Error of polarization targeting by optical radiation in the turbulent atmosphere

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The error introduced by the atmospheric turbulence into the information field of optical radiation polarization of the optical targeting system is investigated. The statistical characteristics of the targeting signal (mean value and dispersion) are considered theoretically. Based on the obtained results, the operation range of the targeting system is estimated.

The targeting systems operating in the atmosphere are exposed to a variety of distortion factors, in particular, the atmospheric turbulence. Generally, the problem of the atmospheric influence on the accuracy of optoelectronic targeting systems is very complicated. This is due to the fact that different targeting systems (depending on the type of the information coming to the receiver) are exposed to different fluctuation effects. For example, intensity fluctuations in the plane of analyzer influence primarily on the accuracy of measurements by amplitude differential techniques employing a 4-platform receiver.[1,2] Wave angle fluctuations introduce the principal error in measurements of angular coordinate of an emitting object by the position-sensitive techniques employing amplitude sensors.[1,2] Angular phase sensors are the most sensitive to wave-front distortions of the incoming radiation. In this paper, the error is calculated, which is introduced by the atmospheric turbulence into the operation of a targeting system specifying direction by forming in space the information field of polarization of the optical radiation.

Define an optical field at laser output $E_0(0,\boldsymbol{\rho})$ in the form of a single-mode Gaussian beam

$$E_0(0,\boldsymbol{\rho}) = E_0(\boldsymbol{\rho}) = E_0 \exp\left[-\frac{\rho^2}{2a_0^2} - \frac{ik}{2R_0}\rho^2\right], \qquad (1)$$

where E_0 is the initial amplitude of the optical field at the laser optical axis; a_0 is the initial value of the laser beam radius; R_0 is the initial value of the wavefront curvature radius in the center of radiating aperture; $k = 2\pi/\lambda$, λ is the wavelength of optical radiation in vacuum; $\boldsymbol{\rho} = \{y, z\}$. Let a linear polarized light beam transmits through an optical wedge in OY-direction. The optical field at output from the wedge can be written as

$$E(0,\boldsymbol{\rho}) = E_0(0,\boldsymbol{\rho}) \exp[i\phi(y)], \qquad (2)$$

where $\phi(y)$ is optical field taper in the wedge. If the optical wedge is made of birefringent crystalline plate cut out in parallel to optical axis of a crystal (OY-axis) and the optical wave incidence is normal to the crystal surface, than two rays (ordinary and extraordinary) propagate through the crystal with different velocities, and electric oscillations in them occur in mutually perpendicular planes. The phase difference $\delta(y)$ appearing between the rays after their transmittance through the optical wedge of thickness $d(y)$ is

$$\delta(y) = \phi_o(y) - \phi_e(y) = -k(n_o - n_e)\, d(y), \qquad (3)$$

where n_o and n_e are the crystal refractive indices for the ordinary and extraordinary rays, respectively. Oscillation amplitudes of electric vectors E_o and E_e of the ordinary and extraordinary rays, respectively, are numerically equal to

$$A = E_0\cos(\theta); \quad B = E_0\sin(\theta),$$

where θ is the angle between the direction of E-vector oscillations in the incident polarized light and the direction of the crystal optical axis (OY). Let $\theta = 45°$, than $A = B = E_0/\sqrt{2}$. Generally, a wave yielded from the optical wedge is elliptically polarized:

$$\begin{cases} E_y(0,\boldsymbol{\rho}) = \dfrac{E_0}{\sqrt{2}} \exp\left[-\dfrac{\rho^2}{2a_0^2} - \dfrac{ik}{2R_0}\rho^2 + ikn_e d(y)\right], \\ E_z(0,\boldsymbol{\rho}) = \dfrac{E_0}{\sqrt{2}} \exp\left[-\dfrac{\rho^2}{2a_0^2} - \dfrac{ik}{2R_0}\rho^2 + ikn_o d(y)\right], \end{cases} \qquad (4)$$

where E_y is the electric field strength of the extraordinary ray (electric vector of the extraordinary ray is OY-directed: $\mathbf{E}_e = \{0, E_y, 0\}$); E_z is the electric field strength of the ordinary ray (electric vector of the ordinary ray is normal to the electric vector of the extraordinary ray: $\mathbf{E}_o = \{0, 0, E_z\}$).

