ON THE STRUCTURE OF LIGHT BEAMS IN ACTIVE MEDIA AND RESONATORS

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The problem of propagation of light beams through a medium with a three–dimensional parabolic profile of the amplification coefficient has been solved. The degree of spatial coherence of the amplified radiation has been found. The spatial structure of the radiation in open resonators filled with an active medium, which has such a three–dimensional amplification coefficient has been described.

Studies of the formation of the spatial structure of light beams in amplifying media without feedback have quite a long history. Though the main features of this process were clarified in Refs. 1–3, the problem has not yet been completely solved. Thus, for example, an analysis of light beam amplification in a medium with a three–dimensional parabolic profile of the amplification coefficient was presented in Ref. 1, but the result for arbitrary shape of the signal was obtained by summing over the components of the Fourier transform, i.e., the problem was solved for the case of independent plane waves. The results of numerical calculations were presented in Ref. 2, and in Ref. 3 calculations for two–dimensional media were made.

There are also certain doubts about the universality of one of the main conclusions made in Refs. 1–3 as well as more recently in Ref. 5, which states that the radius of an amplified beam and its correction length tend toward identical values. In the present paper it is shown that diffraction of the Gaussian pump beam outside the waveguide leads to a quite different result.

In addition, it is of great interest to analyze the spatial mode structure of the beam in an amplifying channel with feedback, i.e., in the resonator of superluminescence lasers and amplifiers based on Raman scattering (RS) as well as in standard lasers with parabolic profile of the amplification coefficient of the active medium.[6]

The exact theory describing the formation of spatially coherent light beams in active media with parabolic profile of the amplification coefficient, including media with feedback, which is developed in the present paper, is based on the use of the method of integration over trajectories.[4] Some of the results presented in this paper were previously published elsewhere,[7,8] but limitations on the lenght of these papers made it impossible to give a comprehensive treatment of the problem.

1. AMPLIFICATION IN AN ACTIVE CHANNEL WITHOUT FEEDBACK

In a prescribed pumping field the amplification of the Stokes wave due to Raman scattering (RS) obeys the equation

$$\left(\frac{\partial}{\partial z} + \frac{i}{2k}\Delta_\perp\right)A(\mathbf{r}, z) = gI_p(\mathbf{r}, z)A(\mathbf{r}, z)\ , \qquad (1)$$

where $A(\mathbf{r}, z)$ is the complex amplitude of the Stokes wave, k is its wave number, $I_p(\mathbf{r}, z)$ is the intensity of the pumping radiation, and the parameter g is determined by the optical nonlinearity of the medium. The wave is assumed to propagate along the z axis. The vector $\mathbf{r}$ is perpendicular to the z axis and $\Delta_\perp$ is the transverse Laplacian.

Let us consider the process of amplification in the field of a focused Gaussian pumping beam. In the case of collinear propagation we have

$$I_p(\mathbf{r}, z) = I_0[a_0/a(z)]^2\exp[-2r^2/a^2(z)]\ , \qquad (2)$$

where

$$a^2(z) = V^2_{\text{col}}(z)a_0^2,\ \ V^2_{\text{col}}(z) = (1\ \ z/f)^2 + (z/l_{dp})^2, \qquad (3)$$

a_0 is the radius of the pumping beam upon entrance into the nonlinear medium (z = 0), $l_{dp} = k_p\,a_0^2/2$ is the diffraction length of the pumping wave, k_p is the wave number of the pumping wave, and f is the focal length of the lens. In the case of counter interaction of the pumping wave an the Stokes wave, the pumping wave is assumed to be prescribed at the exit from the nonlinear medium at $z = l$, where l is the depth of the nonlinear medium, and instead of Eq. (3) we have

$$a^2(z) = V^2_{\text{count}}(z)a_l^2,\ \ V^2_{\text{count}} = \left(1 - \frac{1-z}{f}\right)^2 + \left(\frac{1-z}{l_{dp}}\right)^2. \qquad (4)$$

The subscripts col and count stand for the cases of collinear and counter amplification.

