AUTOMATED SYSTEM FOR STOCHASTIC MODELING OF RADIATION FIELD OF THE ATMOSPHERE

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The "Foton" system which automates stochastic modeling of the atmospheric radiation field is presented. A system's functional scheme, a list of problems solved by the system, as well as a functional scheme and the contents of the database are described. The problem of image transfer in the "atmosphere–underlying surface" system is considered as an example. Two algorithms for estimation of the optical transfer function of the system are presented, as well as the calculated results. In our calculations, we used the models of horizontally homogeneous and horizontally inhomogeneous stochastic atmosphere constructed based on the Palm flows. The values of the optical transfer function obtained were used to correct model "space photographs" of test objects.

1. INTRODUCTION

A study of radiation transfer in natural scattering media from space becomes increasingly urgent because of many problems that arise in remote optical monitoring of natural media from space. In view of a wide scope of scientific and applied problems of such a kind, we have begun to develop a system for automation of numerical experiments conducted using the methods of stochastic modeling aiming at studying the regularities in the radiation field formation in the system of "aerosol – cloudy atmosphere – underlying surface" and the "ocean–atmosphere" system as well. The automated system being developed incorporates a number of algorithms and computer programs that have earlier been created at the Department of Stochastic Modeling in Physics of the Institute for Computational Mathematics and Mathematical Geophysics SB RAS (former Computer Center of SB RAS). Among the specific features of the above problems that hamper their solution there are:

– a large number of factors participating in the formation of the radiation field;

– a large number of boundary conditions. It is especially true regarding the problems, which should take account of spatial inhomogeneity of the albedo and the Earth's surface relief, as well as the stochastic structure of the cloudiness and wind roughness of the sea surface;

– stochasticity of spatiotemporal variations of the majority of optically active atmospheric components.

The state of the art of the stochastic modeling allows one to state and successfully solve the problems of the radiation transfer in the atmosphere and in the "ocean – atmosphere" system in the following general formulation. Given the characteristics of the determinate or random function (model of radiation sources) at the input of a system with preset parameters, which in turn may be determinate or random functions (optical and geometrical model of a medium), the sought are the characteristics of random functions at the system output (the characteristics of the radiation field). By the system we mean here a model developed based on theoretical and numerical studies of the processes of the optical radiation transfer through disperse media. The stages of the development of the model make up a closed iteration cycle, represented by a scheme of control over a scientific experiment. This approach is used as a basis for an automated system for modeling and studying the process of radiative transfer through a stochastic scattering media. It is just this automated system that makes the subject of this study. The system is characterized by the following key features:

– the unique algorithmic content covering a wide scope of the atmospheric optics problems;

– use of a relational database storing the parameters and results of the numerical and physical experiments;

– user's graphic interactive interface with a multilevel pop-up menu;

– open module architecture providing the possibility of extending the classes of problems to be solved.

At the present stage of the development, the system allows a solution of the following problems:

– estimation of the brightness field and calculation of the optical transfer function (OTF) in the systems "atmosphere – underlying surface" and "ocean – atmosphere";

– modeling of the underlying surface images ;

– filtering out of the image distortions caused by scattering of electromagnetic radiation.

There is also a number of related problems: modeling of random fields (topography of the ground,

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ocean or cloud surface) and making fast Fourier transforms.

The system allows one to automate the following functions: preparation and performance of a numerical experiment by Monte Carlo method, data storage and browsing, statistical analysis of the data, estimation of the algorithms' efficiency, generation of different reports, tables, and plots.

By now the system is developed in Delphi environment and may be operated in WINDOWS 95/NT operation systems with databases in the Paradox format. In the future, we plan to use the Oracle database management network system.

2. FUNCTIONAL DIAGRAM

Figure 1 shows the functional diagram of the *Foton* system. The system can be subdivided into the following subsystems: a kernel, database, end-user

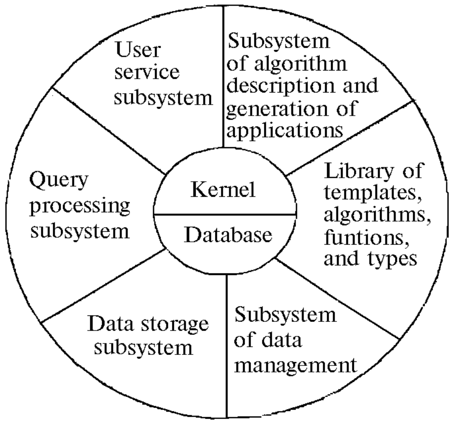


FIG. 1. Functional scheme of the system.

service subsystem; query processing subsystem; subsystem of algorithm description and generation of applications; libraries of algorithm templates, functions, and types; data storage subsystem; information management subsystem.

