Formation and evolution of fogs allowing for synoptic situation

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Numerical model of fog formation is constructed based on the equations of inflow of heat and water vapor in the turbulent atmosphere, which are written in an invariant form. Turbulent and vertical motions are of basic importance in the transport of heat and water vapor from the Earth's surface to the atmosphere and within it. Interchange parameters are determined based on the similarity theory, and the vertical velocity parameters are determined from the continuity equation. According to observations, most frequently the fogs are formed in the areas of decreased pressure [V.A. Lukin, *Analysis of Fog Formation in the South-West of Leningrad Region* (VVV AIU, Voronezh, 1996), issue 18, 411 pp.]. As the characteristics of a synoptic situation (baric field) we use: the geostrophic wind (uniquely connected with the cyclone depth), the wind velocity and its deviation from an isobar near the Earth's surface (at the level of 10 m), the dimension (radius) of a cyclone, the thermal stratification, the shape (roughness) of the Earth's surface, the temperature, and the relative humidity in the initial state. Effect of these characteristics on the thickness and water content of fog, possibility of its formation are estimated quantitatively.

Fogs and hazes relate to a number of the atmospheric phenomena with which the abrupt change of optical and thermal-physical properties, first of all, of the atmospheric near-ground layer is connected. Deterioration of the visibility range, from 10 to 1 km in a haze and below 1 km in a fog, affects the operation of all types of transport: aviation, automobile, river, and sea. For formation of a fog, the effective radiation of Earth's surface decreases abruptly, and, as a result, the probability of occurrence and the intensity of the light spring frosts acting destructively on the fruit and other agricultural crops during their florescence decrease.

In the investigations carried out up to now, the main attention has been given to the effect of radiation, turbulent, and advection inflows of heat on fogs. However, it is well known from observations that the formation of fogs is closely related to synoptic situation. The most essential distinction of the regions of decreased (cyclone, trough) and increased (anticyclone, crest) pressure is that the vertical velocity is different in them: in the former region, the upward motion of air is observed, as a rule, and the downward motion is observed in the latter one.

The primary goal of this paper is to construct a numerical (hydrodynamic) model of formation and evolution of fogs and to estimate quantitatively the effect of various factors on the water content and other characteristics of fogs.

Numerical model of formation and evolution of fog

It is well known[1] that the drop of the air temperature (T) and an increase in the water vapor content play a principal role in the attainment of the state of saturation and consequent condensation of water vapor. In a due course, the change of T and water vapor content occurs under the effect of the turbulent exchange, vertical motions, advection, and process of condensation. As regards the radiation, it has the determining direct effect on the temperature of the underlying surface.

The initial set of equations of inflow (balance) of heat and water vapor, following Ref. 1, we write in the invariant form

$$\frac{\partial \Pi}{\partial t} + w\frac{\partial \Pi}{\partial z} = \frac{\partial}{\partial z} k \frac{\partial \Pi}{\partial z}, \quad (1)$$

$$\frac{\partial s}{\partial t} + w\frac{\partial s}{\partial z} = \frac{\partial}{\partial z} k \frac{\partial s}{\partial z}. \quad (2)$$

In Eqs. (1) and (2) Π and s are the invariant values.

$$\Pi = \theta + Lq/c_p, \quad (3)$$

$$s = q + \delta, \quad (4)$$

where q is the fraction of water vapor mass in the total mass, δ is the mass (specific) water content of fog, $\theta = T(1000/p)^{0.286}$ is the potential temperature, L and c_p are the specific heat of condensation and the specific heat of air at constant pressure, w is the vertical velocity, k is the turbulence factor, z is the height, t is time, and s is the specific moisture content.

The boundary conditions for unknown values T and s are set as follows:

$$T(0, t) = T_0(t);$$

$$s(0, t) = f_0 q_m [T_0(0), p_0] \text{ for } z = 0, \quad (5)$$

$$T(H, t) = T_H(0);\ s(H, t) = q_H(0) \text{ for } z = H. \quad (6)$$

The initial distribution of the same values is

$$T(z, 0) = \varphi(z);$$

$$s(z, 0) = f_0 q_m(T(z, 0), p(z)) \text{ for } t = 0. \quad (7)$$

Here $q_m = 0.622\ E(T)/p$ is the fraction of saturated water vapor in the total air mass; f_0 is the relative

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humidity of air at $t = 0$ which keeps constant within the entire layer from 0 up to H; $p_0 = 1000$ hPa is the pressure of air at $z = 0$; the pressure height distribution $p(z)$ was determined from the equation of static.