The solution of Eq. (1) can be written in terms of the continuum integral

$$A(\mathbf{r}, l) = \int_{-\infty}^{\infty} A_0(\rho)\ \times$$

$$\times \int\exp\left\{-i\,\frac{k}{2}\int_0^l L_1[\mathbf{r}(\zeta),\ \dot{\mathbf{r}}(\zeta)]df\right\}D^2\mathbf{r}(\zeta)\ d^2\rho\ , \qquad (5)$$

$$L_1(\mathbf{r},\ \dot{\mathbf{r}}) = [\dot{\mathbf{r}}(\zeta)]^2 + \frac{2g}{k}\,I_p[\mathbf{r}(\zeta),\ \zeta]\ \ \dot{\mathbf{r}}(\zeta) = \frac{d\mathbf{r}}{d\zeta}. \qquad (6)$$

The differential $D^2\mathbf{r}(\zeta) = D\mathbf{x}(\zeta)\,D\mathbf{y}(\zeta)$ indicates integration over the trajectories that intersect the points with the coordinates $\mathbf{x}(\zeta)$ and $\mathbf{y}(\zeta)$, at the end points $\mathbf{r}(\zeta = 0) = \rho$ and $\mathbf{r}(\zeta = l) = \mathbf{r}$; and the amplitude $A_0(\mathbf{r}) = A(\mathbf{r}, z = 0)$.

We succeeded in our analytical calculations with the help of the paraxial approach in which the pumping beam profile (2) is approximated by a parabolic profile:

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$$I_p(\mathbf{r}, z) \simeq [\alpha_0/\alpha(z)]^2 I_0[1 - 2r^2/\alpha^2(z)] \ . \quad (7)$$

In this case for the collinear interaction between the waves we arrive at the result

$$-i\frac{k}{2}\int_0^l L_1 d\zeta = gI_0\int_0^l V_{\rm col}^{-2}(\zeta)d\zeta - i\frac{k}{2}\int_0^l L[\mathbf{r}(\zeta), \dot{\mathbf{r}}(\zeta)]d\zeta, \quad (8)$$

where

$L[\mathbf{r}(\zeta), \dot{\mathbf{r}}(\zeta)] = [\dot{\mathbf{r}}(\zeta)]^2 - i\mu l_{dp}^{-2} V_{\rm col}^{-2} r^2$, $l = 2gI_0 l_{dp}^2/l_d$, and $l_d = ka_0^2/2$ is the diffraction length.

The integral factor in the first term in Eq. (8) is equal to

$$\int_0^l V_{\rm col}^{-2}(\zeta)d\zeta = l_{dp}\arctan(l/l_{dp})/\left(1 - l/f\right). \quad (9)$$

To calculate the second integral, we transform to the new coordinate $t = z/l_{dp}$ and introduce the quantities $\alpha = l_{dp}/f$ and $\tau = l/l_{dp}$. The greatest contribution to the integral (5) comes from the trajectories that satusfy the Euler equation

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{\mathbf{r}}} - \frac{\partial L}{\partial \mathbf{r}} = 0. \quad (10)$$

Operating on Eq. (8), we obtain the equation

$$[(1 - \alpha t)^2 + t^2]^2\ddot{\mathbf{r}} + i\mu\mathbf{r} = 0, \quad (11)$$

which must be solved with boundary conditions

$$\mathbf{r}(t = 0) = \mathbf{r}, \ \ \mathbf{r}(t = \tau) = \mathbf{r}. \quad (12)$$

For optimal trajectories we have

$$\mathbf{r}_0(t)=[f(t)/f(\tau)\sin g(\tau)]\{\mathbf{r}\sin g(t)-\rho f(\tau)\sin[g(t)-g(\tau)]\},(13)$$

where

$f(t) = [(1 - \alpha t)^2 + t^2]^{1/2}$, $g(t) = M\arctan[t/(1 - \alpha t)]$,

$M = M_0\exp(i\varphi/2)$, $M_0 (1 + \mu^2)^{1/4}$, $\varphi = \arctan\mu$.

