ANALYSIS OF THE EFFECT OF THERMAL SELF-ACTION OF A LASER BEAM ON ECHO-SIGNAL CHARACTERISTICS IN THE TURBULENT ATMOSPHERE

V.M. Buldakov, A.N. Glushkov, and V.V. Pokasov

Institute of Atmospheric Optics, Siberian Branch of USSR Academy of Sciences, Tomsk

Received December 15, 1988

The statistical characteristics of echo-signals in the space behind a receiving lens, in the turbulent atmosphere in the presence of thermal self-action of the probing radiation are calculated on the basis on the phase approximation of Huygens-Kirchhoff method.

Expressions for the energy center shifts and for variations of the image sizes along and transverse to the wind velocity are obtained. It is shown that for some specific cases of the parameters which characterize the transmitter-receiver system and the atmospheric part of the propagation path the effect of turbulence and nonlinearity of the echo-signals is insignificant.

Results of investigations of the propagation of laser radiation along radar paths in the turbulent atmosphere, not taking account of thermal self-action, are presented in sufficient detail in Refs. 1–3.

Use of high-power lasers for radars has led to the necessity of investigating the effects of thermal distortion on the characteristics of backscattering. This has to do with the fact that heating of the air along the path of propagation of the probing beam give rise to such nonlinear effects as, for example, wind refraction and redistribution of intensity[4]. In this paper a bistatic scheme of location is used to study thermal self-action effects (see Fig. 1).

A radiation source and a receiver are located in the plane ZOY of the cartesian coordinate system XYZ. The centers of the radiating and receiving apertures O_1 and O_2 are separated by a distance r_b.

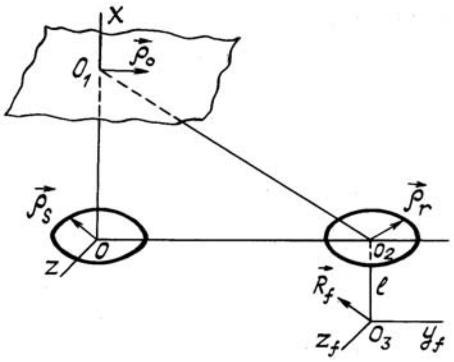

FIG. 1. Geometrical location scheme sounding

The angle between the direction of laser beam propagation OO_1 and the direction of the return signal O_1O_2 is characterized by the angle

$$\varphi = \mathrm{arctg}\,(r_b/L_s)$$

where L_s and L_r are the distances from the source and the receiver to the scattering surface, respectively. At the distance $l = O_2O_3$ from the receiving lens a plane of observation is placed parallel to the ZOY-plane.

The field distribution $U(\vec{R}_f, l)$ at the distance l behind the receiving lens is related to the field incident on the lens $U(\rho_r)$ as follows[5]:

$$U(\vec{R}_f, l) = \frac{k}{2\pi i l}\exp\left\{ikl + \frac{ikR_f^2}{2l}\right\}\int_{-\infty}^{\infty} d^2\rho_r\, T(\rho_r)\times$$
$$\times U_r(\rho_r)\exp\left\{ik/2\left(\frac{1}{l}-\frac{1}{F_L}\right)\rho_r^2 - \frac{ik\mathbf{R}_f}{2}\rho_r\right\} \qquad (1)$$

where $k = \dfrac{2\pi}{\lambda}$ is the wave number; $T(\rho_r) = \exp(-\rho_r^2/2\alpha_L^2)$ is the amplitude coefficient of lens transmission; and F_L and α_L are the focal length and the radius of the receiving lens, respectively;

$$U(\rho_r) = \int_{-\infty}^{\infty} d^2\rho_s d^2\rho_0\, U_0(\rho_s)\, K(\rho_0)\, G_1(L_s, 0; \rho_0, \rho_s)\times$$
$$\times G_2(L_r, 0; \rho_0, \rho_r); \qquad (2)$$

