BACKSCATTERING OF OPTICAL RADIATION IN THE TURBULENT MEDIUM WITH DISCRETE DISSEMINATIONS

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The problem of optical radiation scattering in the turbulent medium with discrete disseminations is treated for the cases in which the expansion of the scattering operator in terms of different multiplicity of scattering in the two–component medium comprising the turbulent inhomogeneities of the refractive index and the ensemble of particles can be approximated by the finite number of terms. Derived expressions for the scattered wave field can be used in theoretical studies of the laser radar techniques for sounding the parameters of atmospheric turbulence and for estimating the measurement error due to the random large–scale pulsations of the refractive index on the path "lidar–scattering volume".

Theoretical study of the laser radar techniques for sounding the parameters of atmospheric turbulence calls for solving the problem of backscattering of optical radiation in the medium with the discrete disseminations. This problem was solved in the approximation of single scattering by the system of discrete disseminations in Ref. 1 for soft particles, in Ref. 2 for ideally reflected disks, and in Refs. 3 and 4 for point–size particles. The introduction into the study of the specific types of scatterers substantially simplifies the mathematical reasoning. For example, in the case of optical wave scattered by ideally reflecting disks this problem reduces to the problem of optical radiation reflected from a surface placed in the turbulent medium.[5] At the same time the results of calculations of the statistical characteristics of scattered light obtained for the specific types of scatterers have a limited range of application. Sequential solution of the problem of optical radiation backscattered in the turbulent atmosphere with the discrete disseminations can be obtained if we start from the theory of multiple–wave scattering.[6] In accordance with this theory, the problem is reduced to writing the scattering operator as an expansion in terms of different multiplicity scattering and to the subsequent analysis of terms of this series. In this paper the problem is studied for three cases, in which the expansion of the scattering operator can be approximated by the finite number of terms. The expansion is performed in terms of different multiplicity of scattering in the two–component medium comprising the turbulent inhomogeneities and the system of particles. The first case corresponds to backscattering of a monochromatic wave in the turbulent media with the discrete disseminations localized in the volume of finite dimensions. The second and third cases, in which we can take into account only the finite number of terms in the series, are the problems of single scattering of a monochromatic wave and of pulsed radiation by the system of particles localized in the turbulent atmosphere. Note that the problem of backscattering of the pulsed optical radiation is of special interest for an analysis of the performance of the laser detection lidar when the representation of the scattered radiation field modulated in time must be well known for writing the equations of lasing of a laser with an external reflector.[7]

BACKSCATTERING OF OPTICAL WAVE IN THE TURBULENT MEDIUM WITH DISCRETE DISSEMINATIONS LOCALIZED IN THE VOLUME OF FINITE DIMENSIONS

In the quasistatic approximation and in the absence of depolarization of an optical wave in the scattering medium, the field amplitude depending on time harmonically

$$E(\mathbf{r},\ t) = u(\mathbf{r},\ \omega) \exp(-i\omega t)$$

is the solution of the wave equation[8]

$$\{\Delta + k^2[1 + \varepsilon(\mathbf{r})]\}\, u(\mathbf{r},\ \omega) = 0\ , \qquad (1)$$

where Δ is the Laplacian operator, k is the wave number, $\varepsilon(\mathbf{r}) = \varepsilon_t(\mathbf{r}) + \varepsilon_p(\mathbf{r})$ are the total dielectric constant fluctuations in the medium, $\varepsilon(\mathbf{r})$ are the turbulent dielectric constant fluctuations, and $\varepsilon(\mathbf{r})$ are the dielectric constant fluctuations caused by the disseminations.

Let us represent the solution of Eq. (1) in terms of the T–operator (scattering operator) in the form[6]

$$u = (I + G_0 T)\, u_i\ , \qquad (2)$$

where I is the unit operator and G_0 is the Green's function for the free space.

