Formation of interference patterns in diffusely scattered fields at a double-exposure microscope-recording of quasi-Fourier and Fourier holograms

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The sensitivity of a holographic interferometer to transversal or longitudinal displacements of a flat surface, diffusively scattering light, is analyzed. It is shown that the interference patterns are located in the hologram plane and far diffraction zone. To record them, spatial filtration of the diffraction field is required. The experimental results agree well with theoretical prerequisites.

It was shown in Ref. 1, that under control of transversal displacement of a flat surface, diffusively scattering light (hereinafter scatterer), at a double-exposure recording of the hologram of the scatterer image, focused with a Kepler tube, a homogeneous displacement of subjective speckles, corresponding to the second exposure, and variations of their slope angles offer scope for localization of interference patterns in two planes, i.e., the hologram plane and Fourier plane. Spatial filtration of the diffraction field in the corresponding planes allows one to determine the interferometer sensitivity. In the case of interference pattern localization in the hologram plane, where the scatterer is imaged, the sensitivity depends on magnification of the double-exposure optical system and the curvature of a spherical wave of the coherent radiation, used to illuminate the scatterer while recording the hologram. For the interference pattern located in the Fourier plane, the interferometer sensitivity depends on the focal length of the telescopic lens, when the coefficient of scatterer image scaling is less than unit. Besides, recording of interference patterns in the planes of their localization is accompanied by the fringe parallax effect.

The localization of the interference pattern in the above planes is also realized in case of a transversal displacement of the scatterer due to extension of subjective speckles, corresponding to the second exposure, in the hologram plane, and their slope angle, radially varying from the optical axis. In this case, the interferometer sensitivity for both localizations depends on the same above-described parameters. In this case, the fringe parallax effect does not accompany the recording of interference patterns in the planes of their localization while spatial filtering the diffraction field.

In this work, peculiarities of formation of interference patterns are analyzed, which characterize transversal or longitudinal displacements of the scatterer during double-exposure recording of quasi-Fourier and Fourier holograms with the help of a collimating microscope in order to determine the interferometer sensitivity.

Consider Fig. 1. Opaque screen *1* in the plane (x_1, y_1) is illuminated by coherent radiation with diverging spherical wave of the curvature radius R. Radiation, diffusively scattered by the screen and passing the microscope optics (positive thin lens L_1 is the objective and positive thin lens L_2 is the ocular), is recorded during the first exposure on the photoplate *2* in the plane (x_4, y_4) by means of the off-axis reference plane wave making the angle θ with the normal to the photoplate plane. Before the second exposure, the opaque screen is displaced in its plane, e.g., to the value a toward the x-axis.

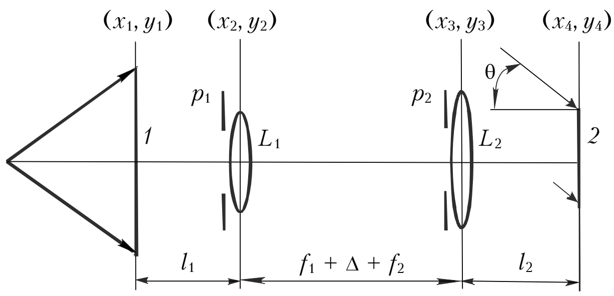


Fig. 1. Schematic view of two-exposure hologram recording: *1* is the opaque screen; *2* is the photoplate; L_1 and L_2 are the positive lens; p_1 and p_2 are the objective apertures.

The distribution of the complex amplitude of the field, corresponding to the first exposure, to the Fresnel approximation with accounting for the diffraction limitedness, in the object channel in the photoplate plane takes the form

$$u_1(x_4,y_4) \sim \iiint\int_{-\infty}^{\infty}\iint t(x_1,y_1)\exp\left[\frac{ik}{2R}\left(x_1^2+y_1^2\right)\right]\times$$

$$\times\exp\left\{\frac{ik}{2l_1}\left[(x_1-x_2)^2+(y_1-y_2)^2\right]\right\}p_1(x_2,y_2)\times$$

$$\times\exp\left[-\frac{ik}{2f_1}\left(x_2^2+y_2^2\right)\right]\exp\left\{\frac{ik}{2(f_1+\Delta+f_2)}\left[(x_2-x_3)^2+\right.\right.$$

