SIMULATION OF THE SEA SURFACE ROUGHNESS AND STUDY OF ITS OPTICAL PROPERTIES BY THE MONTE CARLO METHOD

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A numerical model of the wind–driven sea waves is constructed. This model is used for studying the optical characteristics of the sea surface by the Monte Carlo method. It is shown that simulation of the field of heights of the irregularities of the sea surface makes it possible to determine more accurately the optical characteristics of the wind–driven waves with an account of the effects of radiation rereflection and shading by the surface elements.

1. The experimental data testifies to the fact that the statistical properties of the wind–driven sea waves can be described with high accuracy by the Gaussian randomly uniform field of heights of irregularities about the mean sea level.[1] The statistical characteristics of the wind–driven sea waves are determined by the "frequency" spectrum $S(\mu)$ and "angular" spectrum $Q(\mu, \theta)$. The following relations are considered as spectrum approximation (see, e.g., Refs. 1 and 2):

$$S(\mu)=\begin{cases} 6m_0(\mu_{max}/\mu)^5\,\mu^{-1}\exp\{-1.2[(\mu_{max}/\mu)^5-(\mu_{max}/\mu_1)^5]\}, \\ \qquad \mu\in[0,\mu_1]\,, \\ S(\mu_1)+(S(\mu_2)-S(\mu_1))\dfrac{\mu-\mu_1}{\mu_2-\mu_1},\ \mu\in[\mu_1,\mu_2], \\ 0.0078g^2\mu^{-5}\,,\ \mu\in[\mu_2,\mu_3] \end{cases} \quad (1)$$

$$\Theta(\mu,\theta) = 2\cos^2\theta/\pi,\ \theta\in[-\pi/2,\pi/2]\,. \quad (2)$$

Here $\mu_3 \approx 30\ s^{-1}$ is the maximum frequency of the spectrum of gravitational waves, μ_{max} is the frequency of maximum in the spectrum S, g is the acceleration due to gravity,

$$\tilde\mu_1 = 1.8\mu_{max}\,\tilde\mu^{-0.7}\,,\ \tilde\mu_2 = 2.0\mu_{max}\,\tilde\mu^{-0.7}\,,$$

$$\tilde\mu = \upsilon\mu_{max}/g\,,\ m_0 = 0.00127g^{-2}\upsilon^4\,\tilde\mu^{-3.19}\,,$$

ν is the wind velocity (m/s) at an altitude of 10 m above the sea level. The statistical properties of the wind–driven waves are determined by the wind velocity ν and frequency of the spectral maximum μ_{max} within the scope of model (1)–(2).

Let $w(x, y)$ be the field of heights of irregularities of the sea surface about the mean sea level (at certain moment) with spectral characteristics (1) and (2). Then the function $w(x, y)$ can be treated as a realization of the Gaussian randomly uniform field with spectral power density in polar coordinates ρ, θ:

$$\begin{aligned} &S_{\rho\theta}(\rho,\theta) = 2\cos^2\theta S_\rho(\rho)/\pi\,, \\ &S_\rho(\rho) = 0.5\,(g/\rho)^{1/2}S((g\rho)^{1/2})\,, \\ &\rho>0,\ \theta\in[-\pi/2,\pi/2]\,. \end{aligned} \quad (3)$$

The correlation function of the field $w(x, y)$ is
$Mw(0, 0)\,w(x, y) =$

$$= \int_0^\infty\int_0^\infty \cos(\lambda x+\nu y)\,S_{\rho\theta}(\rho(\lambda,\nu),\theta(\lambda,\nu))\,\rho^{-1}(\lambda,\nu)\,d\nu\,d\lambda\,,$$

where $\rho(\lambda, \nu) = (\lambda^2+\nu^2)^{1/2}$ and $\theta(\lambda,\nu) = \arg(\lambda + i\nu)$.