The contribution of the arbitrary trajectory to the integral can be represented as $\mathbf{r}(t) = \mathbf{r}_0(t) + \rho(t)$, where $\rho(0) = \rho(\tau) = 0$. Then, again operating on Eq. (8) we obtain

$$\int_0^t L dt = \dot{\mathbf{r}}(\tau)\mathbf{r} - \dot{\rho}(0)\rho + \int_0^\tau [\dot{\rho}^2 - i\, V_{\rm col}^{-4}(t)\mu\rho^2]\, dt. \quad (14)$$

Thus Eq. (5) takes the form

$$A(\mathbf{r}, \tau) = C(\tau)\int_{-\infty}^{\infty} A_0(\rho)\exp\left\{gI_0 l_{dp}\arctan\left[\frac{\tau}{1 - \alpha\tau}\right] - \right.$$
$$\left. - i\frac{k}{2l_{dp}}\left([\dot{\mathbf{r}}(\tau)\mathbf{r} - \dot{\rho}(0)\rho]\right)\right\}d^2\rho; \quad (15)$$

where

$$C(\tau) = \int\exp\left\{- i\frac{k}{2l_{dp}}\int_0^\tau [\dot{\rho}^2 - iV_{\rm col}^{-4}(t)\mu\rho^2]dt\right\}D^2\rho(t).$$

The value $C(\tau)$ can be calculated, but it is unessential for further analysis. The second term of Eq. (15) can be written in the form

$$\dot{\mathbf{r}}\mathbf{r} - \dot{\rho}\rho = \mathbf{r}^2\frac{M\cot{\rm an}[g(\tau)] + \alpha^2\tau + \tau - \alpha}{(1 - \alpha\tau)^2 + \tau^2} + \rho^2\{M\cot{\rm an}[g(\tau)] +$$
$$+ \alpha[(1-\alpha\tau)^2+\tau^2]^{-1/2} - 2\mathbf{r}\rho M\sin^{-1}[g(\tau)][(1-\alpha\tau)^2+\tau^2]^{-1/2}\}.(16)$$

In the particular case in which the pumping beam is focused into the center of the nonlinear medium, Eq. (16) becomes symmetric with respect to ρ and $\mathbf{r}$ (in this case $\tau = 2\alpha/(1 + \alpha^2)$) and assumes the following form:

$$\dot{\mathbf{r}}\mathbf{r} - \dot{\rho}\rho = (\mathbf{r}^2 + \rho^2)[M\cot{\rm an}[g(\tau)] + \alpha] - 2\mathbf{r}\rho M/\sin g(\tau). \ (17)$$

In the case of counter interaction between the waves the salient feature of the amplification process is a result of the fact that the pump is assumed to be located at the other end of the amplifying medium (see Eqs. (2) and (4)). Therefore, to find the response of the active channel, it is sufficient to make the substitutions $\rho \to \mathbf{r}$ and $\mathbf{r} \to \rho$ in Eq. (16) and consequently in the exponent of Eq. (15).

The latter circumstance can be used, in particular, for analyzing the optical fields in symmetric resonators as well as in resonators one or several mirrors of which conjugate the phase. Such PC–resonators are very promising since their use provides the possibility of automatic compensation for both static aberrations of the resonator and random phase inhomogeneities.[9,10]

2. AMPLIFICATION OF STRAY WAVES. CORRELATION FUNCTION OF THE FIELD

In what follows we consider the amplification of a spatially incoherent signal whose correlation function has the form

$$\Gamma_0(\rho_1, \rho_2) = \langle A_0(\rho_1)A_0^*(\rho_2)\rangle = G_0\delta(\rho_1 - \rho_2)\ ,$$

where G_0 describes the angular distribution of the spectral power density of the signal.

After some quite cumbersome calculations, in the case of unfocused beams ($f \to \infty$) and collinear interaction we obtain for the entrance spatial correlation function of the wave being amplified the following expression:

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2, l) = C^2(\tau)G_0F(l)\exp\left\{2gI_0 l - i\frac{k}{2l_{dp}}\frac{\tau}{(1+\tau^2)}(r_1^2 - r_2^2) - \right.$$
$$\left. - \frac{r_1^2 + r_2^2}{a_{\rm col}^2(l)} - \frac{(r_1 - r_2)^2}{r_{cl}^2}\right\}, \quad (18)$$

where r_{cl} is the beam correlation length for collinear beams,

$$F(l) = \frac{2l_{dp}}{k\,M_0 H(l)}\,[{\rm ch}^2N_s\sin^2N_c + \cos^2N_c\,{\rm sh}^2N_s]\ ,$$

$$a_{\mathrm{col}}^2(l) = \frac{4l_{dp}}{kM_0}(1+\tau^2)H(l)[(\sin^2 N_c + \mathrm{sh}^2 N_s)\cos\varphi +$$

$$+ \mathrm{sh}^2 N_s - \sin^2 N_c]^{-1}, \quad (19)$$

and

$$r_{cl}^2(l) = 2l_{dp}(1+\tau^2)(kM_0)^{-1}H(l),$$

$$H(l) = \mathrm{sh}2N_s\cos(\varphi/2) - \sin 2N_c\sin(\varphi/2). \quad (20)$$

Here the following notation was used:

$$N_c = M_0\cos(\varphi/2)\arctan\tau, \quad N_s = M_0\sin(\varphi/2)\arctan\tau. \quad (21)$$

First of all let us analyze the ratio of the correlation length r_{cl} to the radius of the beam

$$R = \frac{2r_{cl}^2(l)}{a^2(l)} = (\sin^2 N_c + \mathrm{sh}^2 N_s)\cos\varphi + \mathrm{sh}^2 N_s - \sin^2 N_c. \quad (22)$$

The function $R(r)$ calculated for different values of μ is plotted in Fig. 1.

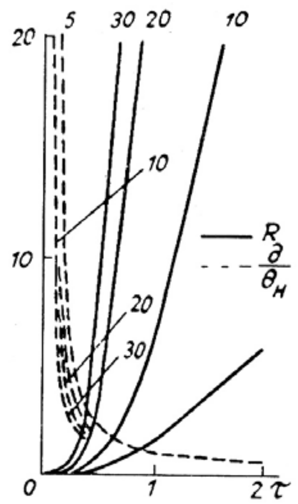

FIG. 1. The doubled square of the ratio of the correlation length of the beam to its radius (solid curves) and the ratio of the angular divergence of the beam to that of the pumping beam (dashed curves) as functions of the normalized propagation depth. Figures above the curves indicate the values of μ *(from 5 to 30).*

For high pumping intensities ($\mu \gg 1$), we obtain

$$R \simeq \mathrm{sh}^2 N_s - \sin^2 N_c. \quad (23)$$

The value of R increases with the argument N_s. At small values of N_c under conditions in which $\tau < 1$

$$R \approx \frac{2}{3}N_s^4 = \frac{2}{3}(gI_0)^2 l^4 l_{dp}^{-2}. \quad (24)$$

It turns out that at small depths $l < l_d < l_{dp}$ the correlation length of the beam is smaller than its radius. This ratio increases with l and at large depths ($l \simeq l_{dp}$) the correlation length of the beam is already greater than its radius. This result is substantially different from the result obtained in Refs. 1–3, where the authors concluded that the correlation length and the beam radius tend toward identical values. According to Eq. (23) at $l \gg l_{dp}$ R starts to saturate.

Let us now consider the behavior of the correlation length and the radius of the beam. At small depths ($\tau \ll 1$) the correlation length is equal to

$$r_{cl} \simeq (8gI_0 l^3/3l_{dp})^{1/2}, \quad (25)$$

i.e., it increases with depth. This equation coincides to within a constant factor with the approximate result obtained in Ref. 2. At depths larger than the diffraction length of the pumping beam ($\tau < 1$) we have

$$r_{cl} \simeq 2^{1/4}(kl_{dp})^{-1/2}\mu^{-1/4} l \exp[\pi^2(gI_0 l_{dp})]^{1/2}. \quad (26)$$

In this case the dependence of the correlation length of the beam on the depth follows the case of free space.

Let us now consider the behavior of the radius of the wave being amplified for $\mu \gg 1$. Then we can write

$$a_{\mathrm{col}}^2(l) = \frac{4l_{dp}(1+\tau^2)}{k\sqrt{2\mu}} \cdot \frac{\mathrm{sh}2N - \sin 2N}{\mathrm{sh}^2 N - \sin^2 N}, \quad (27)$$

where $N = \sqrt{\mu}/2 \arctan\tau$.

