# DETERMINATION OF REFRACTIVE-INDEX FLUCTUATION SPECTRA FROM OBSERVATIONS OF STELLAR OCCULTATION BY THE EARTH'S ATMOSPHERE

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*Received December 12, 1988*

*It is shown that the problem of calculating the spectra of fluctuations of the refractive index in the atmosphere from scintillation measurements during stellar occultation observations from space reduces to the tomography problem in a plane. The case of anisotropic inhomogeneities in the atmosphere is studied in detail. Practical methods for obtaining vertical one-dimensional spectra of the refractive index are investigated. Examples of calculations of spectra from scintillation observations performed on the "Salyut-7" arbiter are presented.*

Explicit expressions for the two-dimensional spectrum $F_I$ of the scintillation of stars observed from a space station (SS) through the Earth's atmosphere are derived in Ref. 1. In the calculation of $F_I$ it was assumed that the relative fluctuations $\nu$ of the refractive index $N$ in the atmosphere

$$\nu=(N-\langle N\rangle)/\langle N\rangle,$$

where $\langle N\rangle$ indicates statistical averaging, are statistically locally isotropic over a sphere and statistically locally homogeneous in altitude. The spatial spectrum $\Phi_\nu$ was assumed to be given for the random field $\nu$. In this paper the practical possibilities for solving the inverse problem of determining the spectral characteristics of the random field from scintillation observations from a space station under the same assumptions regarding the statistical properties of this field are discussed. The problem of determining the spectra of $\nu$ from scintillation observations for locally isotropic inhomogeneities is partially investigated in Ref. 2. For this reason special attention is devoted here to the case of strongly anisotropic, two-dimensional inhomogeneities which, as shown, for example, by the observations, by Grechko et al.[3,4], are typical for the stratosphere.

In scintillation observations from a space station, the light flux $I$ along the trajectory of motion is recorded in one form or another. The velocity of the station is so large that the observed field $I$ can be regarded as "frozen" in time, and the obtained record permits determining experimentally the one-dimensional spatial spectrum of the intensity fluctuations $V_I$. We shall show that $V_I$ and $F_I$ are related by the Radon transform[5] and that the problem of determining $F_I$ from measurements of $V_I$ on the set of trajectories $S$ reduces to solving the tomography problem in a plane[6]. As done in Ref. 7 we shall assume that the effect of the atmosphere is equivalent to a phase screen placed in a plane perpendicular to the true direction to the star and passing through the center of the earth. In scintillation observations, the movement of the observer across is important, since the correlation scale of the fluctuations of the light flux $I$ is significantly larger along the beam than in a plane perpendicular to the beam. Confining our analysis to small angles of refraction, which is almost always the case for observations of stars at visible wavelengths through planetary atmospheres, we shall choose the observation plane parallel to the phase screen. The space station lies in the observation plane at time $t_0$, corresponding to the center of some observation session. In what follows we shall neglect the small deviation (equal to the refraction angle) of the normal, to the observation plane away from the beam at the observation point.

We shall denote by $L$ the distance from the phase screen to the space station at the time $t_0$. We assume further that the statistical average value of the refractive index $\langle N\rangle=\bar{N}(h)$ depends only on the altitude $h$ above the Earth surface which is assume to be a sphere with radius $a_e$. If the fluctuations of the refraction angle are neglected, then it may be assumed that the ray arriving at time $t_0$ at the space station and refracted in the atmosphere lies in the plane passing through the center of the earth. This plane is perpendicular to the phase screen, and it is natural to call it the ray plane. We shall denote by $\alpha$ the angle between the ray plane and the component of the velocity of the space station perpendicular to the ray at the time $t_0$. The one-dimensional spectrum of the fluctuations of the light flux $V_I(æ_s;\ \alpha)$, which can be determined from measurements of $I$ on some segment of the trajectory during the observation session, is related with the two-dimensional spectrum $F_I$, calculated in Ref. 1, of the fluctuations of $I$ observation plane by the following relation:

$$V_I(\kappa_s,\alpha)=\int_{-\infty}^{\infty}F_I\left[\frac{\kappa_s}{\cos\alpha}-\kappa'\sin\alpha,\ \kappa'\cos\alpha\right]d\kappa'. \tag{1}$$

The relation between the spatial frequency $\kappa_s$ and the frequency $f$, the argument of the frequency spectrum $W_I(f)$ of the scintillations calculated from measurements is established by the following equation:

$$\kappa_s=2\pi f/v_\perp, \tag{2}$$

where $V_I$ is the velocity of the space station perpendicular to the arriving ray for the time $t_0$.