$U_0(\rho_s)$ is the source field; $K(\rho_0)$ is the local reflectivity;

$$G_1(L_s, 0; \rho_0, \rho_s) = \frac{k}{2\pi i L_s}\exp\left\{ikL_s + \frac{ik}{2L_s}(\rho_0 - \rho_s)^2 +\right.$$
$$+ ik/2\int_0^{L_s} d\xi\left[i\varepsilon_0(\xi) + \varepsilon_1(\xi, \rho_1(\xi), t) +\right.$$
$$\left.\left. + \varepsilon_2 T'(\xi, \rho_1(\xi), t)\right]\right\};$$

$$G_2(L_r,0;\rho_0,\rho_r) = \frac{k}{2\pi i L_r}\exp\left\{ik\left[L_r - \rho_{0y}\sin\varphi + \rho_{ry}\sin\varphi\right] + \frac{ik}{2L_r}(\rho_0 - \rho_r)^2 + ik/2\int_0^{L_r} d\xi\left[i\varepsilon_0(\xi) + \varepsilon_1(\xi,\rho_2(\xi),t)\right]\right\}$$

are the Green functions;

$$\rho_1(\xi) = \rho_s\frac{L_s - \xi}{L_s} + \rho_0\frac{\xi}{L_s};$$

$$\rho_2(\xi) = \rho_0\frac{L_r - \xi}{L_r} + \rho_r\frac{\xi}{L_r};$$

$k\varepsilon_0(\xi) = \alpha(\xi)$ is the absorption coefficient; t is the time; $\varepsilon_1(\xi, \rho(\xi), t)$ and $\varepsilon_2 T'(\xi, \rho_1(\xi), t)$ are the changes in the dielectric constant due to turbulence and heating of the air by radiation, respectively; and $\varepsilon = -1.9 \times 10^{-6}\ \mathrm{deg}^{-1}$. The expression used in Ref. 6 to describe the temperature variations is:

$$T'(\xi,\rho_1(\xi),t) = \frac{\alpha(\xi)c}{8\pi\rho(\xi)C_p} \times \int_0^t dt'\,\left|U_1(\xi,\rho_1(\xi) - \mathbf{V}_\perp(\xi)(t-t'),t')\right|^2 \tag{3}$$

where

$$U_1(\xi,\rho(\xi),t') = \frac{k}{2\pi i\xi}\exp\left\{-1/2\int_0^\xi d\zeta\alpha(\zeta) + ik\xi\right\} \times \int_{-\infty}^{\infty} d^2\rho_s U_0(\rho_s)\exp\left\{\frac{ik}{2\xi}\left[\rho(\xi) - \rho_s\right]^2\right\}.$$

Here c is the velocity of light in vacuum, $\rho(\xi)$ and C_p are the density and the specific heat, respectively, $\vec{\mathbf{V}}_\perp(\xi)$ is the wind velocity, whose direction coincides with the axis OZ.

For the bistatic location scheme the Green's functions $G_1(L_s, 0; \rho_0, \rho_s, t)$ and $G_2(L_r, 0; \rho_0, \rho_r, t)$ are statistically independent because the dielectric constant is independent of the direction of light propagation.

According to the Ref. 6 the temperature change depends on the average intensity of the initial field. In this case the expression for the average intensity of the reflected signal at the distance 1 behind the receiving lens can be written, taking into account formulas (1) and (2) as follows

$$\langle I(\mathbf{R}_f,l)\rangle = \langle U(\mathbf{R}_f,l)\,U^*(\mathbf{R}_f,l)\rangle = \left[\frac{k}{2\pi l}\right]^2 \times \int_{-\infty}^{\infty} d^2\rho_{s1}d^2\rho_{s2}d^2\rho_{01}d^2\rho_{02}d^2\rho_{r1}d^2\rho_{r2} \times$$
$$\times \langle U_0(\rho_{s1})U_0^*(\rho_{s2})\rangle\langle K(\rho_{01})K^*(\rho_{02})\rangle \times$$
$$\times \langle G_1(L_s,0;\rho_{01},\rho_{s1})G_1^*(L_s,0;\rho_{02},\rho_{s2})\rangle \times$$
$$\times \langle G_2(L_r,0;\rho_{01},\rho_{r1})\;G_2^*(L_r,0;\rho_{02},\rho_{r2})\rangle \times$$
$$\times T(\rho_{r1})\;T^*(\rho_{r2})\;\exp\left\{\frac{ik}{l}\mathbf{R}_f(\rho_{r1} - \rho_{r2}) + ik/2\left[\frac{1}{l} - \frac{1}{F_L}\right](\rho_{r1}^2 - \rho_{r2}^2)\right\}, \tag{4}$$

