REMOTE SENSING OF OPTICALLY ACTIVE MAN-MADE POLLUTANTS

T.A. Sushkevich, A.S. Strelkov, and A.A. Ioltukhovskiĭ

Institute of Applied Mathematics,
USSR Academy of Sciences, Moscow
Received January 27, 1989

The increase the efficiency of methods of remote sensing of optically transfer operator that established the analytical dependence of the Stokes vector of the outgoing radiation on the optical parameters of the medium and underlying surface is constructed. The kernel of this operator, which takes into account completely the effects of multiple scattering, rereflection, and polarization of the radiation, consists of invariant space-frequency characteristics of influence functions, which are the inverse Fourier transforms of the space-frequency characteristics.

To increase the efficiency of remote sensing of main-made, optically active pollutants in the atmospihere[1] it is necessary to refer to the theoretical framework for describing the propagation of radiation in spatially inhomogeneous media. Mathematical modeling, based on the numerical and semianalytical solution of direct and inverse problems in the theory of radiation transfer under conditions closest to natural conditions, offers great possibilities. We propose an approach to the solution of the problem of sensing aerosol pollutants in the atmosphere, bounded by a nonuniform lambertian surface, based on the numerical solution of a boundary-value problem in the theory of radiation transfer and on the optical transfer operator.

The formulation and solution of inverse problems in the theory of radiation transfer require information about the solution of direct problems, making it possible to obtain the angular and spatial distributions of the radiation in scattering and absorbing media. First, if analytical relations between the characteristics of the radiation field and the optical quantity sought can be established, then based on them it is possible to construct inverse operators. Second, direct problems provide a priori information that can be employed not only to construct regularizing algorithms but also to determine the errors introduced by dropping terms of simplifying the radiation transformat ion processes in a multiparameter system when formulating the inverse problems.

In studying the problem of the propagation of solar radiation, taking into account its polarization, in a two-dimensional layer-substrate system with horizontal nonuniformities we shall construct functional expressions for calculating the Stokes vector that establish by means of invariant vector space-frequency characteristics (VSFC), vector influence functions (VIF), or point spread functions (PSF) an analytical relation between the characteristics of the radiation and the coefficients of the vector boundary-value problem of the theory of radiation transfer. The most general functional expressions, in which the effects of several parameters of the problem contributing to the transformations of the angular and spatial structure of the radiation field are separated, are constructed. In Refs. 2–9 we gave several implementations of the proposed approach for scalar and polarization problems for different models of the optical radiation transfer system. These results make it possible to develop a method for separating the contributions of a nonuniform surface and an inhomogeneous atmosphere to the total outgoing radiation and to determine the optical parameters of the atmosphere.

The solution of the vector boundary-value problem for the equation of radiation transfer taking into account the polarization of the radiation in an inhomogeneous flat layer of the atmosphere illuminated by a monodirectional flux and bounded by a nonuniform lambertian surface,

$$\begin{cases} D\Phi_t=\hat{S}\Phi_t, & \Phi_t|_0=\pi S_\lambda\delta(s-s_0)t, \\ \Phi_t|_H=(q\hat{R}I_t)t_H, & \Phi_t=\{I_t,Q_t,U_t,V_t\} \end{cases} \quad (1)$$

with the transfer operator

$$\hat{D}\equiv(s,\mathrm{grad})+\sigma_t(r)$$

$$\sigma_t(r)=\bar{\sigma}_\alpha(z)+\sigma_s(r),\quad \sigma_s(r)=\bar{\sigma}_s(z)+\nu g(z)\tilde{\sigma}(r_\perp),$$

the source function

$$\hat{S}\Phi\equiv\sigma_s(r)\hat{M}\Phi,\quad \hat{M}\Phi\equiv\int_\Omega\hat{P}\Phi ds',$$

$$\bar{S}\Phi\equiv\bar{\sigma}_s(z)\hat{M}\Phi,\quad \hat{\bar{D}}\equiv(s,\mathrm{grad})+\bar{\sigma}_s(z)+\sigma_a(z)-\hat{\bar{S}}$$