$$+ (y_2 - y_3)^2\Big]\Big\} p_2(x_3, y_3) \exp\left[-\frac{ik}{2f_2}(x_3^2 + y_3^2)\right] \times$$
$$\times \exp\left\{\frac{ik}{2l_2}\left[(x_3 - x_4)^2 + (y_3 - y_4)^2\right]\right\} \mathrm{d}x_1 \mathrm{d}y_1 \mathrm{d}x_2 \mathrm{d}y_2 \mathrm{d}x_3 \mathrm{d}y_3, \quad (1)$$

where k is the wave number; $t(x_1, y_1)$ is the complex amplitude of screen transmission, being the random function of coordinates; $p_1(x_2, y_2)$ is the pupil function[2] of the lens L_1 with the focal length f_1; $p_2(x_3, y_3)$ is the pupil function of the lens L_2 with the focal length f_2; Δ is the microscope tube length; l_1 is the distance from the principal plane (x_2, y_2) of the lens L_1 to the screen; l_2 is the distance from the principal plane (x_3, y_3) of the lens L_2 to the photoplate.

Taking into account that the opaque screen is imaged in the frond focal plane of the lens L_2, i.e., $(1/f_1) = 1/l_1 + 1/(f_1 + \Delta)$ and the condition $(1/f_2) = = 1/l_2 + 1/(f_2 + \Delta)$ [Ref. 3], equation (1) takes the form

$$u_1(x_4, y_4) \sim \exp\left[\frac{ik\Delta}{2f_2(f_2 + \Delta)}(x_4^2 + y_4^2)\right] \times$$
$$\times \left\{\exp\left[\frac{ik(f_1 + \Delta + f_2)\Delta}{2ff_2(f_2 + \Delta)}(x_4^2 + y_4^2)\right] \times\right.$$
$$\times \left\{\exp\left[-\frac{ikf_1(f_1 + \Delta)}{2f^2\Delta}(x_4^2 + y_4^2)\right] \times\right.$$
$$\left.\left.\times F^{-1}(x_4, y_4) \otimes P_1(x_4, y_4)\right\} \otimes P_2(x_4, y_4)\right\}, \quad (2)$$

where $\otimes$ denotes the convolution; $f = f_1 f_2/\Delta$ is the focal length of the microscope; $P_1(x_4, y_4)$ is the Fourier transform of the function $p_1(x_2, y_2)$ (with accounting for its parity) with the spatial frequencies $x_4/\lambda f$ and $y_4/\lambda f$ (λ is the wavelength of the coherent light source, used at the stages of hologram recording and reconstruction); $P_2(x_4, y_4)$ is the Fourier transform of $p_2(x_3, y_3)$ with the spatial frequencies

$$x_4\Delta/\lambda f_2(f_2 + \Delta) \text{ and } y_4\Delta/\lambda f_2(f_2 + \Delta);$$

$$F^{-1}(x_4, y_4) = \int\int_{-\infty}^{\infty} t(x_1, y_1) \exp\left[ik(x_1^2 + y_1^2)/2R\right] \times$$
$$\times \exp\left[ik(x_1 x_4 + y_1 y_4)/f\right] \mathrm{d}x_1 \mathrm{d}y_1 =$$
$$= F(x_4, y_4) \otimes \exp\left[-ikR(x_4^2 + y_4^2)/2f^2\right],$$

$$F(x_4, y_4) = \int\int_{-\infty}^{\infty} t(-x_1, -y_1) \times$$
$$\times \exp\left[-ik(x_1 x_4 + y_1 y_4)/f\right] \mathrm{d}x_1 \mathrm{d}y_1.$$

