ADAPTIVE IMAGE CORRECTION UNDER ANISOPLANATISM CONDITIONS FOR A MODEL OF A STRATIFIED ATMOSPHERE

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We present a study of an adaptive imaging system efficiency when viewing an extended object through a layer of a turbulent medium. We have estimated a change in the size of the region where the influence of weak inhomogeneities can be compensated for when placing a corrector into the region of the layer image center. It is shown that the relative improvement in the system performance due to the displacement of the corrector depends on the layer thickness and its position. A modified version of the correction algorithm is proposed that enables extending the region of the system isoplanatism. Analysis has been made using series expansion of the wave front over Zernike polynomials.

1. INTRODUCTION

Anisoplanatism of an adaptive imaging system is one of the factors that restrict making corrections to the images of extended objects viewed through a turbulent medium. This happens because the distorting medium is a three-dimensional one. The effect manifests itself in the fact that one can correct only small portion of an object image, using one plane corrector, within the so called isoplanatism zone. The size of the isoplanatism region of a system depends on the distribution of optical inhomogeneities of the medium along the viewing path and on the corrector position. By positioning the corrector so that its plane coincides with the image plane where most intense fluctuations of the refractive index concentrate one can achieve extending of the isoplanatism region and improvement of the image quality.

Normally, analysis is made of the optical systems in which the corrector used is combined with the receiving aperture. In this case the size of a near-axis region of good quality image is affected by the general, integrated characteristic of turbulence and an improvement in the image quality can only be achieved by eliminating the large-scale aberrations of the wave front (see Ref. 1). If an adaptive imaging system allows the corrector to be moved, it is possible to improve the image by placing the corrector in an optimum position.

If, however, the turbulence is uniformly distributed along the viewing path no essential improvement of the image quality can be achieved by moving the corrector. Nevertheless, even in this case, one may try to improve the quality of imaging the peripheral points of the object while incompletely correcting the central part of the image. The method of tuning the corrector so that the latter can provide improving the image quality when the atmospheric inhomogeneities are uniformly distributed along the viewing path has been considered earlier.[2]

It is known from the experimental studies that vertical distribution of the atmospheric turbulence is nonuniform. Thus, according to Ref. 3 the optical inhomogeneities of the Earth's atmosphere normally localize in a few layers. In that case an adaptive imaging system equipped with several correctors could provide for a significant improvement in the quality of viewing. To achieve this goal, one have to place the correctors in the planes conjugate to those layers, each corrector being tuned in accordance with the character of optical inhomogeneities in the corresponding layer.[4] However, even that complicated system does not provide for a complete correction. The matter is that the atmospheric layers have a finite thickness, on the order of 100–200 m. For this reason the anisoplanatism effect, caused by three-dimensionality of a distorting medium, manifests itself. The magnitude of the residual error will depend on the layer thickness and a distance to it.

The model that is being considered in this paper involves a single distorting layer and a single corrector. The question we address here is the effectiveness of bringing the corrector into the plane conjugate with the central plane of the layer. We also estimate the size of the isoplanatism region as a function of the layer thickness an a distance to it.

In the first part of the paper we use the classical correction technique that uses the distortions of a wave coming from a reference source placed at the center of an object under study.[5] In the second part of the paper we are identifying the situations when a modification of the method that assumes recording of aberrations averaged over the entire region viewed may be useful.

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2. FORMULATION OF THE PROBLEM. RESIDUAL ERROR

The geometry of the problem is shown in Fig. 1. Between the object O viewed and the aperture L of an imaging system there is a distorting layer S of an inhomogeneous medium. The $2d$-thick layer is located on the optical path with the layer center being at the point z_s. The axis z looks from the plane O ($z = 0$) toward the receiving aperture plane L ($z = H$). The system is equipped with a wave-front corrector whose coordinate z_C may be varied. In an actual adaptive system the wave-front corrector is normally being located behind a photodetector aperture in the plane z_C. Ignoring the difficulties, that may arise due to the necessity to make transformations of the corrector coordinate $z'_C(z_C)$ and radius $R'_C(R_C)$ of its aperture we consider here the system in which a corrector of a fixed radius is placed in front of the receiving aperture of the system. We assume that the diffraction limitations are primarily due to the corrector size.