and lambertian boundary

$$\hat{R}\Phi \equiv t_H \hat{R}I = \frac{t_H}{\pi} \int_{\Omega+} I(H, r_\perp, s')\mu' ds'$$

$$q(r_\perp) = \bar{q} + s\tilde{q}(r_\perp), \quad t_H = \{1,0,0,0\}, \quad 0 \le \varepsilon \le 1$$

is sought in the form of Neumann series in different orders of scattering by the variations $\tilde{\sigma}_s(r_\perp)$ and different orders of rereflection from a surface with the albedo $q(r_\perp)$. We separate in Stokes vector

$$\Phi_t = \Phi^0 + \Phi^{(0)} + \Phi_q, \quad \Phi_q = \Phi^{(\bar{q})} + \Phi^{(\bar{q}\ \tilde{q})} \tag{2}$$

the contributions of the unscattered radiation (solution of the Cauchy problem)

$$\{\hat{D}\Phi^0 = 0, \quad \Phi^0|_0 = \pi s_\lambda \delta(s - s_0)t, \quad \Phi^0|_H = 0 \tag{3}$$

the multiply scattered radiation in the atmosphere with an absolutely black bottom

$$\{\hat{D}\Phi^{(0)} = \hat{S}\Phi^{(0)} + \hat{S}\Phi^0, \quad \Phi^{(0)}|_0 = 0, \quad \Phi^{(0)}|_H = 0, \tag{4}$$

and radiation multiply rereflected from the boundary and scattered in the atmosphere

$$\begin{cases} \hat{D}\Phi_q = \hat{S}\Phi_q, \quad \Phi_q|_0 = 0, \\ \Phi_q|_H = q[\hat{R}I_q + \hat{R}I^{(0)} + \hat{R}I^0]t_H \end{cases} \tag{5}$$

The contribution of illumination from the surface can in their turn be separated into the illumination due to the presence of a constant component of the albedo $\bar{q}(r_\perp)$

$$\begin{cases} \hat{D}\Phi^{(\bar{q})} = \hat{S}\Phi_q, \quad \Phi^{(\bar{q})}|_0 = 0, \\ \Phi^{(\bar{q})}|_H = q[\hat{R}I^{(\bar{q})} + \hat{R}I^{(0)} + \hat{R}I^0]t_H \end{cases} \tag{6}$$

and a fluctuating component of the albedo $\tilde{q}(r_\perp)$

$$\begin{cases} \hat{D}\Phi^{(\bar{q}\ \tilde{q})} = \hat{S}\Phi^{(\bar{q}\ \tilde{q})}, \quad \Phi^{(\bar{q}\ \tilde{q})}|_0 = 0, \\ \Phi^{(\bar{q}\ \tilde{q})}|_H = [(\bar{q} + \tilde{q})\hat{R}I^{(\bar{q}\ \tilde{q})} + \tilde{q}(\hat{R}I^{(\bar{q})} + \hat{R}I^{(0)} + \hat{R}I^0)]t_H \end{cases} \tag{7}$$

Functional expressions for the solution of the boundary-value problem (1) in the form of a superposition (2) of the solutions of the problems (3)–(7), which establish the analytical relation with the variations $\tilde{\sigma}(r_\perp)$, $\tilde{q}(r_\perp)$ and the albedo $\bar{q}$ with the help of characteristics of the optical system atmospheric layer + underlying surface that are invariant with respect to these coefficients, were derived with the help of perturbation series expansions in the parameters $\in$ and ν and the integral Fourier transform in the coordinate $r_\perp = \{x, y\}$.