Like in Ref. 3, let the width of the function $P_1(x_4, y_4)$ be about $\lambda f/d_1$ [Ref. 4], where d_1 is the pupil function of the lens L_1 and the phase change of a spherical wave with the curvature $f^2\Delta/f_1(f_1 + \Delta)$ does not exceed π within the domain of function existence. Then take the squared phase factor $\exp\left[-ikf_1(f_1 + \Delta)(x_4^2 + y_4^2)/2f^2\Delta\right]$ out of the integral of convolution with the function $P_1(x_4, y_4)$ in Eq. (2) for an area of $D_1 \le d_1 f\Delta/f_1(f_1 + \Delta)$ in diameter in the photoplate plane and obtain

$$u_1(x_4, y_4) \sim \exp\left[\frac{ik\Delta}{2f_2(f_2 + \Delta)}(x_4^2 + y_4^2)\right] \times$$
$$\times \left\{\exp\left[-\frac{ik\Delta}{2f_2(f_2 + \Delta)}(x_4^2 + y_4^2)\right]\left\{F(x_4, y_4) \otimes\right.\right.$$
$$\left.\left.\otimes \exp\left[-\frac{ikR(x_4^2 + y_4^2)}{2f^2}\right] \otimes P_1(x_4, y_4)\right\} \otimes P_2(x_4, y_4)\right\}. \quad (3)$$

The width of $P_2(x_4, y_4)$ is about $\lambda f_2(f_2 + \Delta)/d_2\Delta$ (d_2 is the pupil function of the lens L_2); therefore, assume that the phase change of a spherical wave with the curvature $f_2(f_2 + \Delta)/\Delta$ does not exceed π within the domain of the function existence. Again, take the squared phase factor $\exp\left[-ik\Delta(x_4^2 + y_4^2)/ /2f_2(f_2 + \Delta)\right]$ out of the integral of convolution with the function $P_2(x_4, y_4)$ in Eq. (3) for a photoplate area of $D_2 \le d_2$ in diameter and obtain

$$u_1(x_4, y_4) \sim F(x_4, y_4) \otimes \exp\left[-\frac{ikR}{2f^2}(x_4^2 + y_4^2)\right] \otimes$$
$$\otimes P_1(x_4, y_4) \otimes P_2(x_4, y_4). \quad (4)$$

It follows from Eq. (4), that the quasi-Fourier transform of $t(-x_1, -y_1)$ is formed in case of $d_1 f_2/(f_1 + \Delta) \le d_2$ in the photoplate plane within the diameter D_1, corresponding to the diameter of the microscope exit pupil at $R \ne \infty$. In this case, each point of the above area is extended up to the size of the subjective speckle, defined by the width of the function $P_1(x_4, y_4) \otimes P_2(x_4, y_4)$.

The distribution of the complex amplitude of the field, corresponding to the second exposure, in the object channel in the photoplate plane is defined by the equation (based on the Fourier transform properties)

$$u_2(x_4, y_4) \sim \exp\left[\frac{ik\Delta}{2f_2(f_2 + \Delta)}(x_4^2 + y_4^2)\right] \times$$
$$\times \left\{\exp\left[-\frac{ik\Delta}{2f_2(f_2 + \Delta)}(x_4^2 + y_4^2)\right] \times\right.$$
$$\times \left\{\exp\left(\frac{kax_4}{f}\right) F(x_4, y_4) \otimes \exp\left[-\frac{ikR}{2f^2}(x_4^2 + y_4^2)\right] \otimes\right.$$
$$\left.\left.\otimes P_1(x_4, y_4)\right\} \otimes P_2(x_4, y_4)\right\}. \quad (5)$$

With accounting for the known identity[5] and the convolution of functions

$$\exp\left[-ikR\left(x_4^2+y_4^2\right)/2f^2\right]\otimes$$
$$\otimes\exp\left[ikR\left(x_4^2+y_4^2\right)/2f^2\right]=\delta(x_4,y_4),$$

where $\delta(x_4,y_4)$ is the Dirac delta function, equation (5) takes the form

$$u_2(x_4,y_4)\sim\exp\left[\frac{ik\Delta}{2f_2(f_2+\Delta)}\left(x_4^2+y_4^2\right)\right]\times$$
$$\times\left\{\exp\left[-\frac{ik\Delta}{2f_2(f_2+\Delta)}\left(x_4^2+y_4^2\right)\right]\exp\left(\frac{ikax_4}{f}\right)\times\right.$$
$$\times\left\{F(x_4,y_4)\otimes\exp\left(-\frac{ikax_4}{f}\right)\exp\left[-\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes\right.$$
$$\otimes\exp\left(-\frac{ikax_4}{f}\right)P_1(x_4,y_4)\otimes\exp\left[\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes$$
$$\left.\left.\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\right\}\otimes P_2(x_4,y_4)\right\}. \quad (6)$$