2.1. Kernel

The basic function of the kernel is to receive messages, recognize them, and transfer to the corresponding subsystems for further processing. The real-time input of the messages allows several users to enter, refine, and output the information simultaneously and independently. The kernel supports the system cache, by which a part of RAM is meant that is reserved by the system for storage of some data set from the database.

2.2. Database

The system's database is one of the basic structure components. It is intended for information support of the problems solved under conditions of shareable resources. The main characteristic feature of the shareable database is its independence of the active programs interacting with it. At present the system uses the relational database in the Paradox format with the direct access to tables. The data are subdivided into the following categories: primary reference tables, models, additional reference tables, computational log file, data of physical measurements and computed results.

2.2.1. Reference Tables

The reference tables contain some support information. They provide for the information processing (a control as well as algorithms and parameters for calculations) and generate references (decoding, communication, relocation). The reference tables can also be used to facilitate data input. Therefore, they perform special functions for information search and retrieval. The primary reference tables reflect the algorithms implemented in the system and serve to facilitate data input. They are protected from modifications by an end user. Models are formed based on the primary reference tables and model parameters. The number of models and their structure are not restricted, so they can be changed by users, if these changes do not contradict user's rights authorized by the system.

Examples of primary reference tables

List of system problems.
List of estimates.
List of scattering phase functions.
List of mathematical models of broken cloudiness.
List of mathematical models of rough sea surface.
List of correlation functions.
List of methods for free path modeling.
List of distribution functions of the underlying surface albedo.

Examples of models

Models of stochastic scattering media.
Models of the underlying surface.
Geometry of computations.
Algorithmic schemes for calculation.
Models of data.

Additional reference tables

List of system users.
List of users' rights.
Mathematical description of algorithms.

2.2.2. Documents and data

By a document it is meant a list of parameters determining numerical experiment. Data are the results of some numerical experiment or physical measurements stored in the database. The documents and the data form the basis of the information processed in the system. They can be accessed, supplemented with new items, or corrected through different screen forms. The principle difference between the data and the reference tables is that it is just the data that comprise the main content of the information to be processed.

2.3. User service subsystem

The user service subsystem forms the interface, with which the users may have an access to the database, and the algorithmic content of the system. This subsystem allows the users to retrieve individual and summarized results of numerical experiments and the data of physical experiments. The data can be presented as tables or plots on a monitor screen, printed out, or stored as separate files in the text or graphical format. The system provides the user interface in two different forms: as an HTML document and as the client screen forms (in the case of operating in a local network). The former one serves, first of all, to browse the sets of dynamically generated records of computed results.

The user interface in a local network provides the following services for a user: to browse the results of numerical experiments in tabulated and graphical forms, to plan and conduct some new experiments, to amend the experiment already conducted, to form statistical reports by subsets of model parameters, and to form a private scope of interests.

The user's interests consist of some topical subset from the system information selected by the user. Figure 2 shows a screenshot of the *Foton* system operation. The figure illustrates the process of choosing some parameter's value of a numerical experiment from the corresponding pop-up menu.