The wind–driven waves were simulated based on the spectrum randomization method.[3] Represented below is the expression for the approximate model $w^*(x, y)$ with spectral power density (3) which was employed in our calculations

$$w^*(x,y) = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\,[r_{ij}\cos(x\rho_i\cos\theta_j + y\rho_i\sin\theta_j + \varphi_{ij}) + $$
$$+ r'_{ij}\cos(x\rho_i\cos\theta_j - y\rho_i\sin\theta_j + \varphi'_{ij})]\,. \quad (4)$$

Here ρ_i are random variables with the probability density function proportional to S_ρ in the corresponding sets A_i, $A_i = [\rho^*(i-1)/(m-1),\ \rho^* i/(m-1)]$, $i = 1, \ldots, m-1$, $A_m = [\rho^*, \infty)$, θ_j are random variables distributed on the interval $B_j = [\pi(j-1)/(2n),\ \pi j/(2n))$ with the probability density function proportional to θ from Eq. (2), r_{ij} and r'_{ij} are the random variables obeying the Rayleigh distribution, and φ_{ij} and φ'_{ij} are uniformly distributed on the interval $[0, 2\pi]$, and

$$a_{ij}^2 = \int_{A_i}\int_{B_j} S_{\rho\theta}(\rho,\theta)\,d\theta\,d\rho\,.$$

The random variables ρ_i were simulated by the method of the inverse distribution function, and θ_j was simulated by the rejection method (with a linear majorant for $j > n/2$ and with a constant majorant for $j < n/2$, where n is even). The results of simulation of the sea surface roughness based on Eq. (4) are shown in Fig. 1.

The simulation algorithm is specified by the three parameters: m, n, and ρ^* which govern the accuracy of approximation. The field w^* has a required spectral power density (3) and is asymptotically Gaussian, as $\max(m, n) \to \infty$, $\rho^* \to \infty$. These conditions are also sufficient for weak convergence of w^* in space of continuously differentiable functions.[4]

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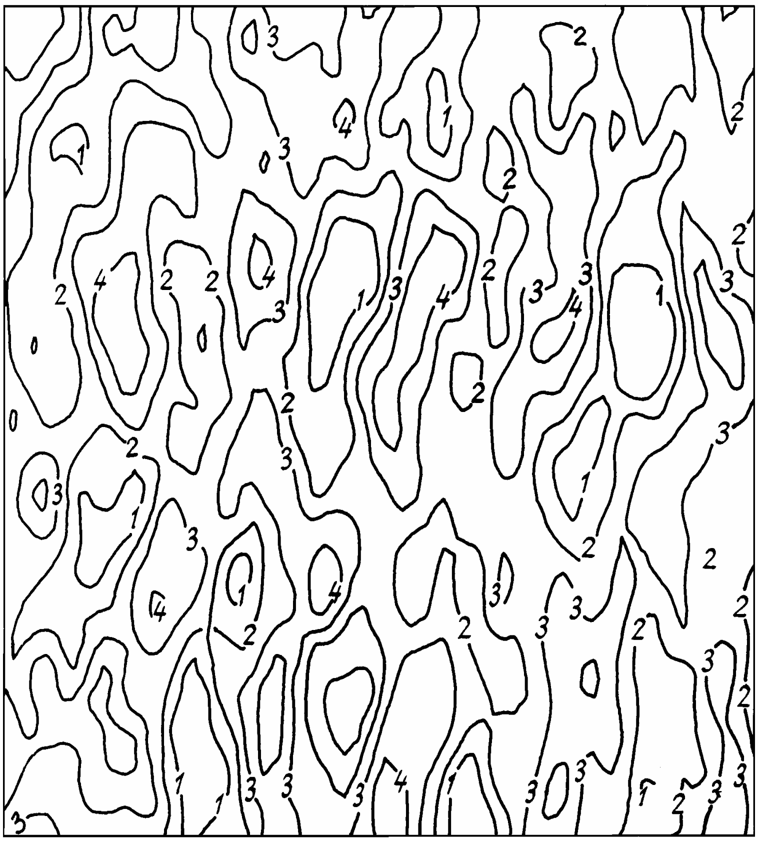

FIG. 1. An example of simulated topography of the sea surface roughness.

2. Below we are concerned with the method for calculating the shaded area of the sea surface, i.e., the part of the surface screened by waves from an incident flux of parallel rays. The estimate of the shaded area is important for solving the problems of remote sensing of the ocean and studying its optical properties.[5] Saunders[6] proposed the method for estimating the shaded area based on the model in which only the surface slope distribution was taken into account and the field of heights of the irregularities was not considered. The results of the theory of overshoots of the random processes were employed for estimating the shaded area in Ref. 7. The methods for approximate simulation of random fields were proposed in Ref. 8 to study the optical characteristics of the sea surface. This approach provides the basis for the method under consideration. An approximate numerical model of the wind–driven waves is first constructed and the shaded area for it is then calculated.