The proof of the identity

$$\exp\left[\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes\exp\left(-\frac{ikax_4}{f}\right)\times$$
$$\times\exp\left[-\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes\exp\left(-\frac{ikax_4}{f}\right)P_1(x_4,y_4)=$$
$$=\exp\left(-\frac{ika^2}{2R}\right)\exp\left(-\frac{ikax_4}{f}\right)P_1\left(x_4+\frac{f}{R}a,y_4\right)$$

follows from the integral representation of convolution operation in Eq. (6). Hence, the distribution of the complex amplitude of the field, corresponding to the second exposure, in the object channel in the photoplate plane within the above area is defined by the equation

$$u_2(x_4,y_4)\sim\exp\left(\frac{ikax_4}{f}\right)\left\{F(x_4,y_4)\exp\left[-\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes\right.$$
$$\otimes\exp\left(-\frac{ik}{2R}a^2\right)\exp\left(-\frac{ikax_4}{f}\right)P_1\left(x_4+\frac{f}{R}a,y_4\right)\otimes$$
$$\left.\otimes\exp\left(-\frac{ikax_4}{f}\right)P_2(x_4,y_4)\right\}. \quad (7)$$

According to Eq. (7), a transversal displacement of the scatterer is accompanied by a variation of slope angle of the subjective speckle-field, corresponding to the second exposure, to the value a/f relative to the speckle-filed of the first exposure. In addition, the subjective speckle component, caused by the diffraction of a plane wave on the microscope objective pupil, homogeneously displaces to af/R. In this case, the displacement value depends on the microscope focal length and the curvature of a spherical wave of the coherent radiation, used for the scatterer illumination while recording the hologram, and the displacement direction depends on the curvature sign.

If the double-exposure quasi-Fourier hologram is recorded at the linear part of the photo-material blackening curve, then the distribution of the complex amplitude of its transmittance, corresponding to the (–1)-st diffraction order, takes the form

$$\tau(x_4,y_4)\sim\exp(-ikx_4\sin\theta)\left\{F(x_4,y_4)\otimes\right.$$
$$\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes P_1(x_4,y_4)\otimes P_2(x_4,y_4)+$$
$$+\exp\left(-\frac{ik}{2R}a^2\right)\exp\left(\frac{ikax_4}{f}\right)\left\{F(x_4,y_4)\otimes\right.$$
$$\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes\exp\left(-\frac{ikax_4}{f}\right)\times$$
$$\left.\left.\times P_1\left(x_4+\frac{f}{R}a,y_4\right)\exp\left(-\frac{ikax_4}{f}\right)P_2(x_4,y_4)\right\}\right\}. \quad (8)$$

Let the diffraction field be spatially filtered at the stage of hologram reconstruction in the hologram plane on the optical axis with a round aperture in the screen p_0 (Fig. 2). In this case, within its diameter, the phase change kax_4/f does not exceed π. Then the distribution of the field complex amplitude at the spatial filter outlet is defined as

$$u(x_4,y_4)\sim p_0(x_4,y_4)\left\{F(x_4,y_4)\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes\right.$$
$$\otimes P_1(x_4,y_4)\otimes P_2(x_4,y_4)+\exp\left(-\frac{ik}{2R}a^2\right)\times$$
$$\times F(x_4,y_4)\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes\exp\left(-\frac{ikax_4}{f}\right)\times$$
$$\left.\times P_1\left(x_4+\frac{f}{R}a,y_4\right)\otimes\exp\left(-\frac{ikax_4}{f}\right)P_2(x_4,y_4)\right\}, \quad (9)$$

where $p_0(x_4,y_4)$ is the transmission function of the spatial filter.[6]

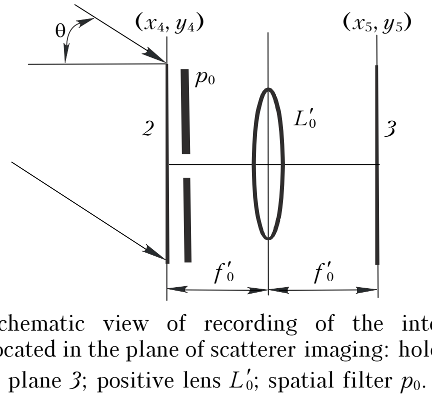


Fig. 2. Schematic view of recording of the interference pattern located in the plane of scatterer imaging: hologram *2*; recording plane *3*; positive lens L'_0; spatial filter p_0.