The Fourier transform of the Stokes vector is determined in terms of the vector space-frequency characteristics in the form of the following functional expression:

$$\check{\Phi}(z,p,s) = [\pi s_\lambda \delta(s - s_0)\exp(-\tau(z)/\mu_0]\ t \sum_{n=0}^{\infty} \hat{\sigma}_0^n(1) + \sum_{n=0}^{\infty} \hat{\sigma}^n(\Phi_0^{(0)}) + \sum_{n=0}^{\infty} \hat{\mathcal{T}}_0^n [\Phi_0^0] + \sum_{n=0}^{\infty} \hat{\mathcal{T}}_1^n \left\{ \frac{\bar{q}(\hat{R}I_0^{(0)} + d)}{1 - \bar{q}\, C_0} W_0 + \frac{\hat{R}I_0^{(0)} + d}{1 - \bar{q}\, C_0} \frac{W\check{\tilde{q}}}{t\bar{t}} + d\bar{q} \sum_{n'=1}^{\infty} \frac{W\hat{\sigma}_0^{n'}(1)}{\bar{t}} + d \sum_{n'=1}^{\infty} \left(\frac{W}{t\bar{t}} \left[\tilde{q} * \frac{\check{\hat{\sigma}}_0^{n'}(1)}{\bar{t}} \right] \right) \right\} \tag{8}$$

where the operators corresponding to the terms of the Neumann series are expressed in terms of the reflection operators

$$\hat{q}_R[\check{f}] \equiv \bar{q} t_H \hat{R}\check{f}^{(1)}(p), \quad \hat{q}_R^+[\check{f}] \equiv t_H \left(\check{\tilde{q}} * \frac{\hat{R}\check{f}}{\bar{t}} \right)^{(1)},$$

$$\hat{\mathcal{T}}_0^n \equiv \left[\hat{\sigma} + \frac{W}{\bar{t}} \hat{q}_R \hat{\sigma} + \frac{W}{t \cdot \bar{t}} \hat{q}_R^+ \hat{\sigma} \right]^n,$$

$$\hat{\mathcal{T}}_1^n \equiv \left[\hat{\sigma}_a + \frac{W}{\bar{t}} \hat{q}_R \hat{\sigma}_q + \frac{W}{t \cdot \bar{t}} \hat{q}_R^+ \hat{\sigma}_q \right]^n,$$

$$d \equiv s_\lambda \mu_0 \exp(-\tau(H)/\mu_0.$$

The operators describing the n-th order scattering by the variations

$$\hat{\sigma}_0^n(1) \equiv \frac{1}{(2\pi)^{2(n-1)}} \int_{-\infty}^{\infty}\!\!\int \check{\tilde{\sigma}}_s(p_n - p_{n-1}) \ldots \check{\tilde{\sigma}}(p_1) \times V_n(z, p_n, \ldots, p_1, s_0) dp_{n-1} \ldots dp_1$$

"generate" the SFC V_n -the solutions of the Cauchy problems

$$\{\hat{L}_0 V_n = -g(z)V_{n-1}, \quad V_n|_0 = 0, \quad V_n|_H = 0,$$

$$\hat{L}_0(p) \equiv \mu_0 \frac{\partial}{\partial z} - i(p, s_\perp);$$

$$\hat{\sigma}_0^n(\Phi_0^{(0)}) \equiv \frac{1}{(2\pi)^{2(n-1)}} \int_{-\infty}^{\infty}\!\!\int \check{\tilde{\sigma}}_s(p_n - p_{n-1}) \ldots \check{\tilde{\sigma}}(p_1) \times \psi_n^{(0)}(z, p_n, \ldots, p_1, s) dp_{n-1} \ldots dp_1$$

"generate" the VSFC $\bar{\psi}_n^{(0)}$ – the solutions of the problems

$$\begin{cases} \hat{L}(p)\psi_n^{(0)} = \hat{\bar{S}}\psi_n^{(0)} + g(z)[(\hat{M} - \hat{E})\psi_{n-1}^{(0)} + V_{n-1}\hat{C}(z)t] + \\ + \bar{\sigma}_s(z)V_n\hat{C}(z)t, \quad \psi_n^{(0)}|_{0,H} = 0; \end{cases}$$

$$\hat{L}(p) \equiv \hat{D}_z - i(p, s_\perp), \quad \hat{D}_z \equiv \mu\frac{\partial}{\partial z} + \bar{\sigma}_s(z) + \bar{\sigma}_a(z), \tag{9}$$

with the starting approximation $\vec{\psi}_0^{(0)} = \vec{\Phi}_0^{(0)}(z, s)$ — the solution of the problem

$$\{\hat{D}_z \Phi_0^{(0)} = \hat{\bar{S}}\Phi_0^{(0)} + \bar{\sigma}_s(z)(z)\hat{C}(z)t, \quad \Phi_0^{(0)}|_{0,H} = 0. \tag{10}$$