The algorithm for estimating the shaded area is as follows. We choose the arbitrary point $(x_0, y_0, z_0 = w^*(x_0, y_0))$ on the surface $\omega^*(x, y)$. A ray in the given direction is traced from this point, i.e., a sequence of points (x_k, y_k, z_k) is calculated along the ray trajectory with close spacing

$$h_k = [(x_k - x_{k-1})^2 + (y_k - y_{k-1})^2 + (z_k - z_{k-1})^2]^{1/2} .$$

If the ray falls under the surface, i.e., $z_k < w^*(x_k, y_k)$ then the point (x_0, y_0, z_0) is considered to be shaded. The area (probability) of shading is estimated by averaging the number of points being in the shadow over realizations of the surface w^*.

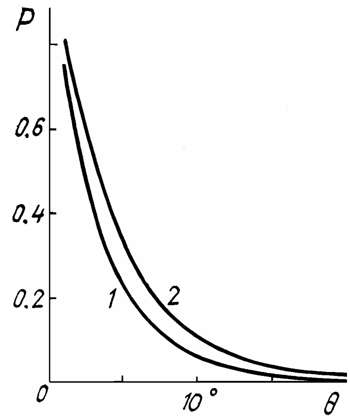

FIG. 2. A plot of the shaded area vs the source elevation angle for $\mu_{max} = 1.0\ s^{-1}$ *at* $\theta = 0°$*: 1)* $\nu = 2$ m/s and *2)* $\nu = 15$ m/s.

One of the basic computational problem is the adjustment of the model, i.e., the choice of parameters the m, n, and ρ^* which affect, on the one hand, the closeness of the approximated model $w^*(x, y)$ to the required field and, on the other hand, the complexity of the algorithm. The choice of the parameters is considered to be satisfactory if their subsequent refinement has no effect on the final result within the limits of admissible error. The analysis of the results of calculation of the shaded area showed that a satisfactory choice of the parameters is $m = 11$, $n = 2$, and $\rho^* = 4\rho_{max}$, where $\rho_{max} = \mu_{max}/g$.

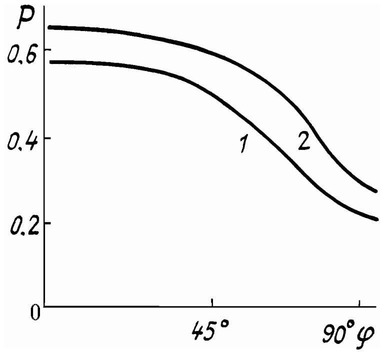

FIG. 3. A plot of the shaded area vs the angle between the wind direction and the azimuthal direction of incident rays for $\mu_{max} = 1.0\ s^{-1}$ *at* $\varphi = 0°$*: 1)* $\nu = 2$ m/s *and 2)* $\nu = 15$ m/s.

The shaded area P of the rough sea surface depends on four parameters, i.e., $P = P(\nu, \mu_{max}, \varphi, \theta)$, where ν and μ_{max} define the statistical characteristics of the wind–driven waves, φ is the angle between the wind direction and the azimuthal direction of incident rays, and θ is the elevation angle of the radiation source. Shown in Figs. 2 and 3 are the calculated φ and θ dependences of the quantity P.

Table I shows the shading probabilities obtained by simulating the sea surface and employing the method from Ref. 6. The results agree well for the wind velocity 2 m/s, while the wind velocity as great as 15 m/s the numerical simulation of the sea surface gives lower shading probabilities than the method described in Ref. 6.

TABLE I. Probabilities of shading of the sea surface (in %) at $\varphi = 0°$.

ν, m/s	θ, deg							
	1	2	4	6	8	10	14	20
2	75	57	30	16	11	7	3	1
	76	57	29	14	6	4	0	0
15	81	66	43	27	17	10	4	2
	88	78	59	44	32	23	10	2

Note: The upper row comprises the values obtained by simulation of the sea surface using models (1)–(4) for $\mu_{max} = 1.0\ s^{-1}$, the lower one – by the method from Ref. 6

3. In this item the sea surface is simulated for calculating the characteristics of the reflected optical radiation field taking into account radiation shading and rereflection by the surface elements. In some cases the calculated results can significantly differ from those obtained based on the "facet" model[9,10] in which the field of heights of the sea roughness is neglected.