Let for definiteness

$$(n_o - n_e)\, d(y) = \begin{cases} 0 & \text{for } y = -l/2; \\ \lambda/4 & \text{for } y = 0; \\ \lambda/2 & \text{for } y = l/2, \end{cases}$$

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where l is the length of the optical wedge in the OY-direction, i.e.,

$$(n_o - n_e)\, d(y) = \frac{\lambda}{4} + \frac{\lambda}{2l} y$$

at $y \in [-l/2,\ l/2]$. Hence, the optical wave has a circular polarization at the optical axis of a targeting system (OX-axis, $y = 0$), linear polarization at the boundaries of the information field $y = \pm l/2$, and elliptical polarization at intermediate values of y.

Waves yielded from the optical wedge (4) propagate in the atmosphere in different directions, diverging increasingly in space, due to different initial slopes of wavefronts, which can cause a degradation of the information field structure at a point of radiation reception. To avoid this, it is necessary to transmit radiation through a system of formation of optical signals, which shifts optical beams with different directions of polarization vectors relative to each other. The shift must be such that the spaced ordinary and extraordinary rays converge at the observation point (or become close to the convergence). Under these conditions, the optical field at the outlet from the forming system can be written as

$$\begin{cases} E_y(0,\boldsymbol{\rho}) = \dfrac{E_0}{\sqrt{2}} \exp\left[-\dfrac{(y-y_e)^2}{2a_0^2} - \dfrac{ik}{2R_0}(y-y_e)^2 - \right. \\ \qquad \left. - \dfrac{z^2}{2a_0^2} - \dfrac{ik}{2R_0} z^2 + ikn_e d(y) \right], \\ E_z(0,\boldsymbol{\rho}) = \dfrac{E_0}{\sqrt{2}} \exp\left[-\dfrac{(y-y_o)^2}{2a_0^2} - \dfrac{ik}{2R_0}(y-y_o)^2 - \right. \\ \qquad \left. - \dfrac{z^2}{2a_0^2} - \dfrac{ik}{2R_0} z^2 + ikn_o d(y) \right], \end{cases} \tag{5}$$

where y_e and y_o are the centroids of beams of the extraordinary and ordinary rays at the output from the system.

Consider the optical radiation field in the turbulent atmosphere at the distance x from the system of formation of optical signals. According to Ref. 3, it can be written as

$$\begin{cases} E_y(x,\boldsymbol{\rho}) = \dfrac{1}{\sqrt{2}} E_e(x,\boldsymbol{\rho}) \exp[ikx + ikn_e d(y)], \\ E_z(x,\boldsymbol{\rho}) = \dfrac{1}{\sqrt{2}} E_o(x,\boldsymbol{\rho}) \exp[ikx + ikn_o d(y)], \end{cases} \tag{6}$$

where

$$E_\ell(x,\boldsymbol{\rho}) = E_{0\ell}(x,\boldsymbol{\rho}) \exp\left[\chi_\ell(x,\boldsymbol{\rho}) + iS_\ell(x,\boldsymbol{\rho})\right];$$
$$\ell = o \text{ or } e.$$

Here $E_{0\ell}(x,\boldsymbol{\rho})$ is the field at the point $\{x,\boldsymbol{\rho}\}$ produced by a source through a homogeneous medium; $\chi_\ell(x,\boldsymbol{\rho})$ and $S_\ell(x,\boldsymbol{\rho})$ are, respectively, fluctuations of the logarithm of amplitude and optical wave phase.