The operators

$$\hat{\sigma}_0^n(\psi_\alpha f) \equiv \frac{1}{(2\pi)^{2n}} \int_{-\infty}^{\infty}\int \check{\tilde{\sigma}}_s(p_n - p_{n-1}) \ldots \check{\tilde{\sigma}}(p_1 - p_0) \times$$
$$\times f(p_0)\ \psi_n(z, p_n, \ldots, p_0, s)dp_{n-1}\ldots dp_0$$

"generate the AFC $\vec{\psi}_n$ — the solution of the problems

$$\{\hat{L}(p)\psi_n = g(z)[\hat{M} - \hat{E}]\psi_{n-1}, \quad \psi_n|_{0,H} = 0 \tag{11}$$

with the starting approximation $\vec{\psi}_0 = \vec{\psi}'_\alpha$. The parametric function

$$C(p) \equiv \hat{R}W^{(1)}(p), \quad C_0 \equiv C(p=0), \quad W_0 \equiv W(p=0),$$

$$H(p) = \frac{C(p)}{1 - qC(p)}, \quad \bar{t}(p) \equiv 1 - \bar{q}C(p),$$

$$t(p) \equiv 1 - \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\check{\tilde{q}}(p')H(p-p')dp'$$

are determined in terms of the linear VSFC $\vec{W}(z, p, s)$, which are solutions of the problem

$$\{L(p)W = 0, \quad W|_0 = 0, \quad W|_H = t_H \tag{12}$$

The Stokes vector is represented in terms of the VIF in the form of Neumann series

$$\Phi_t(z, r_\perp, s) = [\pi s_\lambda \delta(s - s_0)\ \exp(-\tau(z)/\mu_0]\ t\sum_{n=0}^{\infty}\hat{\Sigma}_0^n(1) +$$
$$+ \sum_{n=0}^{\infty}\hat{\Sigma}^n(\Phi_0^{(0)}) + \sum_{n=0}^{\infty}\hat{\mathcal{P}}_0^n\ [\Phi_0^{(0)}] + \sum_{n=0}^{\infty}\hat{\mathcal{P}}_1^n\ (W_0)\ \frac{\bar{q}[\hat{R}I_0^{(0)} + d]}{1 - \bar{q}\ C_0} +$$
$$+ \frac{\hat{R}I_0^{(0)} + d}{1 - \bar{q}\ C_0}\ \sum_{n=0}^{\infty}\hat{\mathcal{P}}_1^n\left[\theta_{\bar{q}} * \frac{\tilde{q}}{v}\right] +$$
$$+ d\sum_{n=1}^{\infty}\hat{\mathcal{P}}_1^n\left[\sum_{n'=1}^{\infty}(\theta_{\bar{q}} * (\hat{G}_{\bar{q}} + \hat{\tilde{G}}_R + \hat{\tilde{G}}_R\hat{\tilde{G}}_{\bar{q}})\hat{\Sigma}_0^{n'}(1))\right] \tag{13}$$

with the operators

$$\hat{\mathcal{P}}_0^n \equiv [\hat{\Sigma}_q + \hat{G}_{\bar{q}}\hat{\Sigma} + \hat{\tilde{G}}_R\hat{\Sigma} + \hat{\tilde{G}}_R\hat{G}_{\bar{q}}\hat{\Sigma}]^n,$$