The first term in the right side of Eq. (2) describes the propagation of the incident optical radiation u_i. The second term is the scattered wave according to the definition

$$u_s = G_0 T\, u_i\ .$$

The field of the incident optical radiation obeys the following equation:

$$\{\Delta + k^2\}\, u_i(\mathbf{r},\ \omega) = 0\ . \qquad (3)$$

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In laser sounding of the atmospheric parameters, we specify the field in the plane of the radiating aperture $u_0(\mathbf{r}, \omega)$. Finding the solution of Eq. (3) with this boundary condition is the Dirichlet problem. When examining the second term in Eq. (2), we introduce the scattering potentials in analogy with the quantum mechanics[6,9] in the following way:

$V(\mathbf{r}) = -k^2\varepsilon(\mathbf{r})$,
$V_t(\mathbf{r}) = -k^2\varepsilon_t(\mathbf{r})$,
$V_p(\mathbf{r}) = -k^2\varepsilon_p(\mathbf{r})$.

Let us write the equations of multiple scattering for the medium, comprising the turbulent atmosphere and the discrete disseminations, in the form

$$T = V + VG_0T , \quad (4)$$

$$T_t = V_t + V_tG_0T_t , \quad (5)$$

$$T_p = V_p + V_pG_0T_p , \quad (6)$$

where T_t and T_p are the operators describing optical wave scattering by the turbulent inhomogeneities and by the ensemble of particles.

The expansion of the T–operator in terms of different multiplicity scattering by the potentials V_t and V_p is given in the form:

$$T = T_t + T_p + T_tG_0T_p + T_pG_0T_t + $$
$$+ T_tG_0T_pG_0T_t + T_pG_0T_t\, G_0T_p +\dots . \quad (7)$$

The first term in the expansion of the T–operator describes scattering of the incident optical radiation by the turbulent inhomogeneities of the medium and the second term describes scattering by the ensemble of particles. These two terms are classified as single scattering and are plotted in Figs. 1*a* and *b*. The circles denote the scattering potentials in Fig. 1, and the arrows indicate the direction of propagation of the optical wave before and after scattering by the given potential. The rest terms in Eq. (7) describe double, triple and so on scattering of the incident wave by the potentials V_t and V_p Fig. 1*c*, *d*, *e*, and *f* show the double and triple scattering corresponding to the next four terms in series (7). For example, triple scattering shown in Fig. 1*e* describes the following sequence of scattering of incident optical radiation by the scattering potential: turbulent inhomogeneities of the refractive index – ensemble of particles – turbulent inhomogeneities of the refractive index.

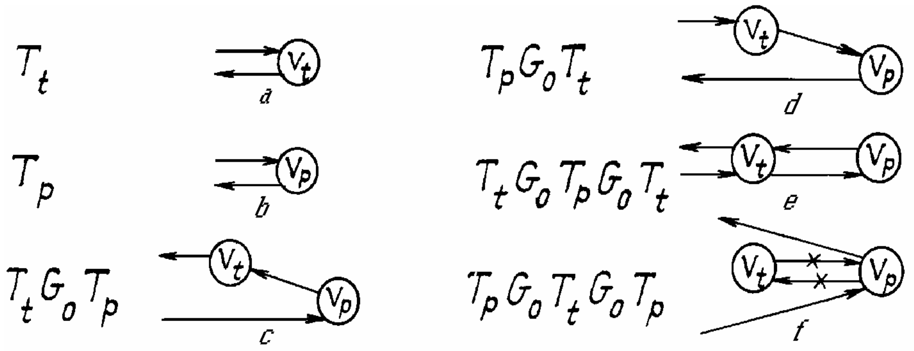

FIG. 1.

The terms of series (7) are of different order of smallness depending on the position of the scatterers in space and on the pattern of optical radiation scattering in the media. The three cases are identified in which the finite number of terms of series (7) approximates the expansion of the T–operator in further study. In this part of the paper the first case is examined describing scattering by the ensemble of particles localized in the volume whose transverse dimensions are much less than the path length. The other two cases are considered in the third and fourth parts of the paper.

The physical phenomenon leading to different orders of terms in series (7) is as follows. As is well khown[5,8] the turbulent atmosphere is the medium with large–scale inhomogeneities whose characteristic dimensions are large in comparison with the wavelength. The scattered waves propagate in the same direction as the incident wave in this medium. Scattering at large angles including backscattering can be neglected completely even if the optical radiation propagates through the column of the Earth's atmosphere. Therefore, we must keep in expansion (7) for the scattering operator only the terms without backscattering by the turbulent inhomogeneities of the medium.