where the brackets <...> denote averaging over the ensemble of realizations. For the partially coherent Gaussian beam we have:

$$\langle U_0(\rho_{s1})U_0^*(\rho_{s2})\rangle = U_0^2\exp\left\{-\frac{k}{2L_s}\left[\frac{1+K_s^2}{2\Omega} + iL_s/F\right]\rho_{s1}^2 - \frac{k}{2L_s}\left[\frac{1+K_s^2}{2\Omega} - iL_s/F\right]\rho_{s2}^2 + \frac{k}{2L_s\Omega_k}\rho_{s1}\rho_{s2}\right\},$$

where $K_s^2 = 1 + \Omega/\Omega_k$, $\Omega = \dfrac{ka^2}{L_s}$, $\Omega_k = \dfrac{ka_k^2}{L_s}$, U_0 is the amplitude of the field at the output of the aperture, a is the effective radius of the radiating aperture, a_k is the effective radius of source coherence, and F is the radius of curvature of the wave front of the initial source field.

The scattering of the atmosphere is described by means of a diffusion screen of infinite size with reflection coefficient coherence function

$$\langle K(\rho_{01})\;K^*(\rho_{02})\rangle = 4\pi/k^2\;\delta(\rho_{01} - \rho_{02}),$$

where $\delta(\rho)$ is the Dirac delta-function.

Averaging over the fluctuations of the dielectric constant ε_1 is done in the same way as in the Refs. 5 and 6:

$$\langle G_1(L_s,0;\rho_{01},\rho_{s1})G_1^*(L_s,0;\rho_{02},\rho_{s2})\rangle = \left[\frac{k}{2\pi L_s}\right]^2 \times$$
$$\times \exp\left\{-\int_0^{L_s} d\xi\alpha(\xi) + \frac{ik}{2L_s}\left[(\rho_{01} - \rho_{s1})^2 - (\rho_{02} - \rho_{s2})^2\right] - \frac{\pi k^2}{4}\int_0^{L_s} d\xi\;H((1-\xi/L_s)(\rho_{s1} - \rho_{s2}) + \xi/L_s(\rho_{01} - \rho_{02})) + \right.$$
$$\left. + \frac{ik}{2}\varepsilon_2\int_0^{L_s} d\xi\left[\langle T'(\xi,(1-\xi/L_s)\rho_{s1} + \xi/L_s\;\rho_{01},t)\rangle - \langle T'(\xi,(1-\xi/L_s)\rho_{s2} + \xi/L_s\;\rho_{02},t)\rangle\right]\right\};$$

here

$$H(\rho) = 2\int_{-\infty}^{\infty} d^2\kappa\ (1-\cos(\kappa\ \rho))\ \Phi_\varepsilon(\xi,\kappa),$$

$$\Phi_\varepsilon(\xi,\kappa\ ) = 0.033C_\varepsilon^2(\xi)\,|\kappa|^{-11/3}\left[1 - \exp\ (-\kappa^2/\kappa_0^2\ )\right]$$

is the spatial spectrum of air turbulence, C_ε^2 is the structure characteristic of the dielectric constant of air, $\kappa_0 = 2\pi/L_0$, and L_0 is the outer scale of turbulence. By analogy with Eq. (5) we have:

$$\begin{aligned}&\langle G_2(L_r,0;\rho_{01},\rho_{r1})G_2^*(L_r,0;\rho_{02},\rho_{r2})\rangle = \left[\frac{k}{2\pi L_r}\right]^2 \times\\ &\times \exp\left\{-\int_0^{L_r} d\xi\alpha(\xi) + ik\sin\varphi\ (\rho_{ry1}-\rho_{ry2}-\rho_{0y1}+\rho_{0y2}) +\right.\\ &+ \frac{ik}{2L_r}\left[(\rho_{01}-\rho_{r1})^2-(\rho_{02}-\rho_{r2})^2\right]-\frac{\pi k^2}{4}\times\\ &\left.\times\int_0^{L_r} d\xi\ H((1-\xi/L_r)(\rho_{01}-\rho_{02})+\xi/L_r(\rho_{r1}-\rho_{r2}))\right\}\end{aligned} \tag{5}$$