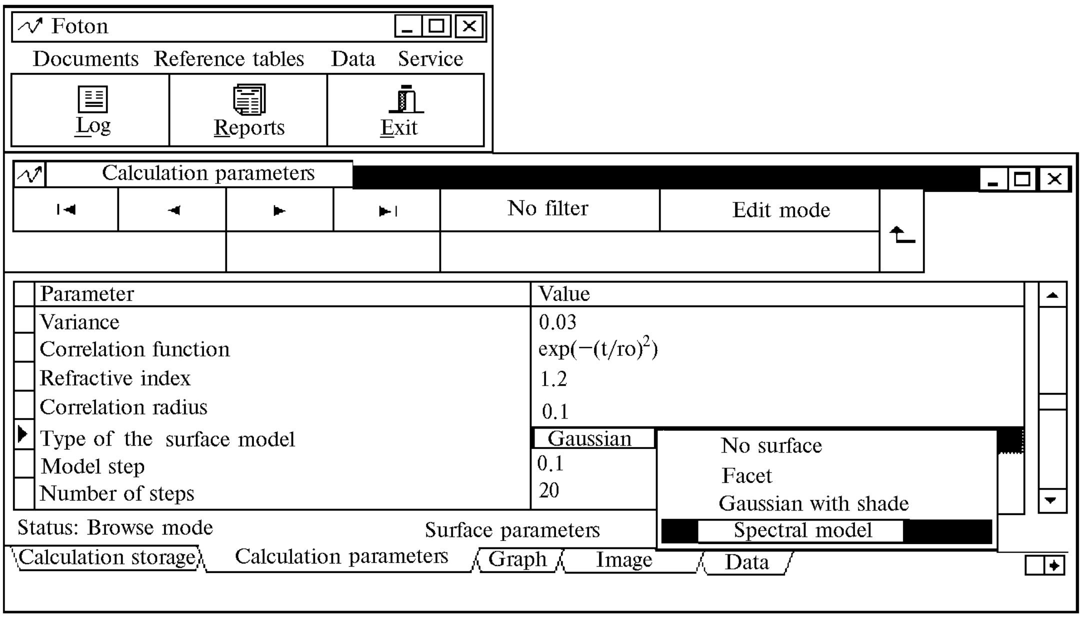


FIG. 2. Screenshot of the system operation (only Russian version available).

2.4. Query processing subsystem

Query processing subsystem is an interface of the data logical presentation inside the system. This subsystem is a program model of the processor, which executes a series of input commands and presents the results in the form of the data tables. An actual required information is being taken from the database during the processing.

2.5. Subsystem for describing the algorithms and generation of applications

The subsystem provides for a sufficiently fast development of new applications. By application it is meant an informational independent area of numerical investigations, which possesses its own set of input data and parameters as well as its own set of objects under study (models), the behavior of which is characterized by these data. Algorithms and models forming the basis of an application are described by the language tools, close to the mathematical language.

2.6. Data storage subsystem and data management subsystem

The data storage and management subsystems are intended for databases service and administration. They provide the following possibilities: import of data of physical measurements from remote databases, screen input and refinement of the data, generation of error log files, accumulation of statistical ensembles.

3. ACCESS TO INFORMATION

In accordance with the problems solved by different users, it should be adequately determined, what kind of information is accessible to each of them. The system of users' access (access right) has several levels: not allowed, read only, full access.

If necessary, full access can be divided into several rights, which could allow users to create and input new information, change or delete available information.

4. REPORTS AND DATA RETRIEVAL

In contrast to data, reports are generated based on retrieval and comparison of various initial data sets, that is, reports are the output documents of the system. The main requirement to the reports in the system is the availability of parameters for the reports relocation and a possibility for users to create new changeable report forms. By now the reports can be conditionally subdivided into two groups:

1. Reports representing the analysis of a number of numerical experiments (for example, comparison of the algorithm efficiency, estimation of the correlation function);

2. Reports representing the results of particular numerical experiments (for example, MTF plots).

The principle difference between these two groups of reports is that the state and the content of data may change when generating reports of the first group, while being unchangeable for the reports of the second group. The reports of the first group are a continuation of a numerical experiment, while the reports of the second group reflect the current state of data.

5. VERSIONS OF USING THE SYSTEM

Automated systems are most efficient when used in a local computer network by different users having different access prioritization. At present the local version of the *Foton* system automates preparing and performing a numerical experiment; data storage and browsing; generating various reports, tables, and plots; and obtaining the data statistics.

The network version of the system is now under development. Its implementation will allow automating the following functions:

– inter-task communications, in particular, an attachment of executable modules; using a program as a server or a client in the ActiveX technology;

– generation of independent programs;

– interface allocation to Intranet, generation of reports in the form of HTML documents;

– enhancement of the database and algorithm types by users.

– support of distributed calculations in a local network.

6. IMAGE TRANSFER PROBLEM

As an example we consider the problem of image simulation and estimation of optical transfer function in the "atmosphere–underlying surface" system to demonstrate the *Foton* system application. Here we present two algorithms for estimation of the optical transfer function and the results calculated. The influence of light scattering on the quality of image transfer is studied in this work based on the principles of the theory of linear systems.[1] The basic equation of image formation in a linear isoplanatic system has the following form:

$$I(\rho) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_0(\rho')\, G(\rho - \rho')\, \mathrm{d}\rho' =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_0(\rho - \rho')\, G(\rho')\, \mathrm{d}\rho, \qquad (1)$$

where $\rho = (x, y)$ and $\rho' = (x', y')$ are the points in the receiver and image plane, respectively; $I(\rho)$ and $I_0(\rho)$ denote the brightness at the point ρ in the receiver and image planes; $G(\rho)$ is the Green's or the point spread function. Having taken the Fourier transform of the convolution equation (1), we obtain the equation in the plane of spatial frequencies:

$$\hat{I}(p) = \hat{I}_0(p)\, \hat{G}(p), \qquad (2)$$

where the caps are used to denote the Fourier transform. The function $\hat{G}(p)$ is referred to as an optical transfer function (OTF) of the system; it is denoted hereinafter as $T(p)$. Thus, the OTF is an optimal filter of distortions of the spatial structure of the image brightness for a linear isoplanatic systems.

