About convergence of partial wave amplitudes of scattering characteristics of optical and microwave discharges

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The convergence of partial wave amplitudes of Mie's series is considered for expanded plasma spheres of microwave and optical discharge. Asymptotic approximations for simultaneous growth of the mode index and diffraction parameter, as well as the table of criteria for a choice of optimal calculation algorithm are presented.

Propagation of high-power super-high frequency (SHF) and laser radiation in an aerosol medium is accompanied by an emergence of plasma microspheres (plasmoids) of the optical breakdown (OB) at the cost of photoionozation of atoms and molecules.[1] Optical properties of plasmoids are defined by their form, spatial distribution of the complex index of diffraction, and the absorption coefficient at a wavelength of the laser or SHW pumping, initiating the breakdown plasmoid formation. The expansion of the breakdown plasma around an aerosol particle is due to generation of the light-detonation waves at the cost of high concentration of electromagnetic fields in the aerosol particle's volume and surface layer.[2]

The formation of the detonation wave in the OB plasma around some seed particle has several time-sequential steps, at which spherical asymmetry of the electron distribution concentration and the homogeneity of the OB plasma diffractive index are observed.[3,4] Moreover, a possibility of such modes of the plasmoid development was earlier shown,[1,3,4] at which optical properties of the seed particle simulate the homogeneous plasma sphere with a complex index of refraction $|m| < 1$.

By the numerical simulating methods, for absorbing plasma spheres in the laser radiation field, the scattering phase functions and distributions of electromagnetic field intensity inside a plasmoid were obtained[1] at a pumping wavelength of 1.06 μm, diffraction parameters of up to 22, and the complex diffractive index $m = 0.7 - i0.47$.

When studying the physical nature of sonoluminescence, the mathematical model of scattering a plane electromagnetic wave by oscillating air bubbles and a plasmoid, formed under impact of periodic shot wave in water to determine the time dependence of the bubble radius and the diffractive index is used,[5,6] where optical properties of the bubbles, such as the intensity and polarization characteristics for radius sizes in a range 15–30 μm at a pumping wavelength 0.55 μm and refractive index between 0.3 and 0.9 were considered without accounting for absorption.

Electromagnetic wave distribution inside and outside a sphere is known from the Mie theory.[7] Difficulties in computing light-scattering characteristics at large diffraction parameters ($\rho \geq 60$, $\rho = 2\pi a/\lambda$, where λ is the incident radiation wavelength, a is the plasmoid radius) and complex indices of refraction have been marked in Refs. 7 and 8. The difficulties concern of the fact that the correct computation requires the accounting for a large number of partial wave amplitudes, composing the series in the Mie theory. Therefore, it is necessary to consider the convergence of partial wave amplitudes in a given range, since basic computations of the extinction and the efficient absorption coefficient are realized in the form of a sequence of Riccati–Bessel functions of first (FRB1) and third (FRB3) kinds with a complex argument calculated by recurrent algorithms.[9,10]

The recurrent formulae were first proposed for dealing with the partial wave convergence in Ref. 11, where Fock's type asymptotics were used for building the control grid as well. With the use of the asymptotic representations, the grid of values was build for FRB1 and FRB3 for an intermediate range of the diffraction parameter in the region, where approximations of geometric optics are not yet work Rayleigh and the approximation is no longer applicable. In this connection, it seems to be reasonable to obtain estimates for partial wave amplitudes for plasmoid and to show correctly their convergence in case of extending plasmoid and oscillating air bubbles in a liquid with complex index of refraction within the intermediate range of the diffraction parameter.

Using the asymptotic formulae,[12] it is possible to show the convergence of partial wave amplitudes for a large extending plasmoid. In the convenient for calculation form, we have[13]:

$$a_n = \frac{\Psi_n(\rho)}{\zeta_n(\rho)}\left[\frac{D_n(m\rho) - mD_n(\rho)}{D_n(m\rho) - mC_n(\rho)}\right]; \qquad (1)$$

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$$b_n = \frac{\Psi_n(\rho)}{\zeta_n(\rho)}\left[\frac{mD_n(m\rho) - D_n(\rho)}{mD_n(m\rho) - C_n(\rho)}\right], \quad (2)$$

where $D_n(\rho)$ and $C_n(\rho)$ are logarithmic derivatives of FRB1 and FRB3, respectively, of the form[14]:

$$D_n(\rho) = \Psi_n'(\rho)/\Psi_n(\rho); \quad (3)$$

$$C_n(\rho) = \zeta_n'(\rho)/\zeta_n(\rho). \quad (4)$$

In their turn, FRB1 and FRB3 are[8]