The first term in Eq. (7) describes backscattering of incident radiation by the turbulent inhomogeneities of the medium. Therefore, it can be neglected. The second term describes scattering of incident radiation by the ensemble of particles. When considering this term, the natural assumption that the backscattered signal is determined by the atmospheric particles, should be taken into account. Therefore, the second term must be taken into account in the subsequent analysis of series (7). The analogous phenomenon arises when considering the third, fourth, and fifth terms in Eq. (7).

Let us consider the term $T_pG_0T_tG_0Tp$. It corresponds to the following sequence of incident radiation scattering: ensemble of particles – turbulent inhomogeneities of the refractive index – ensemble of particles. It is clear from Fig. 1*f* that in the case in which the particles are localized in the volume whose transverse dimensions are much less than the path length, the optical wave is scattered in a direction opposite to that of the incident wave after interaction with the potential V_p. These two waves are marked by crosses in Fig. 1*f*. Note that the optical wave associated with backscattering by turbulent inhomogeneities of the refractive index can disappear in the term $T_pG_0T_tG_0Tp$ if the disseminations will not be localized. An analysis of terms whose multiplicity of scattering is higher than that of $T_pG_0T_tG_0Tp$ shows that we can neglect these terms in the subsequent analysis by the same reasons.

Thus, the main contribution to the backscattered radiation in the case of the ensemble of particles localized in the volume of finite dimensions comes from only the four terms of the series: T_p, $T_tG_0T_p$, $T_pG_0T_t$, and $T_tG_0T_pG_0T_t$. These terms are plotted in Figs. 1*b*, *c*, *d*, and *e*.

Grouping the above–mentioned terms and substituting the obtained expression into Eq. (2) we derive the integral representation for the wave field scattered in the turbulent medium with discrete disseminations localized in the volume of finite dimension

$$u_s(\mathbf{r}, \omega) = \int G_t(\mathbf{r}, \mathbf{r}_1; \omega)\, T_p(\mathbf{r}_1, \mathbf{r}_2; \omega)\, u_{it}(\mathbf{r}_2; \omega)\, \mathrm{d}\mathbf{r}_1 \mathrm{d}\mathbf{r}_2 , \quad (8)$$

where $G_t(\mathbf{r}, \mathbf{r}_1; \omega)$ is the Green's function of the turbulent medium, $T_p(\mathbf{r}_1, \mathbf{r}_2; \omega)$ is the kernel of the T–operator.

In calculating the statistical characteristics of the scattered wave field, it is more convenient to start from the following equation for the Green's function:

$$G_t = G_0 + G_0V_tG_0 ,$$

which takes into account multiple backscattering by the turbulent inhomogeneities and to transform over to its approximate form in the process of calculation. The field $u_{it}(\mathbf{r}; \omega)$ is the solution of the problem of scattering of the optical radiation $u_i(\mathbf{r}; \omega)$ in the turbulent medium, and it obeys the equation

$$\{\Delta + k^2[1 + \varepsilon_t(\mathbf{r})]\}\, u_{it}(\mathbf{r}; \omega) = 0 . \quad (9)$$

It is well known[5,8] that scattering problem (9) may be reduced to the problem of optical wave propagation through the turbulent medium if we ignore the wave backscattered by $\varepsilon_t(\mathbf{r})$. In this approximation, the solution of Eq. (9) may be written in the form of the diffractional Huygens–Kirchhoff integral

$$u_{it}(\mathbf{r}; \omega) = 2ik \int G_t(\mathbf{r}, \mathbf{r}_1; \omega)\, u_0(\mathbf{r}_1; \omega)\, \mathrm{d}S_1 , \quad (10)$$

where $\mathrm{d}S_1$ is the elementary area of the radiating aperture. When deriving Eq. (8), it was assumed that only backscattering by the turbulent inhomogeneities was completely absent. Hence, representation (8) of the scattered wave field takes into account multiple forward scattering of optical radiation by the turbulent inhomogeneities of the medium and multiple scattering by the ensemble of particles.