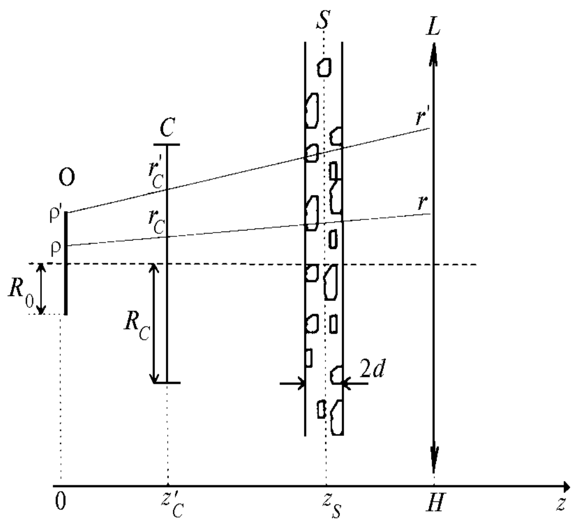

FIG. 1. Optical arrangement of an extended object imaging through the distorting layer of a turbulent medium. Here the object is designated as (O), wave-front corrector (C), layer (S), and the receiving aperture (L).

Let the optical inhomogeneities of the distorting layer of the turbulent atmosphere obey the Kolmogorov statistics and have the spatial spectrum

$$\Phi_n(k) = C_n^2\, k^{-11/3}. \tag{1}$$

Let us choose the structure constant such that turbulent layers of different thickness and coordinate have the same integral characteristic (Fried radius r_0)

$$\left(\frac{1}{r_0}\right)^{5/3} = \frac{2.92}{6.88}\, k_0^2\, C_n^2(d, z_S) \int\limits_{z_S-d}^{z_S+d} \left(1 - \frac{z}{H}\right)^{5/3} \mathrm{d}z = \mathrm{const}\,, \tag{2}$$

where k_0 is the wave number, z_S and $2d$ are the coordinate and thickness of the layer, H is the length of the optical path.

Let $\varphi(\mathbf{r}_C, \rho)$ be the phase distortions of wave coming from the point ρ on the object O, caused by the influence of the layer S and measured in the aperture plane L. Since our analysis is based on the geometrical optics principle, it can be said that $\varphi(\mathbf{r}_C, \rho)$ is the random phase shift introduced by the layer into the ray coming from the point ρ on the object, through the point $\mathbf{r}_C$ in the corrector plane, and measured in the aperture plane, that is, at the point $\mathbf{r}$ (see Fig. 1). Let $u(\mathbf{r}_C)$ be the correcting phase shift introduced by the corrector C. The residual rms distortions averaged over the corrector aperture we shall call the residual error

$$<J^2>(\rho) = \frac{1}{S_C} \int\limits_{S_C} <(\varphi(\mathbf{r}_C, \rho) - u(\mathbf{r}_C))^2> \mathrm{d}^2 r_C, \tag{3}$$

where S_C is the corrector's area.

The magnitude of the residual error (Eq. (3)) depends on the layer thickness and the corrector position with respect to the layer. The distribution of the error over the object is determined by the correction method used, that is by the way of choosing the function $u(\mathbf{r}_C)$. Having chosen the correction method one may find an optimal position of the corrector given layer parameters correspond to the residual error minimum (Eq. (3)).

3. CORRECTION USING A POINT SOURCE

Let us consider the correction method according to which the corrector is adjusted for work with the distortions of a wave coming from a point source located at the object center. In that case the central point of the object is viewed without any distortions

$$u(\mathbf{r}_C) = \varphi(\mathbf{r}_C, \rho). \tag{4}$$

The residual error is equal to zero at the object center and gradually increases as the distance from the center increases

$$<J^2>(\rho) = \frac{1}{S_C} \int\limits_{S_C} <(\varphi(\mathbf{r}_C, \rho) - \varphi(\mathbf{r}_C, 0))^2> \mathrm{d}^2 r_C\,. \tag{5}$$

Let the corrector of the imaging system considered be an ideal modal corrector that enables correction for the basic optical aberrations. That means that the phase distortions are represented as a series over Zernike polynomials $Z_i(\mathrm{r}_C)$ (Ref. 6)

$$\varphi(\mathbf{r}_C, \rho) = \sum_i a_i(\rho)\, Z_i(\mathrm{r}_C). \tag{6}$$

In this representation the rms error, (Eq. (3)), is a sum of errors due to individual aberrations

$$<J^2>(\rho) = \sum_i <J_i^2>(\rho). \tag{7}$$

As is shown below behaviors of different terms in Eq. (7), as functions of ρ, are different. To show the gain that can be achieved due to the corrector displacement, consider each term of the sum separately. The rms error, due to the *i*th aberration, may be represented as follows (Ref. 6):

$$<J_i^2>(\rho) = \frac{1}{S_C^2}\int\limits_{S_C}\int\limits_{S_C} <\{\varphi(\mathbf{r}_C,\rho)-\varphi(\mathbf{r}_C,0)\}\times$$

$$\times\{\varphi(\mathbf{r}'_C,\rho)-\varphi(\mathbf{r}'_C,0)\}> Z_i(\mathbf{r}_C)\,Z_i(\mathbf{r}_C)\,\mathrm{d}^2r_C\,\mathrm{d}^2r'_C. \qquad (8)$$