Optical radiation is detected with two point square receivers reacting to mutually perpendicular values of polarization. Generally, polarization directions of receivers do not coincide with directions of polarization vectors of optical wave. With regard to location of receivers at different points $\boldsymbol{\rho}_1 = \{y_1, z_1\}$ and $\boldsymbol{\rho}_2 = \{y_2, z_2\}$, the photocurrents of the receivers can be written in the following form[4]:

$$\begin{cases} \iota_1 = \eta_1 E_\perp(x,\boldsymbol{\rho}_1 1) E_\perp^*(x,\boldsymbol{\rho}_1), \\ \iota_2 = \eta_2 E_\parallel(x,\boldsymbol{\rho}_2) E_\parallel^*(x,\boldsymbol{\rho}_2), \end{cases} \tag{7}$$

where η_1 and η_2 are quantum efficiencies of the receivers;

$$E_\perp(x,\boldsymbol{\rho}_1) = E_y(x,\boldsymbol{\rho}_1)\cos(\psi) - E_z(x,\boldsymbol{\rho}_1)\sin(\psi);$$
$$E_\parallel(x,\boldsymbol{\rho}_2) = E_y(x,\boldsymbol{\rho}_2)\sin(\psi) + E_z(x,\boldsymbol{\rho}_2)\cos(\psi);$$

ψ is the angle between the polarization vectors of receivers ($E_\perp$, $E_\parallel$) and of optical wave (E_y, E_z).

The value of targeting signal ι can be found as a ratio of difference between the photocurrents of two receivers to their sum:

$$\iota = \frac{\iota_2 - \iota_1}{\iota_1 + \iota_2}. \tag{8}$$

Since the receivers can be arranged right up to each other ($\boldsymbol{\rho}_1 \to \boldsymbol{\rho}_2$), than $d(y_1) \cong d(y_2)$, $S_o(x,\boldsymbol{\rho}_1) - S_o(x,\boldsymbol{\rho}_2) \to 0$ and $S_e(x,\boldsymbol{\rho}_1) - S_e(x,\boldsymbol{\rho}_2) \to 0$. Taking into account a smallness of the difference $|y_e - y_o|$, we obtain for regions of weak fluctuations of the partial optical wave intensity[3]

$$\begin{aligned} \iota \cong \frac{1}{4} \{ & \exp\{i[S_o(x,\boldsymbol{\rho}_1) - S_e(x,\boldsymbol{\rho}_1)] + ik(n_o - n_e)\, d(y_1)\} + \\ & + \exp\{-i[S_o(x,\boldsymbol{\rho}_1) - S_e(x,\boldsymbol{\rho}_1)] - ik(n_o - n_e)\, d(y_1)\} + \\ & + \exp\{i[S_o(x,\boldsymbol{\rho}_2) - S_e(x,\boldsymbol{\rho}_2)] + ik(n_o - n_e)\, d(y_2)\} + \\ & + \exp\{-i[S_o(x,\boldsymbol{\rho}_2) - S_e(x,\boldsymbol{\rho}_2)] - ik(n_o - n_e) d(y_2)\}\} \sin(2\psi). \end{aligned} \tag{9}$$

Since phase fluctuations of the optical wave propagating in the turbulent atmosphere are normally distributed with the mean close to zero,[3,5] then the mean value of the targeting signal ι (9) can be written as

$$\begin{aligned} \langle\iota\rangle \cong \exp\left\{ -\frac{1}{2} \left\langle [S_o(x,\boldsymbol{\rho}) - S_e(x,\boldsymbol{\rho})]^2 \right\rangle \right\} \times \\ \times \cos[k(n_o - n_e)\, d(y)] \sin(2\psi), \end{aligned} \tag{10}$$

where $\left\langle [S_o(x,\boldsymbol{\rho}) - S_e(x,\boldsymbol{\rho})]^2 \right\rangle = D_S(y_o, y_e)$ is the structure function of phase fluctuations of two Gaussian beams with the initial conditions (5). The structure function calculation by the method of smooth perturbations[3,5] shows that for $l_0 < |y_o - y_e| < L_0$

$$D_S(y_o, y_e) \cong 2\,\alpha(\mu, \Omega_0) \left(\frac{|y_o - y_e|}{\rho_0} \right)^{5/3}, \tag{11}$$