The formula (1) is obtained, if the correlation function of the fluctuations of the light flux in the observation plane $B_I(\Delta s;\ \alpha)$ along the projection $S$ of the trajectory of the space station on this plane is written in the spectral representation with the spectrum $F_I(\text{æ}_1,\ \text{æ}_2)$ and $V_I(\text{æ}_s;\ \alpha)$ is then calculated as the Fourier transform of $B_I(\Delta s;\ \alpha)$ with respect to $\Delta S$. Of course, Eq. (1) was derived under the assumption that on the measurement segment the projection of the trajectory can be assumed to be rectilinear and the curvilinear nature of the coordinates $\text{æ}_1$, $\text{æ}_2$ introduced into Ref. 1 can be neglected in the calculations. These assumptions are based on the fact that the height of the atmosphere, characterized by the parameter $H_0/a_e$, where $H\approx 6$–8 km is the scale height of the atmosphere, is small. It is not difficult to prove that the integral $(I)$ is the integral of $F_I$ along a straight line in the $\text{æ}_1$, $\text{æ}_2$ plane. The straight line cuts off the segments $\text{æ}_s/\cos\alpha$ and $\text{æ}_s/\sin\alpha$ on the axes $O\text{æ}_1$ and $O\text{æ}_2$, respectively, where $\text{æ}_s$ is the distance from the straight line to the origin of coordinates. Thus $V_I$ is a direct Radon transform of $F_I$[5,6]. Since an explicit formula for the Radon transform exists, using it and the measurements of $V_I(\text{æ}_s;\ \alpha)$, for all $0\le\alpha\le\pi/2$, $F_I(\text{æ}_1,\ \text{æ}_2)$ can in principle be calculated. If, further, the expressions[1]

$$F_I(\kappa_1,\kappa_2)=4k^2q^{-1}\sin^2\left[\frac{L}{2k}\left(\frac{\kappa_1^2}{q}+\kappa_2^2\right)\right]F_\Psi\left(\frac{\kappa_1}{q},\ \kappa_2\right); \tag{3}$$

$$F_\Psi(\kappa,\kappa_2)=\frac{2\pi a_1H_0\bar N^2}{(1+\kappa_1^2H_0^2)^{1/2}}\int_{-\infty}^{\infty}\Phi_\nu\left(\kappa_1,\ (\kappa_2^2+\kappa_3^2)^{1/2}\right)\times$$

$$\times\exp\left(-\frac{a_1H_0\kappa_3^2}{1+\kappa_1^2H_0^2}\right)d\kappa_3, \tag{4}$$

relating $F$ with the spectrum $\Phi_v$, where $q$ is the average refraction attenuation are employed, the method for solving the inverse problem posed above becomes obvious. But the practical implementation of this method based on measurements of $V_I(\text{æ}_s;\ \alpha)$ followed by the inverse Radon transformation can encounter insurmountable obstacles, owing to the combination of two factors: the mathematically improper nature of the inverse Radon transform and the extreme difficulty of obtaining the data $V_I(\text{æ}_s;\ \alpha)$ for different a when performing measurements on the space station. For this reason the general approach presented above must be regarded primarily as a demonstration of the possibility of solving the inverse problem.

The use of a priori information about the character of the fluctuations of the refractive index in the average atmosphere greatly simplifies the evaluation of its spectral characteristics. It was noted above that the observations[3,4] definitely indicate that the inhomogeneities in the stratosphere are strongly anisotropic. Moreover, observations have shown that the scintillation spectrum rapidly decays at frequencies considerably lower than $(K/L)^{1/2}$. Consequently, in solving the inverse problem the geometric optics approximation can be taken as the starting point. This last point implies that the sine in (2) can be replaced by its argument.