In the most practical problems of observing the Earth's surface through the atmosphere, it is sufficient to characterize the image transfer quality by the modulation transfer function (MTF) defined as:

$$T_a(p) = |T(p)| = \sqrt{(\mathrm{Im}\, T)^2 + (\mathrm{Re}\, T)^2}. \qquad (3)$$

The underlying surface (image plane) is then assumed to emit radiation according to the Lambert law. In this case, $G(\rho)$ has the meaning of relative distribution of the brightness in the image plane along the sighting direction toward the point of a diffuse source of a unit power located in the image plane. Besides, $G(\rho)$ is a solution of the stochastic transfer equation under corresponding boundary conditions.[2]

7. STATEMENT OF THE PROBLEM, GEOMETRY, AND DESIGNATIONS

A plane horizontal layer of a scattering substance is illuminated by a parallel flux of solar radiation of a "unit power" (Fig. 3). Radiation interaction with the substance is specified by setting a random field of extinction coefficient and the scattering phase function. The light is reflected diffusely (according to the Lambert law) from the underlying surface. The position of a particle (radiation quantum) in the phase space is characterized by the radius vector $r = (x, y, z)$ and the unit vector of the motion direction $w = (a, b, c)$. The task is to find a ratio between the underlying surface albedo and the brightness of a light field measured at some altitude above the surface.

Hereinafter we use the following designations: $n = (0, 0, 1)$ for the vector normal to the plane $z = \mathrm{const}$; $P_1^*, P_2^*, \ldots, P_n^*$ for the sighting points; $\sigma(r)$ and $\sigma_a(r)$ for the scattering and absorption cross-sections; $\Sigma(r) = \sigma(r) + \sigma_a(r)$ for the total extinction cross-section; $q(r) = \sigma_a(r)/\Sigma(r)$ for the absorption probability; $\Phi(r, w)$ for the particles' flux density; $f(r, w) = \Sigma(r)\Phi(r, w)$ for the collision density; $\tau(r', r) = \int_0^l \Sigma(r' + ws)\, \mathrm{d}s$ for the optical path length between the points r' and r; $l = |r - r'|$;

$f_l(s) = \Sigma(r(s)) \exp[-\tau(r', r(s))]$, $r(s) = r' + ws$

for the distribution density of the mean free path; $\mu = (\omega', \omega)$ for the cosine of the angle between the directions of motion before and after scattering; $g(r, \mu)$ for the distribution density of the scattering cosine at the point r (scattering phase function) normalized as $\int_{-1}^{+1} g(r, \mu)\, d\mu = 1$; $R(r_\perp, w, w')$ for the brightness coefficient characterizing reflection properties of the Earth's surface; $A(r_\perp)$ for the surface albedo at the point $r_\perp$; $I(r, w)$ for the spectral brightness (intensity) of the radiation;

$$E(r, n) = \int_{(w,n)<0} |(n,w)|\, I(r, w)\, dw$$

for the monochromatic irradiance of a unit area at the point r having the normal n;

$I(P^*,w^*) = \{I(P^*_1,w^*), I(P^*_2,w^*), \dots, I(P^*_n,w^*)\}$

for a space photograph and

$I(P, w^*) = \{I(P_1, w^*), I(P_2, w^*), \dots, I(P_n, w^*)\}$

for a near-surface photograph.

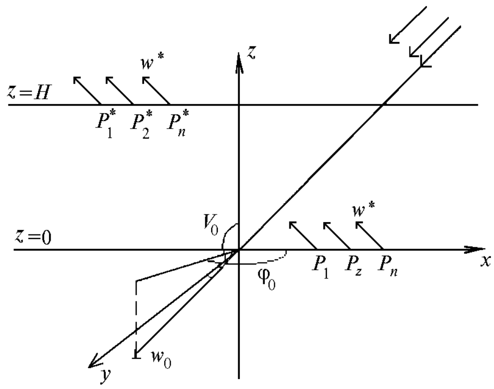


FIG. 3. Geometry of the problem.