$$\Psi_n(z) = \sqrt{\frac{\pi z}{2}} J_{n+1/2}(z); \quad (5)$$

$$\zeta_n(z) = \sqrt{\frac{\pi z}{2}} H^{(2)}_{n+1/2}(z). \quad (6)$$

When calculating the FRB1 and FRB3 derivatives through the Bessel (5) and Hunkel (6) functions of second type, as well as their derivatives of the form[12]:

$$J_{n+1/2}(z) = \left(\frac{z}{2}\right)^{n+1/2} \frac{1}{\Gamma(n+3/2)} \times$$
$$\times \exp[-z^2/4(n+3/2)][1+O(1/n)]; \quad (7)$$

$$H^{(2)}_{n+1/2}(z) = \frac{i}{\pi}\left(\frac{z}{2}\right)^{-(n+1/2)} \Gamma(n+1/2) \times$$
$$\times \exp[z^2/4(n+1/2)][1+O(1/n)], \quad (8)$$

where $\Gamma(n + 1/2)$ is gamma-function, we obtain for Eqs. (3) and (4)

$$D_n(z) = (n+1)/z; \quad (9)$$

$$C_n(z) = -n/z. \quad (10)$$

Using Eqs. (7) and (8) for calculation of FRB1 and FRB3 ratio, write for partial amplitudes

$$a_n = \frac{\pi}{i}\left(\frac{\rho}{2}\right)^{2n+1} \frac{1}{\Gamma(n+3/2)\Gamma(n+1/2)} \times$$
$$\times \exp\left(-\frac{\rho^2}{4(n+3/2)} - \frac{\rho^2}{4(n+1/2)}\right) \frac{(n+1)(1-m^2)}{n+1+m^2 n}. \quad (11)$$

The numerator in Eq. 2 at the cost of asymptotic expressions for logarithmic derivatives of FRB1 and FRB2 becomes equal to zero, consequently, $b_n = 0$ and the limit of the partial amplitude a_n at $n \to \infty$ remains to be considered. Note that equation (7) and (8) are found under the condition

$$|\rho| \le c(n+3/2)^{1/2},$$

$n \to \infty$, $\rho \to \infty$, $c < 1$ is a constant.[12]

Considering the limit of a_n together with the above condition, we obtain that the exponent is bounded at growing n, therefore, the tending to zero takes place at the cost of growing gamma-function. At a proper choice of the majorant, it is easily seen that the sum of squares of modules of the partial wave amplitudes is convergent as well.

This convergence for large plasmoids takes place at the complex index of refraction $|m| \le 1$. Note that at $|m| > 1$ equations (7) and (8) do not work and can be changed by the corresponding Fock or Debay approximations.[11,17] Thus, the application of the widely known calculation algorithms[9,10] in a given range of parameters is also correct. In test estimates of the amplitudes, when calculative instability can appear at the cost of accumulation of errors, equations (7) and (8) are applicable along with Fock's type asymptotics.[11]

The recommended methods for calculation of FRB1 and FRB3 are presented in Table. They can be used in composing algorithms for calculating light scattering characteristics of different objects.

It should be noted that the asymptotic methods can be widely applicable to bodies of any regular shape, because such light scattering characteristics as extinction, scattering, and absorption coefficients are series of proper functions for problems of the electromagnetic wave diffraction, which mostly are reduced to solving the Bessel equation. In this case the partial wave amplitudes, obtainable from the limiting conditions on the rotation body surface, should be known.

Table. Estimate parameters of scattering spheres and recommended methods for calculating FBR1 and FBR3 in order to build optimal algorithms for computing light scattering characteristics[16]

Scattering object	Incident radiation wavelength λ	Radius a of the sphere, μm	Refractive index n of the sphere matter	Diffraction parameter $\rho=2\pi a/\lambda$	Inhomogeneity parameter of the field $m\rho$	Recommended method for calculation of FBR1 and FBR3
Spherical particle of $KTaO_3$ ($SrTiO_3$)	3 cm	500	70.7 (158.1) At T = 4.2 K in vapor of liquid helium	0.1	7.01 (15.8)	Asymptotical (by Debay asymptotics)[17]
Water drop	1.06 μm	10	$1.319-i4\cdot 10^{-6}$	62.8	82.8	Miller–Olver (recurrent)[14,15]
OB plasmoid	10.6 μm	200	$0.5-i5\cdot 10^{-2}$	125.6	62.7	Asymptotical[12]
Air bubble in water	1.06 μm	15	$0.7-i\cdot 10^{-6}$	88.7	62.2	Miller–Olver (recurrent)[14,15]
UHF breakdown plasmoid	3 cm	200	$0.3-i\cdot 10^{-2}$	0.04	0.013	Asymptotical[12,17]

Application of the constructed asymptotics and calculation algorithms is of interest for development of methods of screening laser and high-frequency radiation by plasmoids.

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