$$\hat{\mathcal{P}}_1^n \equiv [\hat{\Sigma}_q + \hat{G}_{\bar{q}}\hat{\Sigma}_q + \hat{\tilde{G}}_R\hat{\Sigma}_q + \hat{\tilde{G}}_R\hat{G}_{\bar{q}}\hat{\Sigma}_q]^n,$$

where

$$\hat{G}_{\bar{q}} \equiv \bar{q}(\theta_{\bar{q}} * \hat{R}f^{(1)}), \qquad \hat{\bar{G}}_R[f] \equiv t_H\left[\theta_{\bar{q}} * \frac{\tilde{q}\hat{R}f^{(1)}}{v}\right].$$

are reflection operators.

The operators describing n-th order scattering by variations

$$\hat{\Sigma}_0^n(1) \equiv \int_{-\infty}^{\infty}\ldots\int\tilde{\sigma}_s(r_{\perp n})\ldots\tilde{\sigma}_s(r_{\perp 1}) \times$$
$$\times\ \theta_n^0(z, r_\perp - r_{\perp n}, \ldots, r_{\perp 2} - r_{\perp 1}, s_0)dr_{\perp n}\ldots dr_{\perp 1}$$

"generate" the influence functions $\Theta_n^0 = \mathcal{T}^{-1}[V_n]$ — the solutions of the Cauchy problems

$$\{\hat{\bar{D}}\Theta_n^0 = -g(z)\Theta_{n-1}^0\delta(r_\perp), \theta_n^0|_{0,n} = 0,$$

The operators

$$\hat{\Sigma}_0^n(\Phi_0^{(0)}) \equiv \int_{-\infty}^{\infty}\ldots\int\tilde{\sigma}_s(r_{\perp n})\ldots\tilde{\sigma}_s(r_{\perp 1})$$
$$\times\ \theta_n^{(n)}(z, r_\perp - r_{\perp n}, \ldots, r_{\perp 2} - r_{\perp 1}, s)dr_{\perp n}\ldots dr_{\perp 1}$$

"generate" the VIF $\Theta_n^0 = \mathcal{T}^{-1}[V_n]$ — the solutions of the problems

$$\begin{cases}\hat{D}\ \theta_n^{(0)} = \hat{\bar{S}}\theta_n^{(0)} + g(z)[(\hat{M} - \hat{E})\theta_{n-1}^{(0)} + \theta_{n-1}^{\sigma}\hat{C}(z)t]\delta(r_\perp) \\ + \bar{\sigma}_s(z)\theta_n^0\hat{C}(z)t, \quad \theta_n^{(0)}|_{0,H} = 0;\end{cases}$$

with the starting approximation $\vec{\theta}_0^{(0)} = \vec{\Phi}_0(z, s)$;

$$\hat{\Sigma}_q^n(\vec{\theta}_\alpha * f) \equiv \int_{-\infty}^{\infty}\ldots\int\tilde{\sigma}_s(r_{\perp n})\ldots\tilde{\sigma}_s(r_{\perp 1})f(r_{\perp 0})$$
$$\times\ \vec{\theta}_n(z, r_\perp - r_{\perp n}, \ldots, r_{\perp 1} - r_{\perp 0}, s)dr_{\perp n}\ldots dr_{\perp 0}$$

"generate" the VIF $\vec{\theta}_n$ — the solutions of the problems

$$\hat{D}\ \theta_n = g(z)(\hat{M} - \hat{E})\vec{\theta}_{n-1}^{(0)} + \theta_{n-1}\delta(r_\perp), \quad \vec{\theta}_n|_{0,H} = 0$$

with the starting approximation $\vec{\theta}_0 = \vec{\theta}_\alpha$.