Let the focal length of the lens L_0' (Fig. 2) be equal to f_0'. Then, on the base of Ref. 7, the distribution of the field complex amplitude in its back focal plane (x_5, y_5) takes the form

$$u(x_5,y_5) \sim \left\{\left[p_1\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5,\frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]+\right.\right.$$
$$+\exp\left(\frac{ika^2}{2R}\right)\exp\left(\frac{ikf}{Rf_0'}ax_5\right)p_1\left(\frac{f}{f_0'}x_5+a,\frac{f}{f_0'}y_5\right)\times$$
$$\left.\times p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5+\frac{f_2(f_2+\Delta)}{f\Delta}a,\frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]\right]\times$$
$$\left.\times t\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)\exp\left[\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]\right\}\otimes P_0(x_5,y_5), \quad (10)$$

where $P_0(x_5, y_5)$ is the Fourier transform of $p_0(x_4, y_4)$ with the spatial frequencies $x_5/\lambda f_0'$ and $y_5/\lambda f_0'$.

If the variation period of $1 + \exp(ika_2/2R) \times \exp(ikfax_5/Rf_0')$ is at least one order of magnitude[8] larger than the width of $P_0(x_5, y_5)$, determining the size of subjective speckle in hologram recording plane *3* (see Fig. 2), within limits of overlap of the functions

$$p_1(fx_5/f_0', fy_5/f_0')p_2\left[f_2(f_2+\Delta)x_5/f_0'\Delta,\ f_2(f_2+\Delta)y_5/f_0'\Delta\right],$$
$$p_1(fx_5/f_0'+a, fy_5/f_0')p_2\left[f_2(f_2+\Delta)\times\right.$$
$$\left.\times x_5/f_0'\Delta+f_2(f_2+\Delta)a/f\Delta, f_2(f_2+\Delta)y_5/f_0'\Delta\right],$$

then we take it out of the convolution integral in Eq. (10). Again, with accounting for smallness of a and $f_2(f_2 + \Delta)a/f\Delta$, the light distribution in the plane (x_5, y_5) is defined by the equation

$$I(x_5,y_5) \sim \left[1+\cos\left(\frac{ka^2}{2R}+\frac{kfax_5}{Rf_0'}\right)\right]\left|p_1\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)\times\right.$$
$$\times p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5,\frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]t\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)\times$$
$$\left.\times\exp\left[\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]\otimes P_0(x_5,y_5)\right|^2. \quad (11)$$

It follows from Eq. (11) that in case of diffraction limitedness of the field, when the diameter D_0 of the illuminated area of the opaque screen (see Fig. 1) satisfies the condition $D_0 \geq d_1$, the subjective speckle structure is modulated by fringes, alternate on the x-axis, in the scatterer-imaging plane (Fourier plane). The fringe period $\Delta x_5 = \lambda R f_0'/af$ is independent of the sign of curvature of the spherical wave front of the coherent radiation, used for scatterer illumination at double-exposure recording of the quasi-Fourier hologram. In addition, the fringe frequency increases with a decrease in R at fixed λ, a, f, and f_0'.

This enhancement of interferometer sensitivity to a transversal displacement of the scatterer is explained by an increase in displacement of the component of the subjective speckle, corresponding to the second exposure, which is caused by the plane wave diffraction on the microscope objective pupil in the hologram plane. When $R = \infty$ and the distribution of the complex amplitude of the field, corresponding to the Fourier transform of the function $t(-x_1, -y_1)$, is formed in the plane of the photoplate *2* (see Fig. 1), an interference pattern does not formed in the Fourier plane at the stage of Fourier hologram reconstruction, where a “frozen” interference pattern is formed, and there is no need in spatial filtration of the diffraction field while recording.