Time variation of temperature $T_0(t)$ on the Earth's surface was described by the D. Brent[2] formula

$$T_0(t) = T_0(0) - D\sqrt{t}, \quad (8)$$

where D is the parameter depending on the effective emission of surface and the thermal-physical properties of soil.

In accordance with Eq. (5) the specific moisture content at $z = 0$ keeps constant and is equal to the fraction of the total mass $q(0, 0)$ at the initial moment. However, as the temperature decreases, the condition of saturation is reached, when in the expression $s = q_m + \delta$ the summand q_m decreases down to values $q_m(0, T) < q(0, 0)$ near the Earth's surface and fog is being formed.

The vertical velocity w and turbulence factor k have major importance in this processes. To find their dependence on height and other parameters, the results of the similarity theory and the dimensionality theory, which have been developed for incompressible liquids (in application to the atmosphere it is equilibrium stratification) by German scientists Prandtle and Karman and generalized for the case of non-equilibrium stratification by A.S. Monin and A.M. Obukhov[3] are used. Sufficiently general relations for the dependence of k, the wind velocity c, temperature T, and the fraction of the total mass q on height have been obtained in Refs. 4 and 5.

Let us write here the formulae for the height distribution of k and T in the near-ground layer:

$$k(z) = \chi L_* u_* [1 - \exp(-z/L_*)], \quad (9)$$

$$T(z) = T_0 + T_* \ln(\eta/\eta_0) - \gamma_a(z - z_0). \quad (10)$$

Here L_* is the Monin-Obukhov scale; $u_* = \dfrac{\chi c_1}{\ln(\eta/\eta_0)}$ is the scale of wind velocity; $T_* = \dfrac{T_3 - T_2 + \gamma_a(z_3 - z_2)}{\ln(\eta_3/\eta_2)}$ is the scale of temperature;

$$\eta(z) = \exp(z/L_*) - 1; \quad (11)$$

T_0, T_2, T_3 are the known (measured) values of the temperature at the levels z_0, z_2, and z_3, respectively; c is the wind velocity at the height z_1; η_0, η_1, η_2, and η_3 are the values of the variable η at the level of surface roughness z_0 and heights z_1, z_2, and z_3, respectively; $\chi = 0.38$ is the Karman constant.

The scale L_* can be represented as

$$L_* = \frac{T_2}{g}\frac{u_*^2}{\chi^2 T_*}, \quad (12)$$

where g is the acceleration of free fall.

The sign of L_* depends on T_*. Since $(T_3 - T_2)/(z_3 - z_2) = -\gamma$, we have $T_* = (\gamma_a - \gamma)(z_3 - z_2)/\ln(\eta_3/\eta_2)$. From this it follows that T_* and L_* are positive under the stable stratification ($\gamma < \gamma_a$) and are negative ($T_* < 0$, $L_* < 0$) for the unstable ($\gamma > \gamma_a$) stratification of the near-ground layer. The closer the stratification is to equilibrium (the less the difference $|\gamma_a - \gamma|$ is) the lager the modulus of L_* is.

If we select the levels z_1, z_2, and z_3 so that $z_2 = z_1/2$ and $z_3 = 2\, z_1$ and use the expressions for u_* and T_*, the relation (12) takes the form

$$\frac{z_1}{L_*}\frac{\ln(\eta_3/\eta_2)}{\ln^2(\eta/\eta_0)} = Dl, \quad (13)$$

where Dl is the dimensionless parameter:

$$Dl = \frac{gz_1}{T_2}\frac{T_3 - T_2 + 3\gamma_a z_1/2}{c_1^2}. \quad (14)$$

If we know the difference $T_3 - T_2$ and the wind velocity c_1, and also z_0/z_1, then from Eq. (14) we find Dl, and from Eq. (13) we find the scale L_*/z_1.

The results on L_*/z_1 calculated by the relation (13) have been presented in Ref. 4.