If $\tau \ll 1$, then the radius of the beam decreases with depth

$$a_{\mathrm{col}}(l) = 4l_{dp}\sqrt{k\mu l} \quad (28)$$

In the opposite case (at $\tau \gg 1$)

$$a_{col}(l) \simeq 2(kl_{dp})^{-1/2}(2\mu)^{-1/4} l. \quad (29)$$

In the latter case the dependence of a_{col} on l follows the case of free space. At same time, the comparison of Eqs. (26) and (29) shows that the diameters of the equivalent diffraction apertures for the correlation length and beam radius are different.

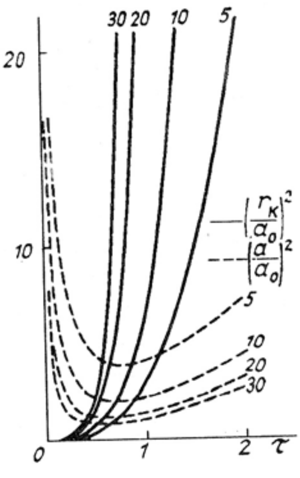

FIG. 2. Squared correlation length (solid curves) and beam radius (dashed curves) normalized by the radius of the pumping beam as functions of the normalized depth. Figures above the curves show the values of μ *(from 5 to 30).*

According to Eqs. (25) and (26), the correlation length of the beam increases monotonically with depth and only the rate of increase varies. At the same time, the dependence of the beam radius on depth appears to be nonmonotonic: the initial decrease of the radius is then followed by its increase (Fig. 2). This conclusion agrees fairly well with the calculated results.[2]

In the case of counter interaction, the correlation function has the form

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2, l) = C(l)G_0(1 + \tau^2)F(l) \times$$

$$\times \exp\left[2gI_0 l - \frac{r_1^2 + r_2^2}{a_c^2(l)} - \frac{(\mathbf{r}_1 - \mathbf{r}_2)^2}{r_{ccl}^2(l)} \right], \quad (30)$$

where $C(l)$ is a constant,

$$a_{\text{count}}^2(l) = a_{\text{col}}^2(l)/(1 + \tau^2),$$

and

$$r_{ccl}^2 = r_{cl}^2(l)/(1 + \tau^2). \quad (31)$$

From Eq. (30) we find that differences between the correlation length and the beam radius for collinear and counter interaction between the waves are observed at $\tau > 1$, i.e., when the effects of diffraction of the pumping wave are noticeable. It can easily be seen that in this case for counter interaction between the waves the correlation length and beam radius become independent of the interaction length. In other words, only the region of the amplifying medium adjacent to its exit and of depth l_{dp} has an appreciable effect on the amplification process.

Our results have been used in the analysis of beam shaping in amplifying media with feedback. The theory developed in this paper can also be used for solving problems of amplification of partially coherent beams. The propagation of such beams through amplifying media with longitudinal inhomogeneities was analyzed in Ref. 5 using the density matrix formalism and the method of integrals of motion. In this analysis the limiting value of the correlation length of the delta–correlated initial beam coincides with the beam radius.

3. MODE STRUCTURE OF THE FIELD IN A RESONATOR WITH AN ACTIVE MEDIUM

In general the problem of describing the electromagnetic fields inside a resonator formed by two mirrors of arbitrary shape reduces to an analysis of an integral equation for the complex amplitude A(ρ) of the field at one of the mirrors, which has the form

$$A(\rho) = \underset{\leftarrow}{a}\,\underset{\rightarrow}{a} \int_{S_1}\int_{S_2} R_1(\rho_1)R_2(\rho_2)A(\rho_2)\, \underset{\rightarrow}{G}(\rho_1, \rho_2) \times$$

$$\times \underset{\leftarrow}{G}(\rho, \rho_1)\, d^2\rho_2 d^2\rho_1, \quad (32)$$

where $\rho = \{x, y\}$ is the vector perpendicular to the z axis, the arrows under the symbols indicate the direction of propagation of the corresponding waves, $\underset{\leftarrow}{a}$ and $\underset{\rightarrow}{a}$ are the eigenvalues of the equation, R_1 and R_2 are the amplitude (in general complex) reflectances of the mirrors, $G(\rho, \rho_1)$ are the Green's functions, and the integrals are taken over the surfaces of the mirrors.