For the average profile of the induced temperature of the steady-state regime '$(t\to\infty)$ one can derive the following expressions:

$$\begin{aligned}&\langle T(\xi,\rho(\xi),t)\rangle = 0.5R_N\sqrt{\pi}\ \mathrm{erfc}\left[-\frac{\rho_{sz}(1-\xi)+\rho_{0z}\xi}{a}\right]\times\\ &\times\exp\left\{-\left[\frac{\rho_{sy}(1-\xi)-\rho_{0y}\xi}{a}\right]\right\},\end{aligned} \tag{6}$$

where $R_N = \dfrac{U_0^2 c\alpha\varepsilon_2}{8\pi\rho C_p}\dfrac{a}{\langle V_\perp\rangle}(ka)^2$ is the parameter of nonlinearity, α is the coefficient of molecular absorption, and ρ and C_p are the density and specific heat of the air, respectively.

In order to carry out the analytical calculations which are to follow we can make use of the nonaberrative (or paraxial) approximation. Its limitations are discussed in Ref. 7.

From our analytical calculations we obtain the following expression for the average intensity of the reflected signal image:

$$\begin{aligned}&\langle I(\mathbf{R}_f,l)\rangle = U_0^2\exp\left\{-2\int_0^L d\xi\ \alpha(\xi)\right\}\left[\frac{a\ a_L}{Ll}\right]^2\times\\ &\times\exp\left\{-\left[\frac{R_{cz}}{a_z}+\frac{R_{fz}}{a_z}\right]^2-\left[\frac{R_{cy}}{a_y}+\frac{R_{fy}}{a_y}\right]^2\right\},\end{aligned} \tag{7}$$

where $L = L_s = L_r$, $\mathbf{R}_f = (R_{fz}, R_{fy})$ is a transverse vector in the reflected signal image plane, $\alpha(\xi)$ is the change of the molecular absorption coefficient along the path, $R_{cz} = l\dfrac{R_N}{4(ka)^2}$ and $R_{cy} = r_b l/L$ are shifts of the energy center of the image on the Z and Y axes, respectively, are the sizes of the image on the Z and Y axes, respectively,

$$a_z = \frac{l}{L}B,\quad a_y = \frac{l}{L}\left[B^2+\frac{\sqrt{\pi}R_N L}{6k^2a}\left(\frac{\sqrt{\pi}R_N L}{6k^2a^3}-2(1-L/F)\right)\right]^{1/2}$$

H is the height of the source above the earth's surface, $C_\varepsilon^2(h(\xi L))$ is the structure characteristic of the dielectric constant as a function of hight $(h(\xi L)$,

$$h(\xi L) = H + R_E\left[\left(\frac{\xi L}{R_E}\right)^2 + 2\frac{\xi L}{R_E}\cos\theta + 1\right]^{1/2} - R_E,$$

R_E is the earth's radius, and θ is the zenith angle. This expression shows that the distribution of the average intensity of the backscattered signal in the observation plane is determined by the parameters of the transmitting and receiving systems as well as by thermal self-action effects and the atmospheric turbulence. In this case the energy center coordinates of the backscattered signal in the observation plane along the Z-axis are determined by the effect of wind refraction while these coordinates along the Y-axis are determined by the distance between the source and the receiver r_b. The difference between the dimensions of the image along the Z and Y axes is due to the increase of the transverse size of the laser beam cross-section relative to the Y-axis because of the interaction of the high-energy radiation and the medium moving along the Z-axis. However, according to our calculations the effect of turbulence and nonlinearity can under certain conditions be negligible. In particular, for

$\lambda=1.06\times(10^{-6}\div 10^{-5})$m, $a=0.5$m, $L=10\div10^3$m, $L/F=0\div1$,

$r_b=1$m, $a_L=0.5$m, $\alpha=10^{-4}$m^{-1}, $H=2.5$m,

$C_\varepsilon^2(H)=10^{-14}\div10^{-17}m^{-2/3}$ and $R_N=10\div10^3$,

the influence of these factors leads to a decrease of the maximum intensity by not more than 6% and to changes in the sizes of the image and the shifts of the energy center coordinates of not more than 2%. These results offer possibilities of determining the characteristics of the initial high-power radiation field from the backscattered signal.

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