8. MODEL OF A STOCHASTIC MEDIUM

Let us consider the models of cloudiness, the only random parameter in which is the indicator field of a cloud substance $\zeta(r)$. The parameters of scattering are constant inside and outside a cloud. The subscript 1 is used for the atmospheric scattering parameters, while the subscript 2 denotes the scattering parameters of a cloud:

$$\zeta(r) = \begin{cases} (0,1), & r \in G, \\ (1,0), & r \notin G, \end{cases}$$

where G is a random set of points in the layer $\{0 < z < H\}$. Then $\Sigma(r) = (\Sigma_1, \Sigma_2)\zeta(r)$ is the extinction coefficient of the medium; $g(\mu, r) = (g_1(\mu), g_2(\mu))\zeta(r)$ is the scattering phase function of the medium. Here Σ_1 is the extinction coefficient of the atmosphere, Σ_2 is the extinction coefficient of the cloud, $g_{1,2}(\mu)$ is the scattering phase function of the atmosphere and the cloud, respectively. The statistics of the optical parameters is completely specified by the statistical characteristics of the indicator field $\zeta(r)$.

9. CONSTRUCTING THE INDICATOR FIELD

9.1. One-dimensional model

The Palm flux $z_k = z_{k-1} + \eta_k$, $z_0 = 0$ is constructed to propagate along the z-axis at $0 < z < H$. Here $\{\eta_k\}$ are independent random quantities obeying the Poisson distribution law with the density $f_\eta(t) = \lambda\exp\{-\lambda t\}$. The layer $\{0 \le z \le H\}$ is thus divided into m random layers $\tau_i \le z \le \tau_{i+1}$, $\tau_0 = 0$, $\tau_m = H$. The indicator field ζ_i is set in each layer as:

$$\zeta_i = \begin{cases} (0,1)^T, & \text{with probability } p, \\ (1,0)^T, & \text{with probability } 1 - p. \end{cases}$$

The normalized correlation function for such a field is equal to $\exp(-\lambda z)$ (Ref. 2), and, consequently, the correlation length is $1/\lambda$.

9.2. Three-dimensional model

The above point fluxes are constructed independently along every axis within a parallelepiped $\{0 < x < H_x, 0 < y < H_y, 0 < z < H\}$:

along x τ_i, $i = 0,1, \dots, m_x$ layers;

along y t_j, $j = 0,1, \dots, m_y$ layers;

along z l_k, $k = 0,1, \dots, m_z$ layers.

The indicator field ζ_{ijk} is then set in every of the m_x, m_y, m_z parallelepipeds as:

$$\zeta_{ijk} = \begin{cases} (0,1)^T, & \text{with probability } p, \\ (1,0)^T, & \text{with probability } 1 - p. \end{cases}$$

The random field is statistically homogeneous, nonisotropic with the normalized correlation function $\exp(-\lambda_z z - \lambda_x x - \lambda_y y)$, where λ_z, λ_x, and λ_y are the parameters of fluxes along the coordinate axes. Some clouds have the form of parallelepipeds with the exponential size distribution function along every coordinate axis.

10. COMPLEX EQUATION FOR OTF

Direct application of the principles of linear systems to the systems of image formation in an inhomogeneous medium bounded by the underlying surface is unjustified. Such systems are nonlinear due to multiple reflections of radiation between the medium and the underlying surface. The principle of isoplanaticity is violated for every realization of a horizontally inhomogeneous medium. If a random field of the extinction coefficients is homogeneous, then the system is isoplanatic, on the average, otherwise it can be considered invariant to a shift only within locally homogeneous zones.[3]

In Ref. 4 the surface albedo is presented as $A(r_\perp, z, w) = \bar{A} + \tilde{A}(r_\perp)$, and thus the relation is derived: $I(r_\perp, z, w) = \bar{I}(z, w) + \tilde{I}(r_\perp, z, w)$, where $\bar{I}(z, w)$ is the brightness of the system with the albedo $\bar{A}$; $\tilde{I}(r_\perp, z, w)$ is the horizontally inhomogeneous component of the brightness in the single scattering approximation (neglecting multiple reflections). In this case, the albedo is related to $\tilde{I}(r_\perp, z, w)$ as follows:

$$\hat{\tilde{I}}(z, p, w) = ET(z, p, w)\, \hat{\tilde{A}}(p), \qquad (4)$$

where E is the mean irradiance of the surface; the cap symbol is used for the Fourier transform; $T(z, p, w)$ is the atmospheric OTF, which satisfies the complex "transfer equation"

$$\mu \frac{T(z, p, w)}{z} + \hat{\Sigma}(z)\, T(z, p, w) =$$

$$= \sigma(p) \int_\Omega T(z, p, w)\, g[z, (w'\, w)]\, dw' \qquad (5)$$

with the boundary conditions

$$T(z, p, w)\Big|_{z=H,\ w\in\Omega_-} = 0,$$

$$T(z, p, w)\Big|_{z=0,\ w\in\Omega_+} =$$

$$= \frac{\bar{A}}{\pi} \int_\Omega T(0, p, w)\,|(n, w')|\, dw' + 1.$$

Here $\hat{\Sigma} = \Sigma(z) - i(p, w_\perp)$, $w_\perp = (\sqrt{1-\mu^2}\cos\varphi, \sqrt{1-\mu^2}\sin\varphi)$, $p = (p_x, p_y)$ is the vector of spatial frequencies.