We shall study the case when the observed star lies in the plane of the orbit of the space station. This corresponds to $\alpha = 0$. In the geometric optics approximation it is not difficult to obtain the following expression for $V_I(\text{æ}_1)$ by substituting Eq. (3) into Eq. (1) and performing the integration over $\text{æ}_2$ and $\text{æ}_3$:

$$V_I(\kappa_1)=\frac{2\pi L^2\bar\Psi^2}{\kappa_1H_0}\int_0^{\infty}\Phi_\nu\left(\frac{\kappa_1}{q},\kappa\right)\exp\left(-\frac{\kappa^2a_1q^2}{2\kappa_1^2H_0}\right)\times$$

$$\times\left\{\left(\frac{\kappa_1^2}{q}+\frac{\kappa^2}{2}\right)^2I_0\left(\frac{\kappa^2a_1q^2}{2\kappa_1^2H_0}\right)+\left(\frac{\kappa_1^2}{q}+\frac{\kappa^2}{2}\right)\kappa^2I_1\left(\frac{\kappa^2a_1q^2}{2\kappa_1^2H_0}\right)+\right.$$

$$\left.+\frac{\kappa^4}{8}\left[I_0\left(\frac{\kappa^2a_1q^2}{2\kappa_1^2H_0}\right)-I_2\left(\frac{\kappa^2a_1q^2}{2\kappa_1^2H_0}\right)\right]\right\}\kappa d\kappa, \tag{5}$$

where $\bar\Psi$ is the average value of the eikonal $\Psi=(2\pi a_eH_0)^{1/2}\bar N$. In $(\xi)$ is a Bessel function with imaginary argument. In Eq. (5) it is assumed that $\text{æ}_1H_0\gg 1$. In addition, the spectrum $\Phi_v(\text{æ}_1,\ \text{æ}_2)$ decays quite rapidly for large æ, i.e., there exists an $\text{æ}_m$ such that for $\text{æ}>\text{æ}_m$

$$\kappa^5\Phi_\nu(\kappa_1,\kappa)\to 0,\quad \kappa_m\ll(k/L)^{1/2}. \tag{6}$$

Generally speaking, the specific value of $\text{æ}_m$ can depend on $\text{æ}_1$. The last inequality in Eq. (6) is one condition for the applicability of geometrical optics for calculating the spectra of fluctuations of the light flux $I$. For the frequency range

$$(k/L)^{1/2}\gg\kappa_1\gg\kappa_m \tag{7}$$

the expression (5) for $V_I(æ_1)$ simplifies:

$$V_I(\kappa_1)=\frac{2\pi\kappa_1^3\bar{\Psi}^2L^2}{q^2H_0}\times$$

$$\times\int_0^\infty \Phi_\nu\left(\frac{\kappa_1}{q},\kappa\right)\exp\left(-\frac{\kappa^2a_1q^2}{2\kappa_1^2H_0}\right)I_0\left(\frac{\kappa^2a_1q^2}{2\kappa_1^2H_0}\right)\kappa d\kappa. \tag{8}$$

For the further analysis it is best to introduce the new integration variable $æ' = (qæ/æ_1)(a_1/2H_0)^{1/2}$.

$$V_I(\kappa_1)=\frac{4\pi\kappa_1^5\bar{\Psi}^2L^2}{q^4a_1}\times$$

$$\times\int_0^\infty \Phi_\nu\left(\frac{\kappa_1}{q},\ \frac{\kappa_1}{q}\left(\frac{2H_0}{a_1}\right)^{1/2}\kappa'\right)\exp(-\kappa'^2)I_0(\kappa'^2)\kappa'd\kappa'. \tag{9}$$

Using the well-known properties of the function $I_0(\xi)$ $I_0(\xi) = 1 + 0(\xi)$ at $\xi \ll 1$;

$$\xi\exp(-\xi^2)I_0(\xi^2) \to (2\pi)^{-1/2} \text{ at } \xi \to \infty, \tag{10}$$

and (6) the following expression can be obtained from (9) in the frequency range Eq. (7):

$$V_I(\kappa_1)=\frac{\kappa_1^3\bar{\Psi}^2L^2}{q^2H_0}V_\nu\left(\frac{\kappa_1}{q}\right), \tag{11}$$

where

$$V_\nu(\kappa_1)=2\pi\int_0^\infty \Phi_\nu(\kappa_1,\kappa)\kappa d\kappa. \tag{12}$$

Under the above assumptions about the statistical structure of the random field ν Eq. (12) is an expression for the vertical one-dimensional spectrum $V_\nu(æ_1)$ in terms of $\Phi_\nu(æ_1, æ_2, æ_3)$, the three-dimensional spectrum of this field. The physical meaning of the vertical one-dimensional spectrum $V_\nu$ follows from the fact that it can be determined from measurements of ν with the help of a sounding balloon rising or falling in the atmosphere. Using Eqs. (11) and (12) $V_\nu$ can be determined from scintillation measurements performed on the space station.

For the example of calculations of the scintillation frequency spectra, obtained in Ref. 8, we shall calculate $V_\nu(æ_1)$ under the assumption that the inhomogeneities of the refractive index for vertical sizes ranging from 10 m to 1 km are strongly anisotropic.