Equation (8) involves the correlation function $<\varphi(\mathbf{r}_C,\rho)\,\varphi(\mathbf{r}'_C,\rho')>$. Technically, it is the correlation function in the receiving aperture plane L between the phase shifts along the rays from points ρ and ρ′ on the object coming through the points $\mathbf{r}_C$ and $\mathbf{r}'_C$ in the corrector plane, respectively (see Fig. 1). Let us express the correlation function in terms of the spectrum of optical inhomogeneities of the turbulent layer (Eq. (1))

$$<\varphi(\mathbf{r}_C,\rho)\,\varphi(\mathbf{r}'_C,\rho')> = 2\pi\,k_0^2\int\limits_0^H\int\int\Phi_n(k)\times$$

$$\times\exp(\iota\,2\pi\mathbf{k}\mathbf{r}(z))\,\mathrm{d}^2k\,\mathrm{d}z\,, \qquad (9)$$

where

$$\mathbf{r}(z) = \left(1-\frac{z}{z_C}\right)(\rho-\rho') + \frac{z}{z_C}(\mathbf{r}_C-\mathbf{r}'_C). \qquad (10)$$

For the inhomogeneities with Kolmogorov spectrum and at $\mathbf{r}(z) = 0$ the integral (9) diverges. However, this does not affect the outcome, since the phase dispersion does not enter into the final expression. By applying the Fourier transform to the Zernike polynomials we obtain, for the residual error, that

$$<J_i^2>(\rho) = \left(\frac{HD_C}{z_C\cdot r_0}\right)^{5/3}\int\limits_0^H\tilde{C}_n^2(d,\,z_S,\,z)\times$$

$$\times\int\limits_0^\infty 3.8954(n+1)\,J_{n+1}^2(x)\,x^{-14/13}\left(\frac{z}{H}\right)^{5/3}\times$$

$$\times\left\{2-2\left\{J_0\left(x\,\frac{1-z/z_C}{z/z_C}\,\frac{\rho}{R_C}\right)\mp(-1)^m\times\right.\right.$$

$$\left.\left.\times J_{2m}\left(x\,\frac{1-z/z_C}{z/z_C}\,\frac{\rho}{R_C}\right)\cos(2m\varphi_\rho)\right\}\right\}\mathrm{d}x\,\frac{1}{H}\,\mathrm{d}z. \qquad (11)$$

Here $i = (n, m, l)$ is the number of a Zernike mode, $\rho = (\rho, \varphi_\rho)$ is the point on the object in polar coordinates, $J_n(\xi)$ is the Bessel function of the *n*th order, $D_C = 2R_C$ is the corrector diameter, z_C is the corrector coordinate. The function $\tilde{C}_n^2(d, z_S, z)$ provides for the fulfillment of the equality (2). The latter one shows that normalizing of the structure constant is being done in such a way that the Fried radius r_0 keeps the same for any position and thickness of the layer, that is

$$\tilde{C}_n^2(d,\,z_S,\,z) = \begin{cases} 0, & z<z_S-d,\ z>z_S+d, \\ \left(\displaystyle\int\limits_{\frac{z_S-d}{H}}^{\frac{z_S+d}{H}}(1-\eta)^{5/3}\mathrm{d}\eta\right)^{-1}, & z\in(z_S-d,\,z_S+d). \end{cases} \qquad (12)$$

This normalization allows the results to be represented in the general form while the residual error to be expressed in units of $[HD_C/(z_C\,r_0)]^{5/3}$. The quantity HD_C/z_C has the meaning of effective beam aperture in this corrector plane. By displacing the corrector towards an object one increases the effective aperture. Correspondingly, the phase dispersion and absolute value of the error increase.