Below we present two algorithms of the Monte Carlo method. The first one is developed to estimate spatial distribution of the intensity of outgoing radiation. It was used for simulation of photographs taken from space. The algorithm consists in estimation of a solution to the boundary-value problem of the transfer theory for various boundary conditions by the method of conjugate wandering and correlated sampling. The MTF estimation by this algorithm is based on the physical interpretation of its values as a ratio of the modulation index in an image of a sinusoidal test object to the modulation index of its near-ground contrast. The second algorithm is based on interpretation of the complex "transfer equation" (5) as a boundary-value problem of the transfer theory in a medium with the complex extinction coefficient and homogeneous isotropic source of a unit power in the plane $z = 0$.

11. THE FIRST ALGORITHM: SIMULATION OF A PHOTOGRAPH TAKEN FROM SPACE AND ESTIMATION OF THE MTF BY SOLVING THE TRANSFER EQUATION UNDER SPECIAL BOUNDARY CONDITIONS

Let us consider an arbitrary trajectory $\{(r_0, -w^*, W_0), (r_1, w_1, W_1), \ldots, (r_N, w_N, W_N)\}$, where $r_i(x_i, y_i, z_i)$ is the point of ith collision, $(r_0 = P^*)$; w_i is the unit vector of a photon motion after the ith collision; W_i is the weight of a photon after the ith collision ($W_0 = 1$), r_N is the point of Nth collision prior to escape.

The estimation uses one set of the trajectories corresponding to P_1^*. For an arbitrary point P_1^* we denote $I_i = \tilde{I}_1 = \tilde{I}(P_1^*, w^*)$ in the system with the characteristics $\tilde{A}$, $\tilde{\Sigma}$, $\tilde{q}$, $\tilde{g}$ (this system results from the initial system due to a shift by the vector $P_1^* - P_1^*$). From this point on $\tilde{P} = P - P_1^* + P_i^*$.

The trajectories of particles are constructed following the standard scheme.[2] In accordance with the principle of the weighting method of dependent trials, in order to obtain an unbiased estimate for the brightness $I_i = \tilde{I}_1$, the photon weight should be multiplied by the corresponding weights: $q(\tilde{r})$ after collision in the medium and $A(\tilde{r}_\perp)$ after reflection from the plane $z = 0$:

$$\frac{\Sigma(\tilde{r}_{i+1})}{\Sigma(r_{i+1})} \exp\left\{-\int_0^l [\Sigma(\tilde{r}_i + tw_i) - \Sigma(r_i + tw_i)]\, dt\right\}$$

after the transition $r_i \to r_{i+1} = (r_{\perp,i+1}, z > 0)$;

$$\exp\left\{-\int_0^{|r_{i+1}-r_i|} [\Sigma(\tilde{r}_i + tw_i) - \Sigma(r_i + tw_i)]\, dt\right\}$$

after the transition $r_i \to r_{i+1} = (r_{\perp,i+1}, z = 0)$;

$$\frac{g[\tilde{r}_i, (w_{i-1}, w_i)]}{g[r_i, (w_{i-1}, w_i)]}$$

after modeling of the scattering angle upon the collision at the point r_i.

The random quantity, which determines the value of trajectory contribution into the sought value of the brightness, can be estimated as $M\xi = I(P^*, w^*)$,

$$\xi = \sum_{i=1}^N \frac{e^{-\tilde{\tau}_i}\, \tilde{\varphi}_i\, \tilde{W}_i}{2\pi},$$

where

$$\tilde{\tau}_i = \int_0^{(H-z_i)/(-w_0,n)} \Sigma(\tilde{r} - tw_0)\, dw;$$

$$\tilde{\varphi}_i = \begin{cases} g[\tilde{r}_i, (w_{i-1}, -w_0)], & \text{for } r = (r_\perp, z > 0), \\ 2A(\tilde{r}_{\perp,i}), & \text{for } r = (r_\perp, z = 0). \end{cases}$$