The turbulence factor determined by the formula (9) grows with the increasing height within the near-ground layer. It follows from the definition of the scale that the thickness of the near-ground layer h is proportional to L_*. By supposing that $h = L_*$ for $\gamma < \gamma_a$ and $h = -L_*$ for $\gamma > \gamma_a$, by the formula (9) and allowing for u_*, we obtain the following expression for the turbulence factor $k(h) = k_h$ at the upper boundary of the near-ground layer:

$$\frac{k_h}{z_1 c_1} = \frac{\chi^2\beta}{\ln(\eta_1/\eta_0)}\frac{L_*}{z_1}, \quad (15)$$

where $\beta = (e - 1)/e$ for $\gamma < \gamma_a$ and $\beta = 1 - e$ for $\gamma > \gamma_a$ (here $e = 2.72$ is the natural logarithmic base). At higher h the factor k remains practically constant. The wind velocity at the level h equals to

$$c_h = c_1 B, \quad (16)$$

where $B = \ln[(e - 1)/\eta_0]/\ln(\eta_1/\eta_0)$ for $\gamma < \gamma_a$ and $B = \ln[(1 - e)/(e\eta_0)]/\ln(\eta_1/\eta_0)$ for $\gamma > \gamma_a$.

In the upper part of the boundary layer ($z \ge h$), where $k = k_h$, the solution of the equations of motion has a form of the well-known Hackman expressions.

To determine the vertical velocity w, the continuity equation is used

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (17)$$

where u and v are the projections of the wind velocity (vector) on the horizontal coordinate axes x and y.

Integrating expression (17) over height and using the equations of motion for $k = k_h$, we obtain for the vertical velocity averaged over the area of a cyclone σ

$$w = \frac{1}{\sigma}\iint\limits_{(\sigma)} w\,\mathrm{d}\sigma, \quad (18)$$

the following expression:

$$w(z) = \frac{1}{2\omega_z \rho\sigma} \int_l (\tau_{01} - \tau_1) dl. \qquad (19)$$

Here l is the contour of the area σ; τ_1 and τ_{01} are the projections of the turbulent stress of friction at the height z and on the Earth's surface, respectively; $2\omega_z$ is the Coriolis parameter; ρ is the air density.

Assuming that $\sigma = \pi r^2$ and $l = 2\pi r$ (the circle with radius r), we obtain the following expression for the vertical velocity on the height z:

$$w(z) = c_g z_1 (G - G_z)/r + w_h, \qquad (20)$$

where c_g is the geostrophic wind velocity; w_h is the vertical velocity at $z = h$.

Analysis of the values G and G_z shows that $w(z)$ depends on four dimensionless parameters: $R_0 = c_1/(\omega_z z_1)$, c_1/c_g, z_0/z_1, and z_1/L_*, and the parameter $c_g z_1/r$ with the dimension of velocity. In performing calculations, the parameter z_1/L_* has been set, though it is uniquely related by the Eq. (13) to the parameters Dl and z_0/z_1.

The function $\varphi(z)$ introduced in Eq. (7) and describing the height distribution of temperature at zero time has remained indeterminate.

It is natural for the near-ground layer to consider this distribution to be described by formula (10), in this case the difference $T_3(0) - T_2(0)$ included in the expression for the scale T_* should be known (measured).

The levels $z_2 = z_1/2$ and $z_3 = 2z_1$ and the difference of temperatures $T_3 - T_2$ are introduced because under actual conditions the temperature T_0 at the level z_0 is unknown and practically cannot be measured. In the model calculations T_0 can be considered as known (given). Therefore, the formula for the scale T_* can be determined without introducing the temperatures T_2 and T_3, but only for the difference of temperatures T_1 (at the height z_1) and T_0 (at z_0). If we write the formula (10) for the level z_1, we find from it

$$T_* = \frac{T_1 - T_0 + \gamma_a (z_1 - z_0)}{\ln(\eta_1/\eta_0)}. \qquad (21)$$

The temperature distribution $T(z, 0)$ at initial moment has been described with the formula (10) for T_* determined by the relation (21). Since a fog is normally formed under conditions of stable (as a rule, inversion) stratification of the near-ground layer the difference $T_1 - T_0$ should not exceed 0.1°C (the gradient T from z_0 up to z_1 = 10 m is less then the dry adiabatic one). According to expression (10) the distribution essentially differs from a linear one at small heights ($z < L_*$). Above the near-ground layer ($z > L_*$) the third summand in Eq. (10) begins to play the principal role (the gradient of T closer approaches γ_a). Since under actual conditions above the near-ground layer the gradient of T is considerably less then γ_a, in the formula (10) the gradient γ has been introduced instead of γ_a because the change of T comes nearer to it with height at $z >> L_*$.

In addition to the difference $T_0 - T_1$ at the initial moment the relative humidity f_0 assumed constant at all heights (from z_0 up to H) is introduced.