The dependence of the residual error on the corrector position, for several points on the object that differ by the distance ρ from the center while being on the same line ($\varphi_\rho = 0$), is shown in Fig. 2. The curves have been plotted assuming the astigmatism aberrations ($n = 2$, $m = 2$, $l = 1$). The layer is at the path center, $z_S = 0.5H$, and has the thickness $2d = 0.02H$ (see Fig. 2*a*) and $2d = 0.1H$ (see Fig. 2*b*). Let us note that there is an optimum in the corrector position and it is near the layer center what gives minimum to the residual error. Therefore the minimum is sharper the thicker is this turbulent layer.

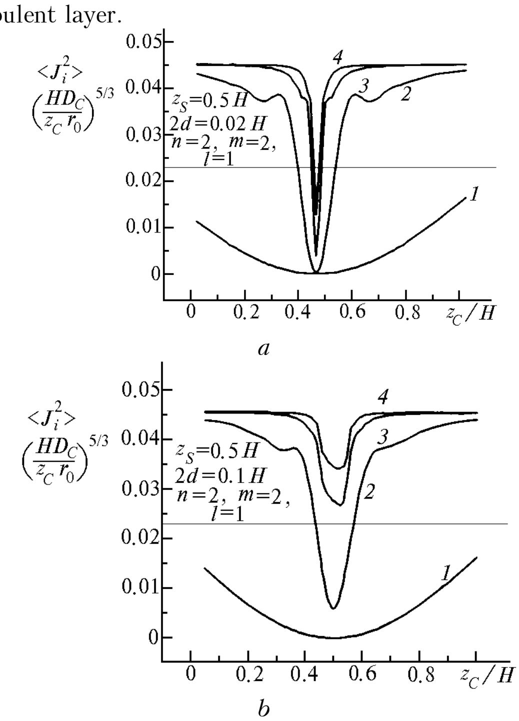

FIG. 2. Dependence of the residual error on the corrector position for different points on the object: $\rho = 0.5R_c$ *(1),* $\rho = 5R_c$ *(2),* $\rho = 20R_c$ *(3),* $\rho = 40R_c$ *(4). The layer is in the middle of the path,* $z_s = 0.5H$ *and has the thickness* $2d = 0.02H$ *(a) and* $2d = 0.1H$ *(b).*

The distribution of the error over the object image for a thin layer, $2d = 0.02H$, is shown in Fig. 3. For a comparison two positions of the corrector are considered, namely, the position at the center of the layer image (curves *1–3*) and in the receiving aperture plane (curves *4–6*). The distortion level with no any correction applied is depicted in figures with the horizontal lines. This level equals to the value of the Noll index, $<a_0^2>$, that is to the variance of the phase distortion expansion coefficients (see Ref. 6). The image quality improvement is achieved at the points where the residual error is below this level. This near-axis region is called the isoplanatism zone for the *i*th aberration. The residual error exceeds the Noll index value at the points outside this zone, i.e., additional aberrations are introduced into the distant points of an image when trying to correct its near-axis portion. One may assess how the isoplanatism zone increases when positioning the corrector at the center of a layer for different positions of the latter with respect to the object viewed. The isoplanatism zone becomes larger when moving the layer from the object and positioning the corrector at the image of this layer. The size of the isoplanatism zone, in units of R_C, (for the tilt and astigmatism aberrations) at different positions of the corrector regarding the image of a thin layer, $2d = 0.1$, are given in Table I.

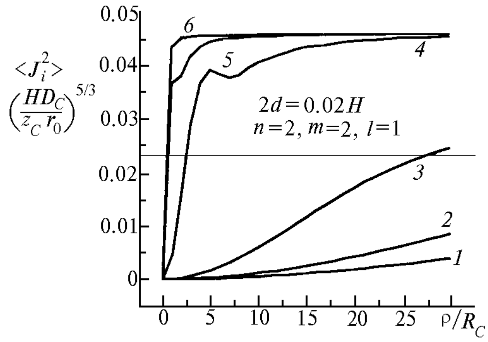

FIG. 3. Residual error as a function of the distance from a point on the object to its center. The corrector is positioned at the layer image center (1–3) and in the receiving aperture plane (4–6): $z_S = z_C = 0.8H$ (1); $z_S = z_C = 0.5H$ (2); $z_S = z_C = 0.2H$ (3); $z_S = 0.8H$, $z_C = H$ (4); $z_S = 0.5H$, $z_C = H$ (5); $z_S = 0.2H$, $z_C = H$ (6).

TABLE I. Size of the isoplanatism zone (in R_C units).