SINGLE SCATTERING OF OPTICAL WAVE BY THE SYSTEM OF PARTICLES LOCALIZED IN THE TURBULENT MEDIUM

Let us proceed to an analysis of the terms in series (7) for the case of single scattering of an optical wave by the ensemble of particles localized in a volume of arbitrary dimensions. Let us represent the scattering potential of the ensemble of particles as a sum of the scattering potentials of the individual particles,[10] i.e.,

$$V_p(\mathbf{r}) = \sum_{m=1}^{N} V_m(\mathbf{r}) ,$$

where N is the number of particles. The form of the scattering operator of the individual particle is assumed to be known. This operator obeys the equation

$$T_m = V_m + V_mG_0T_m .$$

The expansion of the T–operator in terms of different multiplicity of scattering by the ensemble of particles can be written in the form

$$T_p = \sum_m T_m + \sum_m \sum_{m\neq n} T_mG_0T_n + \sum_m \sum_{n\neq m} \sum_{k\neq n} T_mG_0T_nG_0T_k + \dots . \quad (11)$$

In the single–scattering approximation only the first sum in series (11) provides a good approximation for the expansion of the T–operator. In this case scattering by an individual particle occurs independently on the other disseminations. The scattered wave field is the sum of the wave fields scattered by the individual particles localized in the turbulent medium. So far as the size of the atmospheric particles is much less than the path length, the pattern of the optical wave scattering by the particle placed in the turbulent medium is the same as that in the turbulent atmosphere with the discrete disseminations localized in the volume of finite dimensions. It follows from formula (8) after the substitution $T_p(\mathbf{r}_1, \mathbf{r}_2; \omega) \to T_m(\mathbf{r}_1, \mathbf{r}_2; \omega)$ that the wave field scattered by the particle placed in the turbulent atmosphere has the form

$$u_{sm}(\mathbf{r}, \omega) = \int G_t(\mathbf{r}, \mathbf{r}_1; \omega)\, T_m(\mathbf{r}_1, \mathbf{r}_2; \omega)\, u_{it}(\mathbf{r}_2; \omega)\, \mathrm{d}\mathbf{r}_1 \mathrm{d}\mathbf{r}_2 . \quad (12)$$

In what follows first, we will assume that particles are localized in the far–diffraction zone. In this case the amplitude of scattering by the individual particle has been already formed and is equals to

$$A_m = \frac{1}{2ik} \int \exp(-ik\mathbf{q}\mathbf{r}'_1) T_m(\mathbf{r}_m + \mathbf{r}'_1, \mathbf{r}_m + \mathbf{r}'_2; \mathrm{w}) \exp(ik\mathbf{p}\mathbf{r}'_2)\, \mathrm{d}\mathbf{r}'_1 \mathrm{d}\mathbf{r}'_2 ,$$

$$\mathbf{q} = \frac{\mathbf{r} - \mathbf{r}_m}{|\mathbf{r} - \mathbf{r}_m|} , \quad \mathbf{p} = \frac{\mathbf{r}_3 - \mathbf{r}_m}{|\mathbf{r}_3 - \mathbf{r}_m|} ,$$

where $\mathbf{r}_m$ is the coordinate of the particle centre. Second, A_m is assumed to be a smooth function of its arguments $\mathbf{q}$ and $\mathbf{p}$. Third, the characteristic size of particles are much less than the spatial scales of variation of the Green's function $G_t(\mathbf{r}_1, \mathbf{r}_2; \omega)$ which are determined by the turbulent medium. The first two assumptions are conventional when the problem of single scattering of optical wave by the discrete disseminations[8,11] is considered. The third assumption is due to the scattering pattern of optical wave transmitted through the layer of the turbulent medium. The essence of this assumption can be understood starting from the following reasoning. The coherence radius, the intensity fluctuation correlation length, and other quantities determined by the higher–order correlation functions for the spherical wave field in the turbulent medium are the spatial scales of variation of the

Green's function. Therefore, the statement that the characteristic size of particles is much less than the coherence radius means that the particle is in the coherent field. The analogous statement for the intensity fluctuation correlation length means that the particle is within the single dark or bright spot of the speckle structure caused by the turbulent pulsations of the refractive index. Because of the substantial difference between the characteristic size of particles and these scales, the third assumption does not impose stringent restrictions on its use. When the three given assumptions are satisfied, Eq. (12) may be written in the form

$$u_{sm}(\mathbf{r}, \omega) = A_m\, G_t(\mathbf{r}, \mathbf{r}_m; \omega)\, u_{it}(\mathbf{r}_m; \omega) \ . \qquad (13)$$