Solution of the system (1)–(4) under boundary conditions (5)–(6), for the height distribution of temperature and humidity of air at the initial moment, in accordance with Eqs. (7), (10), and (21) and the change with height of the turbulence factor and vertical velocity by the formulae (9) and (20), which are included in the initial system is obtained with the numerical implicit double-sweep method for the temporal step 60 s. Approximation of the height derivatives by the well known regularity is taken into account: the closer the level to the Earth's surface the faster all values vary with height. For this purpose the height step is taken variable

$$z_{j+1} - z_j = \beta(z_j - z_{j-1}), \qquad (22)$$

where β is the constant which is taken to be equal to 1.5, 1.25, and 1.0 (for a comparison of the accuracy of calculations). The following relations approximate the height derivatives:

$$\frac{\partial f}{\partial z} = \frac{f_{j+1}}{2\Delta z_{j+1}} + \frac{f_{j-1}}{2\Delta z_{j-1}}, \qquad (23)$$

$$\frac{\partial^2 f}{\partial z^2} = 2\left(\frac{f_{j+1}}{2\Delta z_{j+1}^2} - \frac{f_j}{\Delta z_j^2} + \frac{f_{j-1}}{2\Delta z_{j-1}^2}\right), \qquad (24)$$

where

$$\Delta z_{j+1} = z_{j+1} - z_j;\ \Delta z_j = z_j - z_{j-1};\ \Delta z_{j-1} = z_{j-1} - z_{j-2}.$$

Calculated results

Let us first present information on the dynamic characteristics: the angle of deviation of wind from an isobar in the near-ground layer (α_0), the turbulence factor (k_H), and the vertical velocity (w_H) close to the upper boundary (H) of the boundary layer.

For fixed $R_0 = c_1/(\omega_z z_1) = 3\cdot10^3$, $L_*/z_1 = 5$, and $z_0/z_1 = 0.1$ the angle α_0 increases when the ratio c_1/c_g grows:

c_1/c_g	0.3	0.4	0.5	0.6	0.7	0.8
α_0, deg	40	59	71	79	85	90

When the parameter L_* increases, the stratification of the near-ground layer comes closer to the dry neutral one, and the turbulent exchange is intensified. For this reason at $R_0 = 3\cdot10^3$, $z_0/z_1 = 0.1$, and $c_1/c_g = 0.5$ the angle α_0 decreases when L_*/z_1 grows

L_*/z_1	5	10	20	50
α_0, deg	71	45	27	5

For fixed $c_1/c_g = 0.5$, $z_0/z_1 = 0.1$, and $L_*/z_1 = 50$ the angle α_0 increases when R_0 grows

$10^{-4}R_0$	1	1.5	2	2.5	3	4	4.5
α_0, deg	24	33	41	47	53	64	74

For $c_1/c_g = 0.8$, $z_0/z_1 = 0.1$, and $L_*/z_1 = 10$ the values of α_0 for various R_0 are the following:

$10^{-3}R_0$	4.5	5	6	7	8
α_0, deg	64	69	79	88	97

Interpretation of the dependence of α_0 on R_0, c_1/c_g, z_0/z_1, and L_*/z_1 is given in Ref. 4.

Let us now present the data that directly refer to fogs. Apart from the dynamic parameters (R_0, c_1/c_g, z_0/z_1, and L_*/z_1) mentioned above, the temperature and humidity of air in the initial state affect on the fog formation. Characteristics of fogs presented below have been obtained for the following values that are the same in all cases considered: $T_{z0}(0) = 15°C$; $D = 2°C/h^{1/2}$ (temperature at the level z_0 decreases by 2°C during the first hour from the initial moment); $T_{z1}(0) - T_{z0}(0) = -0.06°C$; $z_1 = 10$ m; $c_g z_1/r = 5\cdot10^{-4}$ m/s (r is the radius of a cyclone); $\gamma = 0.6\cdot10^{-2}$°C/m is the temperature gradient describing the temperature distribution for $z >> L_*$.

All other parameters have varied within the limits typical under natural conditions. The basic criterion is the angle α_0; its values should be limited between 0° and 90°.

Since the air temperature decreases most considerably near the Earth's surface (at the level z_0) and the maximum water content is also in this layer it is exactly here the formation of the fog starts at. For the relative humidity at the initial moment $f(z, 0) = 0.9$ and values $z_0/z_1 = 0.1$ and $L_*/z_1 = 10$ the fog has been formed in 3 h from the initial moment. The data on the thickness of the fog for various c_1/c_g and two values R_0 are presented in Table 1, according to which for fixed R_0 the thickness of a fog, especially in 8 h, grows with the increase of the ratio c_1/c_g.