The layer position along the path	Tilt (n = 1, m = 1, l = 1)		Astigmatism (n = 2, m = 2, l = 1)	
	Position of the corrector			
	near the receiving aperture	at the layer image center	near the receiving aperture	at the layer image center
0.2H	2.1	95.1	0.16	5.8
0.4H	5.4	187.9	0.42	11.3
0.6H	12.2	281.3	0.95	16.9
0.8H	33.0	374.6	2.55	22.5

Let us now elucidate the gain that may be achieved owing to the displacement of the corrector into the layer image plane for aberrations of different types. To account for the mean contribution coming from a group of aberrations of same radial index n into the distortions, average the residual error over the central-symmetric circular area of the radius R_0.

Thus averaged residual error is shown in Fig. 4 as a function of the averaging area radius. The index n of the radial component is here a parameter. Every curve is normalized by the corresponding Noll-index. Therefore the unit level in the figure corresponds to the value of error without a correction. In this case a thin layer, $0.1H$ thick, is at the path's center. Two typical cases are considered: the phase corrector is in the receiving aperture plane (Fig. 4*a*) and at the center of a distorting layer image (Fig. 4*b*). It is seen that for all aberrations, except for the tilt group (n = 1), the correction zone in the former case is small and with its size approximately equal to one corrector radius. The zone slightly widens as the radial component index decreases. In the latter case the correction zone significantly increases as compared to the first case (Fig. 4*a*) and the dependence of its size on the aberration number becomes more noticeable.

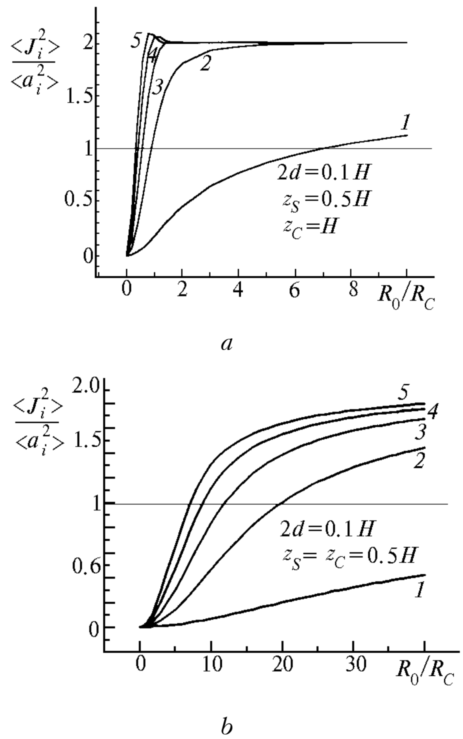

FIG. 4. Averaged, over circular area S_0, residual error as a function of the radius of averaging zone R_0 for groups of aberrations with different radial component index n = 1 (1), n = 2 (2), n = 3 (3), and n = 4 (4). The Layer thickness $2d = 0.1H$. The layer is in the middle of path, the corrector is in the receiving aperture plane (a) and at the center of the distorting layer image (b).

The effect of anisoplanatism becomes stronger as the layer thickness increases and the size of the correction zone decreases. The corresponding values of the isoplanatism zone size for three groups of aberrations, depending on the layer thickness $2d$, are given in Table II. The layer center and the corrector are in the middle of the path.

TABLE II. Izoplanatism zone for different layer thickness (in the units of R_C).

Layer thickness, $2d$	Tilts (n = 1)	Defocusing, astigmatism (n = 2)	Coma (n = 3)
$0.05H$	468.3	28.1	14.7
$0.1H$	234.6	14.7	7.4
$0.2H$	118.2	7.2	3.7
$0.3H$	79.8	4.9	2.6
$0.4H$	60.8	3.8	2.0
$0.5H$	49.7	3.1	1.7

One of the possible ways to widen the isoplanatism zone is to use a new correction algorithm. In the next section we consider a modified algorithm when the corrector is adjusted to the mean, over some area, distortion.

4. MODIFIED CORRECTION METHOD

To widen the isoplanatism zone in the case of a thick distorting layer, other method of the corrector adjustment may be proposed, for example, to the distortion averaged over some region. In this case the central point of the image is corrected only incompletely, but the area, where the residual error is smaller than without a correction, becomes wider.