The relation for the field of the wave singly scattered by the particles in the turbulent medium is a sum of Eq. (13) over the index m

$$u_s(\mathbf{r}, \omega) = 2ik \sum_{m=1}^{N} A_m\, G_t(\mathbf{r}, \mathbf{r}_m; \omega)\, u_{it}(\mathbf{r}_m; \omega) \ . \qquad (14)$$

Formula (14) can be considered as the generalization of the well–known representation for the field of the wave singly scattered by the particles[8,14] to the medium comprising the turbulent atmosphere and the particles. Note that multiple–forward scattering by the turbulent inhomogeneities of the refractive index are completely taken into account in Eq. (14).

SINGLE SCATTERING OF PULSED RADIATION BY THE SYSTEM OF PARTICLES LOCALIZED IN THE TURBULENT ATMOSPHERE

When solving the problem of single scattering of pulsed radiation by the system of particles localized in the turbulent medium, we may use the results of the preceding part of the paper. For this purpose let us represent the relation between the scattered wave field, depending on time harmonically, and the field of the scattered pulsed radiation in the form

$$E_s^p(\mathbf{r}, t) = \frac{1}{2\pi} \int u_s(\mathbf{r}, \omega)\, e^{-i\omega t}\, d\omega \ , \qquad (15)$$

where t is time. Let us separate the envelope $u_s^p(\mathbf{r}, t)$ and the carrier frequency ω_0 of the scattered pulsed radiation field

$$E_s^p(\mathbf{r}, t) = u_s^p(\mathbf{r}, t)\, e^{-i\omega_0 t} \ .$$

Let us represent the initial field distribution $u_0(\mathbf{r}, \omega)$ as a product of two functions

$$u_0(\mathbf{r}, \omega) = f(\omega - \omega_0)\, u_0(\mathbf{r}) \ ,$$

where $u_0(\mathbf{r})$ is the function describing the spatial distribution of the field of the source and the function $f(\omega - \omega_0)$ is given by the formula

$$f(\Omega) = \frac{1}{2\pi} \int \beta(t)\, e^{i\Omega t}\, dt \ .$$

Here the function $\beta(t)$ describes the temporal amplitude–phase modulation of incident radiation in the plane of the source. The pulse duration is assumed to be sufficiently great (several picosecond and more), on the one hand, and on the other, it is less than the time of variation of the state of the atmosphere ($10^{-1} - 10^{-3}$ s). Under these conditions, the temporal deformation of the pulse propagating through the turbulent atmosphere can be ignored. Note that under these assumptions the problem of refraction of pulsed radiation from the surface placed in the turbulent medium was solved in Ref. 12. Taking into account the given limitations on the pulse duration, substitution of formula (14) into relation (15), and necessary transformations lead to the following relation for the envelope of the pulsed radiation singly scattered by the system of particles localized in the turbulent atmosphere:

$$u_s^p(\mathbf{r}, t) = 2ik_0 \sum_{m=1}^{N} A_m\, G_t\big(\mathbf{r}, \mathbf{r}_m; \omega_0\big) = u_{it}^p\left(\mathbf{r}_m,\ t - \frac{1}{c}\,|\mathbf{r} - \mathbf{r}_m|\right) , \qquad (16)$$

where

$$k_0 = \omega_0/c \ ,$$

$$u_{it}^p(\mathbf{r}_m, t) = 2ik_0 \int \left(t - \frac{1}{c}\,|\mathbf{r}_m - \mathbf{r}_1| \right) G_t(\mathbf{r}_m, \mathbf{r}_1; \omega_0)\, u_0(\mathbf{r}_1)\, dS_1 . \qquad (17)$$

The random nature of the output signal of the lidar systems is determined by the statistical properties of the scattered radiation. Formulas (8), (14), and (16) are the basis for the determination of the statistical parameters of the scattered radiation in the turbulent medium with the discrete disseminations. These parameter can be used to solve the following problems: to study theoretically the laser radar techniques for sounding of the parameters of atmospheric turbulence and to evaluate the error due to the randomly large–scale pulsations of the refractive index on the path "lidar–scattering volume" from the data of measurements. The discussions of the above–mentioned problems will be the subject of future publications.

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