Let us assume that the radiation interaction follows the laws of geometric optics (see Ref. 9, p. 299). We introduce the following notation: $r_\perp = (x, y)$ is the projection of the radial distance $r = (x, y, z)$ onto a horizontal plane, $z = w(r_\perp)$ is the equation describing the sea surface roughness, $k = (0, 0, 1)$, and Ω_+ and Ω_- are unit hemispheres of directions with $\langle\omega, k\rangle \in [1, 0]$ and $\langle\omega, k\rangle \in [-1, 0)$, respectively. When the ray propagating in the direction ω' interacts with the surface at the point $r = (r_\perp, w(r_\perp))$ it undergoes, with the probability $R(\omega', s)$, a specular reflection in the direction $\omega = \omega' - 2\langle\omega', s\rangle s$ and, with the probability $1 - R(\omega', s)$, refraction. Here s is the external normal to the surface at the point r and $R(\omega, s)$ is the Fresnel reflectance depending on the local angle of incidence:[8]

$$R(\omega, S) = (|A| - B)^2 (A^2B^2 + C^2)(|A| + B)^{-2} (|A|B + C)^{-2},$$

where $A = \langle\omega, s\rangle$, $B = (n^2 - 1 + A^2)^{-1/2}$, $C = 1 - A^2$, n is the refractive index of water with respect to air (in calculations $n = 1.33$). The normal to the surface is calculated from the formulas

$$s_x = -w_x s_z,\ s_y = -w_y s_z,\ s_z = (1 + w_x^2 + w_y^2)^{-1/2},$$

where w_x and w_y are the derivatives of the function $w(x, y)$ with respect to x and y.

Let us consider the layer $-H \le z \le H$ of the three–dimensional space and let $z = w(x, y)$ be such a realization of the wind–driven waves that $|w(x, y)| < H$. The boundary of the layer is illuminated by a parallel flux of photons in the direction $\omega_0 \in \Omega$. The intensity of radiation $I(r, \omega; w)$ at the point r in the direction ω with the assigned realization of the surface $w(x, y)$ satisfies the well–known equation given in Ref. 8. In what follows only the field of reflected radiation is considered, i.e., absorption and scattering in the medium are neglected. By the term reflected radiation we understand the radiation with the intensity $\langle I(r, \omega; w)\rangle = I(\omega)$, where $\omega \in \Omega_+$, $z = H$, and averaging is performed over realizations of the surface. Because of the assumed statistical uniformity of the sea surface roughness, its dependence on $r = (x, y, z)$ for $z = H$ after averaging vanishes. The problem is to estimate the integral characteristics of the reflected radiation

$$J = \int_{\Omega_1} I(\omega)\, d\omega,\ \Omega_1 \subset \Omega_+.$$

Let us consider the trajectory $\{(r_0, \omega_0), (r_1, \omega_1), ..., (r_m, \omega_m)\}$, where r_i is the point of the ith intersection of the ray with the surface $w(r_\perp)$ and ω_i is the unit vector specifying the direction of the reflected ray after the ith collision. The sought–after brightness is estimated by averaging of the random variable

$$\xi(\Omega_1) = \begin{cases} \prod_{i=1}^{m} R(\omega_i, s_i), & \text{for } \omega_m \in \Omega_1, \\ 0, & \text{for } \omega_m \notin \Omega_1. \end{cases}$$

over the trajectories and realizations of the random field.

In tracing a trajectory, each subsequent point is found from the condition

$$r_i = r_{i-1} + \Delta t \omega_{i-1} = (r_{\perp i}, w(r_{\perp i})). \quad (5)$$

To determine the values $r_{\perp i}$, the relation

$$\langle k, r_{i-1} + \Delta t\omega_{i-1}\rangle < w((r_{i-1} + \Delta t\omega_{i-1})_\perp)$$

was checked from the point r_{i-1} in the direction ω_{i-1} with the step Δt. After having found the point r'_{i-1} at which $z'_{i-1} < w(r'_{\perp i-1})$, we obtain the approximate solution of Eq. (1) with the prescribed accuracy using the halving method. The vector of the normal s is then calculated and new direction is recalculated based on the aforementioned formulas. Thus the entire path of the ray is traced before exiting the level H.