Let the spatial filtration of the diffraction field be performed on the optical axis in the scatterer-imaging plane (x_5, y_5) (Fig. 3) at the stage of reconstruction of the double-exposure quasi-Fourier hologram. Then, based on the integral convolution representation, the distribution of the field complex amplitude of the (–1)-st diffraction order at the hologram output is written in the form

$$u(x_4,y_4) \sim \exp\left[-\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\left\{\exp\left[\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes\right.$$
$$\left.\otimes\, t\left(-\frac{R}{f}x_4,-\frac{R}{f}y_4\right)\right\}\otimes P_1(x_4,y_4)\otimes P_2(x_4,y_4)+$$
$$+\exp\left(\frac{ik}{2R}a^2\right)\exp\left(\frac{ikax_4}{f}\right)\exp\left[-\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\times$$
$$\times\left\{\exp\left[\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes t\left(-\frac{R}{f}x_4,-\frac{R}{f}y_4\right)\right\}\otimes$$
$$\otimes P_1\left(x_4+\frac{f}{R}a,y_4\right)\otimes P_2(x_4,y_4). \quad (12)$$

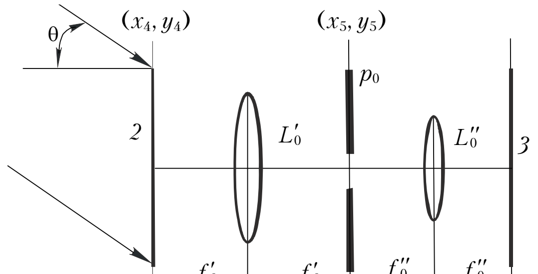


Fig. 3. Schematic view of recording of the interference pattern localizing in the hologram plane: hologram *2*; recording plane *3*; positive lenses L_0' and L_0''; spatial filter p_0.

Hence, neglecting the spatial limitedness of the field, define the distribution of its complex amplitude in the plane (x_5, y_5) (see Fig. 3) by the equation

$$u(x_5,y_5) \sim p_1\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5,\frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]\times$$
$$\times\left\{\exp\left[\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]\otimes F_1(x_5,y_5)\exp\left[-\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]\right\}+$$

$$+\exp\left(\frac{ika^2}{2R}\right)\exp\left(\frac{ikf}{f_0'R}ax_5\right)p_1\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)\times$$

$$\times p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5,\frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]\left\{\left\{\frac{ikf^2}{2Rf_0'^2}\left[\left(x_5-\frac{f_0'}{f}\right)^2+y_5^2\right]\right\}\otimes\right.$$

$$\left.\otimes F_1(x_5,y_5)\exp\left[-\frac{ikf^2}{2Rf_0'^2}\right]\left(x_5^2+y_5^2\right)\right\}, \qquad (13)$$

where $F_1(x_5, y_5)$ is the Fourier transform of the function $t(-R\xi/f, -R\eta/f)$ with the spatial frequencies $x_5/\lambda f'_0$ and $y_5/\lambda f'_0$.

If the phase change $(kfax_5/f'_0R) \le \pi$ within the diameter of the filtering aperture of the spatial filter p_0 (see Fig. 3), than the distribution of the field complex amplitude at its outlet takes the form

$$u(x_5,y_5)\sim p_0(x_5,y_5)\left\{\left\{\exp\left[\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]\otimes\right.\right.$$

$$\left.\otimes F_1(x_5,y_5)\exp\left[-\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]\right\}+\exp\left(\frac{ika^2}{2R}\right)\times$$

$$\times\left\{\exp\left\{\frac{ikf^2}{2Rf_0'^2}\left[\left(x_5-\frac{f_0'}{f}a\right)^2+y_5^2\right]\right\}\otimes\right.$$

$$\left.\left.\otimes F_1(x_5,y_5)\exp\left[-\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]\right\}\right\}, \qquad (14)$$

where $p_0(x_5, y_5)$ is the transmission function of the spatial filter.