This algorithm can be used for simulation of a photograph taken from space and estimation of the MTF by the same trajectories. To estimate MTF, we present the function $A(r_\perp)$ as $A(r_\perp) = \bar{A} + \tilde{A}\cos(2\pi pr_\perp)$. Then $I(r_\perp, z, w) = \bar{I}(z, w) + \tilde{I}(r_\perp, z, w)$ is the radiation brightness at the level z. Hereinafter $w^* = (0, 0, 1)$, then $T(z, p, w) = |T(z, p, w)| = \sqrt{(\mathrm{Im}T)^2 + (\mathrm{Re}T)^2}$. The OTF can be presented as $T(z, p, w) = e^{-\tau(z)} + T'(z, p, w)$, where $T'(z, p, w)$ is the OTF for the scattered radiation, then $\tilde{I}(r_\perp, z, w) = E\bar{A}\,[e^{-\tau(z)} + T'(z, p, w) \times \cos(2\pi pr_\perp)]$. A random estimate ξ of the sought value $T(z, p, w)$ on this trajectory is determined by the expression $M\xi(z, p, w) = T(z, p, w)$:

$$\xi(z, p, w) = \frac{\tilde{\xi}(z, p, w) - \bar{\xi}(z, p, w)}{\tilde{A}\ \xi_E},$$

where $\tilde{\xi}$ and $\bar{\xi}$ are the estimates of the brightness at the point $P^* = [r^* = (0, 0, H);\ w^* = (0, 0, 1)]$ for the albedo equal to $A(r_\perp, w) = \bar{A} + \tilde{A}\cos(2\pi p r_\perp)$ and $\bar{A}$, respectively; $\xi_E = \sum_{j=1}^{M} W'_j$ is the estimate of the mean irradiance of the underlying surface; M is the number of reflections from the plane $z = 0$; W'_j is the particle weight at the jth reflection $M\xi(z, p, w) = T(z, p, w)$.

The algorithm allows the values of $T(z, p, w)$ for an arbitrary number of the p parameters to be estimated by one and the same set of trajectories, as well as the values of $\bar{I}(z, w)$ and E.

12. THE SECOND ALGORITHM: MTF ESTIMATION BY SOLVING THE COMPLEX TRANSFER EQUATION

Let us consider an arbitrary trajectory $\{(z_0, w_0, W_0), (z_1, w_1, W_1), \ldots, (z_N, w_N, W_N)\}$. Here z_i $(i = 0, \ldots, N)$ is the vertical coordinate of the point of the ith collision of a particle moving along the direction w_i, $(z_0 = 0)$; N is the number of the collision preceding the escape; $F_{w_0}(w) = (w, n)/\pi$ is the density of the distribution w_0, $n = (0, 0, 1)$.

Because the extinction coefficient is a complex variable, the mean free path l of a "particle" moving along the direction w_i, after the collision at a point z_i, should be chosen in accordance with the density

$$f(l) = \sigma\,[z_i + (w_i, n)\,l]\,\exp\left\{-\int_0^l \sigma\,[z_i + (w_i, n)\,t]\,\mathrm{d}t\right\}.$$

Mass in this case is recalculated by the expression

$$W_{i+1} = W_i \exp\left\{-\int_0^l \sigma_c\,[z_i + (\omega_i, n)\,t]\,\mathrm{d}t\right\},$$

$\sigma_c = \Sigma(z) - \sigma(z) - i(p, w_\perp)$.

The random estimate ξ of the sought value of $T(z, p, w)$ along the given trajectory can be estimated as $M\,\xi(z, p, w) = T(z, p, w)$,

$$\xi(z, p, w) = \sum_{i=0}^{N} \frac{1}{|(w_i, n)|}\, w_i\, \varphi_i \times$$

$$\times \exp\left[-\frac{1}{(w, n)}\int_{z_1}^{z} \hat{\Sigma}(t, p, w_1)\,\mathrm{d}t\right]\Delta(z_i),$$

where

$$\varphi_i = \begin{cases} \dfrac{g[z_i, (w_i, w)]}{2\pi}, & 0 < z_i < H \\[2mm] \dfrac{\bar{A}}{\pi}\,|(w_i, n)|, & z_i = 0 \end{cases},$$

$$\Delta(z_i) = \begin{cases} 1, & z_i < z,\ w \in \Omega_+, \\ & z_i > z,\ w \in \Omega_-, \\ 0, & \text{in other cases.} \end{cases}$$

The algorithm allows one to estimate $T(z, p, w)$ by the same set of trajectories for an arbitrary number of the p parameters and an arbitrary sets of z and w values.

13. NUMERICAL EXPERIMENT

Below we present some computed results on the MTF mean values for the "atmosphere − underlying surface" system, as well as model photographs of the underlying surface taken from space and their filtered images (Fig. 4). The one-dimensional and three-dimensional models were used for the indicator field of the cloud substance. The former corresponds to some one-dimensional stochastic medium; it is simulated based on a vertical point flux of the intensity λ. The values of the field between points of the flux are assumed to be constant. The transition to the three-dimensional model is performed by similar randomization of the density over the horizontal layers of the vertical subdivision.