The roughness of the Earth's surface essentially affects the fog characteristics. According to data presented in Table 2 which are the calculated values at $R_0 = 9\cdot10^3$, $f(z, 0) = 0.9$, $L_*/z_1 = 10$, and $c_1/c_g = 0.6$ the thickness of the fog decreases by more than 2 times when the roughness parameter decreases by 2 times. When z_0/z_1 decreases the water content of a fog decreases also, though more slowly than the fog thickness. The essential increase of $10^{-3}R_0$ (from 7–9 up to 20), i.e., the wind velocity c_1, is accompanied by the decrease of the thickness of a fog. It is explained by that at the increasing c_1 the turbulent exchange causing the transport of vapor to higher levels and the decrease of its content in the near-ground layer is intensified.

Table 1. Thickness of the fog (m) for various values of the ratio c_1/c_g

Time, h	$10^{-3}R_0$	c_1/c_g 0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
3	7	–	8.4	8.5	9.0	9.0	9.0	9.0	10
	9	8.0	8.5	8.7	9.0	9.5	–	–	–
	20	–	3.8	4.0	4.3	4.5	4.6	–	–
6	7	–	30	35	36	40	45	48	50
	9	29	30	35	39	41	–	–	–
	20	–	20	22	26	27	30	–	–
8	7	–	–	80	100	130	150	170	200
	9	60	72	85	100	115	–	–	–
	20	–	55	56	70	76	88	–	–

Table 2. Thickness and water content of the fog for z_0/z_1 equal to 0.1 (numerators) and 0.05 (denominators)

Time, h	Thickness, m	Water content, g/kg, at the height 3.5 m
3	10/4	0.58/0.27
6	41/20	1.62/1.26
8	115/45	2.15/1.78

Table 3 presents the water content height distribution for a fog at fixed moments at different values of the ratio c_1/c_g. Since the temperature decreases most considerably near the Earth's surface, the largest values of the water content are observed here too.

Table 3. Water content distribution (g/kg) of a fog by a height for $R_0 = 3\cdot10^3$, $f(z, 0) = 0.9$, $L_*/z_1 = 10$, $z_0/z_1 = 0.1$, and various c_1/c_g

c_1/c_g	3 h		6 h				8 h				α_0, deg
	Height, m										
	3.8	8.2	3.8	11.3	25.8	33.3	3.8	11.3	33.3	86.9	
0.1	–	–	–	–	–	–	–	–	–	–	0
0.2	–	–	–	–	–	–	–	–	–	–	0
0.3	0.55	0.00	1.56	0.73	0.11	0.00	2.08	1.25	0.43	0.00	25
0.4	0.56	0.00	1.58	0.76	0.15	0.00	2.10	1.29	0.50	0.00	40
0.5	0.56	0.01	1.59	0.78	0.19	0.01	2.11	1.32	0.54	0.01	45
0.6	0.57	0.02	1.59	0.80	0.21	0.05	2.12	1.34	0.58	0.01	47
0.7	0.57	0.02	1.60	0.82	0.24	0.07	2.13	1.36	0.61	0.06	48
0.8	0.57	0.03	1.61	0.83	0.26	0.09	2.14	1.37	0.64	0.09	47
0.9	0.58	0.03	1.61	0.84	0.27	0.11	2.14	1.37	0.66	0.13	46

With the increase of height the water content for fixed t and c_1/c_g decreases. At a given height the water content of a fog increases slowly with the growth of c_1/c_g.

In summary we formulate brief conclusions:

a) All parameters included in the model (c_1, c_g, z_0, $f(0)$, $T(0)$, L_*, and r) affect the intensity of turbulent exchange and the vertical velocity, and correspondingly the formation time, thickness, and water content of a fog;

b) For fixed values of all other parameters the thickness and water content of a fog increase with the growth of the ratios c_1/c_g, z_0/z_1, z_1/L_*, the relative humidity $f(0)$, and the temperature $T(0)$ at the initial moment;

c) The relative humidity, the roughness of the Earth's surface, and the scale of the near-ground layer (thermal stratification) most strongly affect the thickness and water content of a fog;

d) Since the formation of a fog is determined by many factors, the broad range of conditions is observed, at which the fog can be formed.

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