Assume, for brevity, that hereinafter the focal length f''_0 of L''_0 is equal to f'_0 (see Fig. 3). Then the distribution of the field complex amplitude in its focal plane (x_6, y_6) is defined by the equation

$$u(x_6,y_6)\sim\left\{\exp\left[-\frac{ikR}{2f^2}\left(x_6^2+y_6^2\right)\right]\left\{\exp\left[\frac{ikR}{2f^2}\left(x_6^2+y_6^2\right)\right]\otimes\right.\right.$$

$$\left.\otimes t\left(\frac{R}{f}x_6,\frac{R}{f}y_6\right)\right\}+\exp\left(\frac{ik}{2R}a^2\right)\exp\left(-\frac{ikax_6}{f}\right)\times$$

$$\times\exp\left[-\frac{ikR}{2f^2}\left(x_6^2+y_6^2\right)\right]\left\{\exp\left[\frac{ikR}{2f^2}\left(x_6^2+y_6^2\right)\right]\otimes\right.$$

$$\left.\left.\otimes t\left(\frac{R}{f}x_6,\frac{R}{f}y_6\right)\right\}\right\}\otimes P_0(x_6,y_6), \qquad (15)$$

where $P_0(x_6, y_6)$ is the Fourier transform of $p_0(x_5, y_5)$ with the spatial frequencies $x_6/\lambda f'_0$ and $y_6/\lambda f'_0$.

If the variation period of $1 + \exp(ika^2/2R) \times \times \exp(-ikax_6/f)$ in Eq. (15) is at least one order of magnitude larger than the width of $P_0(x_6, y_6)$, determining the size of subjective speckle in recording plane *3* (see Fig. 3), then take it out of the integral of convergence with the function $P_0(x_6, y_6)$. Then, using the integral representation of the convergence, write the light distribution in the plane (x_6, y_6) in the form

$$I(x_6,y_6)\sim\left[1+\cos\left(\frac{ka^2}{2R}-\frac{kax_6}{f}\right)\right]\left|F(-x_6,-y_6)\otimes\right.$$

$$\left.\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_6^2+y_6^2\right)\right]\otimes P_0(x_6,y_6)\right|^2. \qquad (16)$$

It follows from Eq. (16) that an interference pattern in the form of fringes, alternating on the x-axis, is formed in the hologram-imaging plane, modulating the subjective speckle structure, when building a hologram image with the use of Kepler telescopic system with special filtration of the diffraction field in its partial plane. In this case, the fringe period $\Delta x_6 = \lambda f/a$ is independent of the curvature of a spherical wave of the coherent radiation, used for scatterer illumination at the stage of hologram recording, while the interferometer sensitivity to transversal displacement of the scatterer depends on the coefficient, determining the scale of the Fourier transform of the function $t(-x_1, -y_1)$. Besides, if the diameter of collimated beam at the stage of hologram reconstruction exceeds the above D_1 value, which is larger than the lens L'_0 diameter, then spatial extension of the interference pattern, localizing in the hologram plane, is limited to the domain of existence of the Fourier transform of the function $t(-x_1, -y_1)$.

To analyze the behavior dynamics of the fringes, equidistance on the x-axis, in the planes of their localization (Fourier and hologram planes), assume that the spatial filtration of the diffraction field in the hologram plane is fulfilled beyond the optical axis, i.e., the filtering aperture in Fig. 2 is centered at $(x_{04}, 0)$. Its diameter is much larger than the domain of existence of the function $P_1(x_4, y_4) \otimes P_2(x_4, y_4)$ and the phase change $(kax_4/f) \le \pi$ within the diameter; hence, the distribution of the field complex amplitude at the spatial filter outlet takes the form

$$u(x_4,y_4)\sim p_0(x_4,y_4)\left\{F(x_4+x_{04},y_4)\otimes\right.$$

$$\otimes\exp\left\{-\frac{ikR}{2f^2}\left[\left(x_4+x_{04}\right)^2+y_4^2\right]\right\}\exp\left(\frac{ikx_{04}x_4}{f}\right)P_1(x_4,y_4)\otimes$$

$$\otimes\exp\left(\frac{ikx_{04}x_4}{f}\right)P_2(x_4,y_4)+\exp\left(\frac{ikax_{04}}{f}\right)\exp\left(-\frac{ik}{2R}a^2\right)\times$$

$$\times F(x_4+x_{04},y_4)\otimes\exp\left\{-\frac{ikR}{2f^2}\left[\left(x_4+x_{04}\right)^2+y_4^2\right]\right\}\otimes$$

$$\otimes\exp\left(\frac{ikx_{04}x_4}{f}\right)\exp\left[-\frac{ik(x_4+x_{04})a}{f}\right]P_1(x_4,y_4)\otimes$$

$$\left.\otimes\exp\left(\frac{ikx_{04}x_4}{f}\right)\exp\left[-\frac{ik(x_4+x_{04})a}{f}\right]P_2(x_4,y_4)\right\}. \qquad (17)$$