The atmosphere is illuminated by a vertical parallel flux of unit power. The observation is being conducted along the nadir direction. Mean optical depth of the system is equaled to unity. Upon a collision in the layer $0 < z < H$, a particle is scattered with the unit probability following the Rayleigh law; the reflection from the plane obeys the Lambert law. The extinction cross-section coincides with the scattering cross-section. The trajectories are simulated in a homogeneous medium with $\Sigma(r) = \sigma = 1$. The bias of an estimate is compensated for by the corresponding weighting factors. The statistical error of calculations does not exceed 5%.

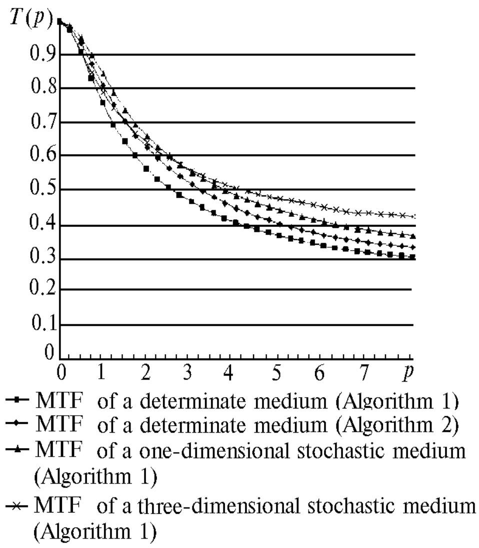
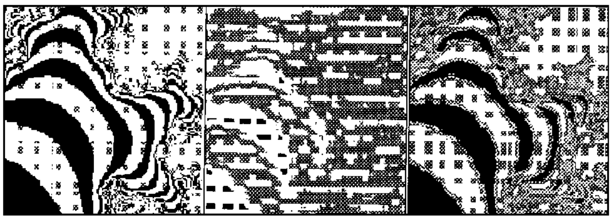


FIG. 4. MTF of stochastic scattering media.

The calculations for a determinate medium were performed by the method of boundary conditions (Algorithm 1) with regard for the multiple reflections from the underlying surface and by the algorithm of solving the complex "transfer equation" (Algorithm 2) with regard for the reflections from the homogeneous component of the albedo. The computed results for the media with sufficiently isotropic scattering and unit optical depth show that multiple reflections lower the values of the normalized MTF by 5 to 10%.

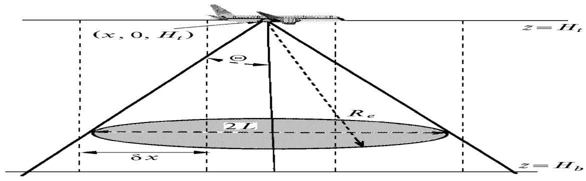
For stochastic models, the calculations were performed by the method of special boundary conditions with regard for multiple reflections from the underlying surface (Algorithm 1). The following model parameters were used in calculations: intensity of a point flux $\lambda = 1$ when constructing vertical and horizontal subdivision of the scattering layer; unconditional probability of the cloud existence $p = 0.5$; scattering cross-section for a cloud $\Sigma_1 = 1.4$; scattering cross section for the atmosphere $\Sigma_2 = 0.6$. The computed results show that the values of normalized MTF increase, on the average, when passing from the determinate model to a stochastic one-dimensional model, and then to the three-dimensional one.

The simulated "space photograph" was estimated by the method of boundary conditions for the determinate medium (Fig. 5) and for the stochastic one-dimensional medium (Fig. 6). The photographs are 256×256 matrices. Matrix elements are the values of brightness along a given direction at the corresponding quantization points (pixels) of the image.



a *b* *c*

FIG. 5. Initial image of the test object (the Mandelbrojt set) (a), "space photograph" (b), filtered image (c). The determinate medium.



a *b* *c*

FIG. 6. Initial image of the test object (the Mandelbrojt set) (a), space photograph (b), filtered image (c). The stochastic one-dimensional medium.

The filtering consisted in taking the two-dimensional Fourier transform of the quantization matrix, multiplying the Fourier transform obtained by the function inverse to MTF, and taking the inverse Fourier transform. The filtering gives good reconstructibility of the low-frequency pattern of an image and somewhat worse reconstructibility of its high-frequency counterpart. This is caused by cutting-off the image spectrum at quantization of the albedo distribution function.

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