The linear VIF $\vec{\theta}_{\bar{q}} = \mathcal{T}^{-1}\left[\frac{W}{1 - \bar{q}C(p)}\right]$ is the solution of the problem

$$\{\hat{\bar{D}}\theta_{\bar{q}} = 0, \quad \vec{\theta}_{\bar{q}}|_0 = 0, \quad \vec{\theta}_{\bar{q}}|_H = [\bar{q}\hat{R}\theta_{\bar{q}}^{(1)} + \delta(r_\perp)]t_H$$

Thus, the solution of the starting, five-dimensional (in phase space) boundary-value problem (1) reduces to a collection of solutions of one-dimensional spatial problems (9), (10), (11), and (12) which have different sources and are solved numerically by the same iteration method of characteristics[11]. The functional representations (8)–(13) take into account completely all effects associated with multiple scattering, rereflection, and polarization of radiation in the system atmospheric layer + underlying surface. In solving the problems of reconstructing the optical parameters of the medium and the surface low-order approximations to the Neumann series written out above are, naturally, employed. In our papers Refs. 4–10 several such particular solutions were written out for the optical transfer operators (8) and (13) in which different orders of the parameters of the radiation transfer system $\tilde{\sigma}_s(r_\perp)$, $\tilde{q}(r_\perp)$, and $\bar{q}$ were taken into account. Methods for solving inverse problems based on the reconstruction of the albedo of the surface and the scattering characteristics and influence functions are described in Ref. 10.

REFERENCES

1. V.E. Zuev, Optika Atmosfery, **1**, No. 1, 5 (1988).
2. T.A. Sushkevich, *Taking the Albedo into Account in Problems with Local and Distributed Sources*, Preprint No. 48, Institute of Applied Mathematics of the USSR Academy of Sciences, Moscow (1986).
3. T.A. Sushkevich, *On Using Neumann Series for Solving Boundary-Value Problems in the Theory of Transfer with a Nonuniform Lambertian Boundary,* Preprint No. 154, Institute of Applied Mathematics, Academy of Sciences of the USSR, Moscow (1986).
4. S.A. Strelkov and T.A. Sushkevich, *Taking the Contribution of a Lambertian Surface into Account Analytically in the Solution of the Polarization Problem by the Method of Space-Frequency Characteristics and Influence Functions,* Preprint No. 200, Institute of Applied Mathematics of the USSR Academy of Sciences, Moscow (1987).
5. A.A. Ioltukhovskiĭ, S.A. Strelkov, and T.A. Sushkevich, *Algorithms for Solving Polarization Problems with a Horizontally Nonuniform Lambertian Substrate by the Method of Space-Frequency Characteristics and Influence Functions,* Preprint No. 227, Institute of Applied Mathematics, Academy of Sciences of the USSR, Moscow (1987).
6. T.A. Sushkevich, *Semianalytical Method forSolving the Radiation Transfer Equations for Solar Radiation in an Inhomogeneous Flat Atmosphere*, Preprint No. 38, Institute of Applied Mathematics, Academy of Sciences of the USSR, Moscow (1988).
7. T.A. Sushkevich, *Semianalytical Solution of Polarization Problems for a Flat Layer with Horizontal Nonuniformities,* Preprint No. 63, Institute of Applied Mathematics, Academy of Sciences of the USSR. Moscow (1988).
8. S.A. Strelkov and T.A. Sushkevich, *Semianalytical Method for Solving the Radiation Transfer Equations for Polarized Solar Radiation in an Inhomogeneous Atmosphere*, Preprint No. 65, Institute of Applied Mathematics, Academy of Sciences of the USSR, Moscow (1988).
9. T.A. Sushkevich, *Neumann Series for Semianalytical Solution of the Polarization Problem with Horizontal Nonuniformities*, Preprint No. 72, Institute of Applied Mathematics, Academy of Sciences of the USSR, Moscow (1988).
10. A.A. Ioltukhovskiĭ, *On the Formulation and Solution of the Inverse Problem of Atmospheric Optics,* Preprint No. 84, Institute of Applied Mathematics, Academy of Sciences of the USSR, Moscow (1988).
11. T.A. Sushkevich, *Numerical Method for Solving Radiation Transfer Equations with a Complex \ Function*, Preprint No. 136, Institute of Applied Mathematics, Academy of Sciences of the USSR, Moscow (1980).