Let us consider the central-symmetric area S_0 whose plane coincides with the object plane, with the center being on the optical axis (Fig. 1). Let the phase distortions of waves, coming from the points within this area, be known. The correction function $u(\mathbf{r}_C)$ is chosen in accordance with the averaged over the area S_0 distortion

$$u(\mathbf{r}_C) = \frac{1}{S_0} \int_{S_0} \varphi(\mathbf{r}_C, \rho) \mathrm{d}^2\rho. \tag{13}$$

The expression (13) is the correction phase in the modified correction method. At $S_0 \to 0$ this method reduces to the classical one that uses for correction the central point. By substituting Eq. (13) into the expression for the residual error, Eq. (3), and making transformations similar to those in Sect. 3 we obtain

$$<J_i^2>(\rho) = \left(\frac{HD_C}{z_C \cdot r_0}\right)^{5/3} \int_0^H \tilde{C}_n^2(d, z_S, z) \int_0^\infty 3.8954(n+1) \times$$

$$\times J_{n+1}^2(x)\, x^{-14/13} \left(\frac{z}{H}\right)^{5/3} \left\{1 - 2F\left(x\kappa \frac{R_0}{R_C}\right) \times \right.$$

$$\left. \times \left\{ J_0\left(x\kappa \frac{\rho}{R_C}\right) + (-1)^m\, l\, J_{2m}\left(x\kappa \frac{\rho}{R_C}\right) \cos(2m\varphi_\rho) \right\} + \right.$$

$$\left. + F^2\left(x\kappa \frac{R_0}{R_C}\right) \right\} \mathrm{d}x \frac{1}{H} \mathrm{d}z, \tag{14}$$

where

$$F\left(x\,\kappa \frac{R_0}{R_C}\right) = J_0\left(x\,\kappa \frac{R_0}{R_C}\right) + J_2\left(x\,\kappa \frac{R_0}{R_C}\right) = \frac{1 - z/z_C}{z/z_C},$$

where R_0 is the radius of region S_0 over which the averaging is performed.

Let us now elucidate how the size of the isoplanatism zone varies depending on the size of the zone of averaging used in the modified correction method. In so doing we use, as an example, the tilt aberrations (Fig. 5). The correction method is determined by the size of the averaging region R_0. The distorting layer with the thickness $2d = 0.4H$ is in the middle of the path. At $R_0 = 0$ (correction made using the central point) the isoplanatism zone is smaller than at $R_0 > 0$, but the central point image is completely corrected. The correction zone widens with increasing R_0 but at the expense of the central point image deterioration. At an infinite growth of the averaging region the curve approaches the Noll-index line, that means the value of phase distortions in the case when no any correction is used.

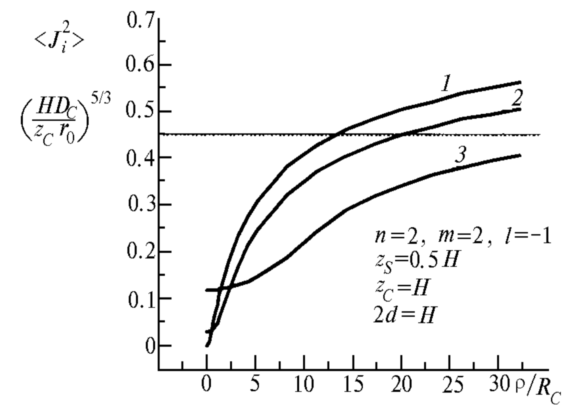

FIG. 5. Distribution of the error over the object when using the modified correction method at different size of the averaging zone. $R_0 = 0$ (1) (correction using the central point), $R_0 = 2R_C$ (2), and *$R_0 = 10R_C$ (3).*

As can be seen from Fig. 5 the region where the residual error is smaller than the Noll index exceeds in size the averaging regions S_0. It can be shown that this situation takes place for all aberration types (n, m, l), at any radius of the averaging region R_0. Consequently, the isoplanatism zone is larger than the region of the distortions averaging in the case of the summed error, Eq. (7) as well. That means that an object whose size does not exceed the region of averaging, S_0, is entirely inside the isoplanatism zone and, as a result the quality of imaging of all its points improves.

5. CONCLUSION

Thus, if the corrector is positioned at the plane where the system images the distorting layer center the isoplanatism zone will be wider as compared to that in the case when the corrector is placed in the receiving aperture plane. The increase of this zone size takes place for aberrations of any type. Therefore, an adaptive imaging system with a displaced corrector provides a compensation for a larger number of aberrations.

The size of the isoplanatism zone in the case of the corrector placed at the layer image center inversely decreases with the layer thickness increase. For thick layers the region of a high-quality image is small and only slightly depends on the corrector position.

To form the image of an extended object whose size exceeds the isoplanatism zone, a modified correction method is proposed. When using this approach the region with lower residual rms error, as compared to that when no correction is used, is always wider than the region over which the distortions are averaged.

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