If a medium with parabolic profile of the complex extinction coefficient completely fills the volume of the resonator, then the Green's function is given by:

$$G(\rho, \rho_1) = h\, \exp(-\alpha\rho^2 - \beta\rho_4^2 - 2\gamma\rho\rho_1), \quad (33)$$

given that the optical axis of the resonator coincides with the axis of symmetry of a medium. Here h is a constant, and α, β, and γ are complex coefficients. Let us assume for simplicity that the mirror apertures greatly exceed the transverse dimension of the amplifying active medium channel. In addition we shall assume that the resonator is symmetrical not only about its axis but also about the origin of the x, y, z coordinate system. In this case $\alpha = \beta$, the variables x and y separate and taking Eq. (33) into account, Eq. (32) can be represented in the form

$$A(x) = ah\int_{-\infty}^{\infty} A(x_1)\, \exp(-\beta(x^2 + x_1^2) - 2\gamma x x_1)\, dx_1, \quad (34)$$

where the constant factor of the reflectance is contained in the constant a, and the phase delay caused by a possible parabolic (spherical in the paraxial approximation) shape is contained in the coefficient β.

We will show that the modes which take the form of a polynomial of degree m multiplied by a Gaussian factor

$$A_m(x) = (C_m x^m + C_{m-1}x^{m-1} + \ldots + C_0)e^{-qx^2} =$$

$$= \left(\sum_{j=0}^{m} C_j x^j\right)e^{-qx^2} \quad (35)$$

are the solutions of Eq. (34) for Re $q > 0$. By substituting Eq. (35) into Eq. (34) and by making use of the identity

$$\int_{-\infty}^{\infty} A(x^m)\exp(-Px^2 - Qx)dx = (-1)^m \sqrt{\frac{\pi}{P}}\frac{\partial^m \exp(Q^2/4P)}{\partial Q^m}, \quad (36)$$

we see that both sides of this equality contain polynomials of degree m multiplied by the factor $\exp(-qx^2)$ and the condition

$$q^2 = \beta^2 - \gamma^2, \ (\text{Re } q > 0) \quad (37)$$

must be fulfilled. It also follows from Eq. (36) that the coefficients C_m are the Hermite polynomials with complex variables.

Thus, the problem is reduced to finding the eigenvalues a_m by direct substitution from the condition of the identity of the polynomials. At the same time, it should be the case that the eigenfunctions of Eq. (34) are determined only to within a constant factor and that for even m the odd coefficients C_{2m-1} are equal to zero, and conversely for odd m the even coefficients vanish.

Let us now present some calculational results (the serial order corresponds to m).

Fundamental mode:

$$a_0 = h^{-1}\pi^{-1/2}(\beta + q)^{1/2}, \quad (38)$$

where C_0 is the normalization coefficient.

The first mode:

$$a_1 = -(\gamma h)^{-1}\pi^{1/2}(\beta + q)^{3/2} , \quad (39)$$

where C_1 is also the normalization coefficient.

The second mode:

$$a_2 = h^{-1}\gamma^{-2}\pi^{-1/2}(\beta + q)^{5/2} , \quad (40)$$

where $C_0/C_2 = -1/4q$.

Generalization of Eqs. (38)–(40) leads to an expression for the eigenvalue of an arbitrary mode m

$$a_m = (-1)^m h^{-1}\gamma^{-m}\pi^{-1/2}(\beta + q)^{m+1/2} . \quad (41)$$

Finally, the following interesting fact should be noted. The volume–distributed action of a medium with parabolic profile of the amplification coefficient can always be replaced by corresponding free space zones concentrated in a certain plane. This follows from the identity of the integral relation between the entrance and exit waves written in terms of the Green's function. As applied to our analysis of a symmetric resonator, this means that instead of a resonator filled with a medium with parabolic profile of the amplification coefficient, it is possible to consider the same resonator but filled with a homogeneously amplifying medium at the center of which a thin lens is located whose aperture is limited by a diaphragm with soft edges and whose amplitude transmission coefficient profile has Gaussian shape in the transverse direction. It is possible that such a simple model will be useful for the grafic interpretation of our results.

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