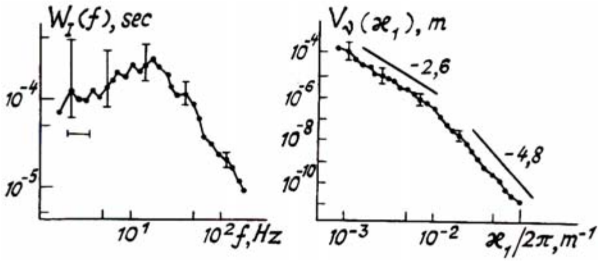


*FIG. 1. The frequency spectrum $W_I(f)$ of scintillations and the spatial spectrum $V_\nu(æ_1)$ of the relative fluctuations of the refractive index. The horizontal bar is the spectral window; the vertical bars indicate the 90% confidence interval for the determination of the spectral density.*

Figure 1 shows the scintillation frequency spectrum $W_I(f)$ obtained from observations of the star Arcturus during a session on September 11, 1985. The spectrum $W_I(f)$ was calculated from a two-second segment of the light-flux record with the average altitude of the perigee of the ray equal to 33 km. At this altitude the scintillations are much stronger than the shot noise of the photodetector and at the same time their relative dispersion is small: the scintillation index equals 13%. As found in Ref. 8 the effect of chromatism at this altitude can be neglected down to the smallest scales for which the spectrum was measured. The direction to the star makes an angle 22° with the plane of the orbit at the time of the observation. In order to transfer from the frequency $f$ to the spatial frequency $æ_1$ appearing in the formulae (11) and (12) it was assumed that the inhomogeneities of the refractive index are strongly extended along the earth's surface. This made it possible to neglect, under the conditions of the observations performed the displacement of the ray arriving at the observer along the earth's surface and to use instead of the general formula (2), the simplified formula $æ_1 = 2\pi f/v_v$, where $v_v$ is the projection of the velocity of the space platform on the vertical- the straight line formed by the intersection of the observation plane and the ray plane. Since the frequency spectrum was calculated as a Fourier transform over positive frequencies we obtain the formula:

$$V_I(\kappa_1)=(v_v/4\pi)W_I(f) \tag{13}$$

Figure 1 shows, together with the spectrum $W_I(f)$, one-dimensional spatial spectrum $V_\nu(æ_1)$ of the relative fluctuations of the refractive index calculated from the spectrum $W_I(f)$ using Eqs. (11) and (12). In the spectrum obtained two sections with different rates of decay of the spectral density $V_\nu$ as the spatial frequency $æ_1$ increases can be separated. As low frequencies $0.8\times10^{-3} \le æ_1/2\pi \le 0.8\times10^{-2}$ m$^{-1}$ the one-dimensional spectrum $V_\nu(æ_1)$ is proportional to $æ_1^{-2,6}$, while at higher frequencies $0.2\times10^{-2} \le æ_1/2\pi \le 1.0\times10^{-1}$ m$^{-1}$

it is proportional to $æ_1^{-4,8}$. The characteristic frequency $(æ_1)_0/2\pi \approx 10^{-2}m^{-1}$ is the boundary between the two sections with different rates of decay of the spectral density $V_v$. At an altitude of 33 km, to which the reconstructed spectrum refers, the atmosphere has stable thermal stratification, and it was suggested in Ref. 8 that $2\pi/(æ_1)_0$ corresponds to the Bolgiano-Obukhov scale[10,11].

At optical wavelengths the refractive index $N$ is proportional to the ratio of the pressure to the air temperature. Based on the assumption that small-scale relative fluctuations of the atmospheric pressure are considerably weaker than the small-scale relative fluctuations of the temperature the reconstructed spectrum $V_v(æ_1)$ can also be regarded as a one-dimensional spectrum of the relative temperature fluctuations in the stratosphere. In Ref. 12 it was suggested, based on analysis of the scintillation record obtained at different geographic points, that the relative temperature fluctuations in the stratosphere not vary greatly with altitude and are virtually independent of the geographic coordinates. Consequently, the spectrum $V_v(æ_1)$ shown in Fig. 1 is apparently quite typical for the earth's stratosphere. It should also be noted that the scintillation spectra $V_v(æ_1)$ reconstructed from the observations were averaged over a large space, and therefore the spot structure characteristic of turbulence in stably stratified media is not manifested in them.

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