To study the field of optical radiation reflected from the sea surface, in addition to the field of heights of irregularities $w(x, y)$ it is necessary to simulate its derivatives $w_x(x, y)$ and $w_y(x, y)$. Since the variables $w(x, y)$ and $(w_x(x, y), w_y(x, y))$ are independent for the Gaussian uniform field we assume

$$w_x(x, y) = \xi \ , \ w_y(x, y) = \eta \ , \qquad (6)$$

where (ξ, η) are the Gaussian vectors with zero mean and, in accordance with Ref. 11, we have

$$D\xi = 0.003 + 0.00192\,\upsilon \ , \ D\eta = 0.00316\,\upsilon \ , \ M\xi\eta = 0 \ , \qquad (7)$$

provided that the wind direction is coaxial with the direction of the OX axis.

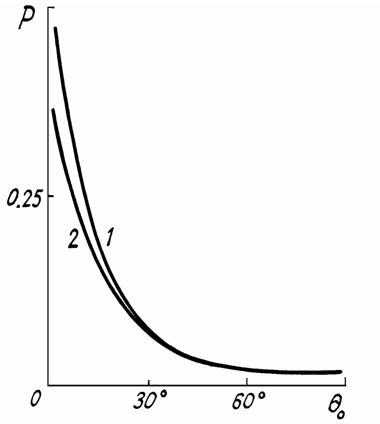

FIG. 4. A plot of a portion of radiation reflected from the sea surface vs the source elevation angle: 1) facet model and 2) model (1)–(4) and (6)–(7).

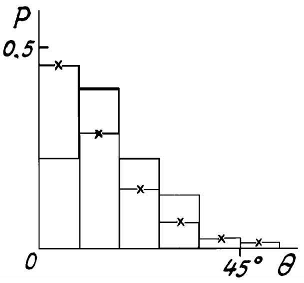

FIG. 5. A histogram of the phase function of reflection from the sea surface at the source elevation angle $\theta_0 = 10°$: 1) facet model and 2) model (1)–(4) and (6)–(7).

Let θ_0 be the angle of the source elevation above the horizon, φ_0 be the angle between the azimuthal direction of incident rays and the direction of wind, and θ be the angle of elevation of the reflected ray above the horizon. As can be seen from the intercomparison of the model (1)–(4), (6), and (7) and the facet model (which corresponds to $w(x, y) \equiv 0$), at small angles θ_0 the facet model noticeably overestimates a portion of the radiation reflected from the sea surface (Fig. 4) as well as strongly distorts the phase function of reflection (Fig. 5). The average number of rereflections is shown in Fig. 6 as a function of θ_0. The results depicted in Figs. 4–6 were obtained based on the facet model (6)–(7) and the model (1)–(4) and (6)–(7) at $\varphi_0 = 0°$ for $v = 15$ m/s and the parameters $\mu_{max} = 1.0$ s^{-1}, $m = 11$, $n = 2$, and $\rho^* = 4\mu_{max}/g$. Though the simulation of the field of heights of the sea surface roughness makes the model more complicated, it makes it possibile to refine the optical characteristics of the wind–driven waves taking into account the radiation rereflection and shading by the elements of the surface.

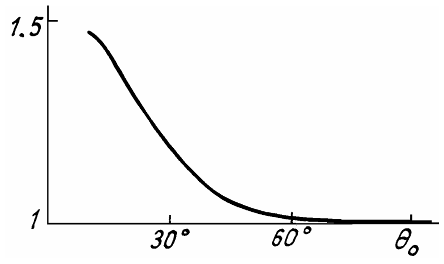

FIG. 6. The mean number of reflections of the ray from the sea surface as a function of the source elevation angle (model (1)–(4) and (6)–(7)).

In conclusion it should be noted that the proposed approach to numerical simulation of the wind–driven waves can be employed for solving various optical problems. In particular, the model of the sea surface with an account of variations in time

$$w_n(x, t) = \sum_{j=1}^{n} a_j r_j \cos(\langle\lambda_j\, \mathbf{x}\rangle + \mu_j t + \varphi_j) \ ,$$

where μ_j are related to λ_j in terms of the dispersion relation, is considered to be promising for solving the problems in laser sensing of the sea surface by numerical simulation.

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