < 10$ rad^2 (that corresponds to $\sigma \approx \lambda/2$) in a sector to 30 seconds of arc with the minimum bellow $\sigma^2 = 4$ rad^2 (that corresponds to $\sigma \approx 0.3\lambda$). Moreover, $\sigma^2 < 36$ rad^2 (corresponds to $\sigma \approx \lambda$) can be obtained in a sector to 2′ minutes of arc using a Rayleigh source at an altitude of 1 km.

3. CONCLUSION

It is shown that it is possible to reach the marked improvement of the parameters of an optical system in its field of view using only partial adaptive correction. The improvement of more than one order of magnitude can be achieved. The problem of focus isoplanatism and its practical aspect will be also studied and described in the next paper. Particular attention in that case will be paid to the differential tilts in the covered field of view.

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