$$\alpha(\mu,\Omega_0) = \\ = \frac{3}{8}\frac{|-\mu(1-\mu)+\Omega_0^{-2}|^{5/3}+|\mathrm{Re}\{[-\mu(1-\mu)+\Omega_0^{-2}+i\Omega_0^{-1}]^{5/3}\}|}{[(1-\mu)^2+\Omega_0^{-2}]^{5/3}},$$

where l_0 and L_0 are inner and outer scales of turbulence, respectively[3,5]; $\mu = x/R_0$ is the beam focusing parameter; $\Omega_0 = ka_0^2/x$ is the Fresnel number of the transmitting aperture;

$$\rho_0 = \left(2^{-5/3}\frac{18}{5}0.033\,\pi^2\,\Gamma\left(\frac{7}{6}\right)\Big/\Gamma\left(\frac{11}{6}\right)C_\varepsilon^2 k^2 x\right)^{-3/5}$$

is the coherence radius of a plane optical wave at a point of radiation reception; C_ε^2 is the structural parameter of fluctuations of the turbulent atmosphere dielectric permittivity.[3,5]

Calculations by the formula (11) for infrared radiation (for example, for $\lambda = 10.6$ μm) at horizontal, vertical, and slant paths with lengths not more than tens of kilometers at $|y_o - y_e| \le 10^{-2}$ m show, that the condition $D_S(y_o, y_e) \ll 1$ is always fulfilled for collimated ($\mu = 0$) or diverging ($\mu < 0$) optical beams, i.e. the mean value of the targeting signal (10) agrees with ι in a homogeneous medium:

$$\langle\iota\rangle \cong \cos[k(n_o - n_e)\,d(y)]\sin(2\psi). \quad (12)$$

Therefore, $\langle\iota\rangle = 0$ at the optical axis of the system (circular polarization), and takes its maximal values

$$\langle\iota\rangle \cong \pm\sin(2\psi)$$

at the boundary of the information field (linear polarization).

Normalized variance of fluctuations can serve a measure of the random signal modulation. Calculated under the same assumptions as for the mean value of the targeting signal, it can be written as

$$\sigma_\iota^2 = \frac{\langle\iota^2\rangle - \langle\iota\rangle^2}{\langle\iota\rangle^2} \cong$$

$$\cong D_S(y_o, y_e)\tan^2[k(n_o - n_e)\,d(y)] + \frac{1}{2}D_S^2(y_o, y_e). \quad (13)$$

Due to zero mean value of the targeting signal (10), $\sigma_\iota^2 \to \infty$ in the center of information field (circular polarization). The targeting signal variance (13) reaches its minimum

$$\sigma_\iota^2 \cong \frac{1}{2}D_S^2(y_o, y_e)$$

at the boundary of the information field.

Variance of atmospheric turbulence-induced fluctuations of the targeting signal is not higher that 10% practically all over the information field of the polarization targeting system for infrared radiation at $|y_o - y_e| \le 10^{-2}$ m along horizontal, vertical, and slant paths of $x \le 10$ km length. The only exception is a small vicinity of the system's optical axis, where the level of fluctuations can be significant. Linear dimension (l_{sp}) of the vicinity can be estimated from the condition

$$\sigma_\iota^2 \le 0.1.$$

The following expression for l_{sp} can be then derived from Eq. (13):

$$l_{sp} \le l_{field}\,\frac{\sqrt{10}}{\pi}\sqrt{D_S(y_o, y_e)},$$

where l_{field} is the linear dimension of the targeting field at the measurement point.

Estimates show, that $l_{sp}/l_{field} \le 0.1$ at vertical and slant paths with the length $x \cong 10$ km under any atmospheric conditions. The worst situation ($l_{sp}/l_{field} \cong 0.5$) is realized at a horizontal surface path of the length $x \cong 10$ km (at the optical radiation transmission height of about 1 m above the underlying surface) under conditions of maximal turbulence.[3,5]

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