As a result of the Fourier transformation, this distribution in the Fourier plane (x_5, y_5) is defined as

$$u(x_5,y_5) \sim \Bigg\{p_1\left(\frac{f}{f_0'}x_5 - x_{04}, \frac{f}{f_0'}y_5\right)\times$$
$$\times p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5 - \frac{f_2(f_2+\Delta)}{f\Delta}x_{04}, \frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]\times$$
$$\times t\left(\frac{f}{f_0'}x_5, \frac{f}{f_0'}y_5\right)\exp\left[\frac{ikf^2}{2Rf_0'^2}(x_5^2+y_5^2)\right]\times$$
$$\times \exp\left[\frac{i2kx_{04}x_5}{f_0'}\right] + p_1\left(\frac{f}{f_0'}x_5 + a - x_{04}, \frac{f}{f_0'}y_5\right)\times$$
$$\times p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5 + \frac{f_2(f_2+\Delta)}{f\Delta}(a - x_{04}), \frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]\times$$
$$\times \exp\left(\frac{ikfax_5}{Rf_0'}\right)\exp\left(-\frac{ik}{f}ax_{04}\right)\times$$
$$\times \exp\left(-\frac{ikax_{04}}{R}\right)\Bigg\} \otimes P_0(x_5,y_5). \quad (18)$$

It follows from this and the accounting for $x_{04} \gg a$ that the light distribution in the recording plane *3* (see Fig. 2) takes the form

$$I(x_5,y_5) \sim \left[1+\cos\left(\frac{ka^2}{2R} + \frac{kfax_5}{Rf_0'} - \frac{kax_{04}}{f} - \frac{kax_{04}}{R}\right)\right]\times$$
$$\times \Bigg| p_1\left(\frac{f}{f_0'}x_5 - x_{04}, \frac{f}{f_0'}y_5\right)\times$$
$$\times p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5 - \frac{f_2(f_2+\Delta)}{f\Delta}x_{04}, \frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]\times$$
$$\times t\left(\frac{f}{f_0'}x_5, \frac{f}{f_0'}y_5\right)\exp\left[\frac{ikf^2}{2Rf_0'^2}(x_5^2+y_5^2)\right]\times$$
$$\times \exp\left[\frac{i2kx_{04}x_5}{f_0'}\right] \otimes P_0(x_5,y_5)\Bigg|^2. \quad (19)$$

The comparison of Eqs. (11) and (19) shows that when the filtering aperture center is displaced on the x-axis in the hologram plane, the interference pattern (along with pupil images of the lens L_1 and L_2 (Fig. 1)) is displaced relative to a static image of the scatterer due to the fringe parallax, caused by homogeneous displacement of the above component of the subjective speckle, corresponding to the second exposure, and properties of subjective speckles. In this case, the magnitude of displaced fringes depends not only on the curvature of a spherical wave of the coherent radiation, used for scatterer illumination while recording the hologram, but also on its sign. In a particular case of convergent spherical wave with $R = f$, the light distribution in the plane (x_5, y_5) (see Fig. 2) is defined by the equation

$$I(x_5,y_5) \sim \left[1+\cos\left(-\frac{ka^2}{2f} - \frac{kax_5}{f} + \frac{kax_{04}}{f}\right)\right]\times$$
$$\times \Bigg| p_1\left(\frac{f}{f_0'}x_5 - x_{04}, \frac{f}{f_0'}y_5\right)\times$$
$$\times p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5 - \frac{f_2(f_2+\Delta)}{f\Delta}x_{04}, \frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]\times$$
$$\times t\left(\frac{f}{f_0'}x_5, \frac{f}{f_0'}y_5\right)\exp\left[\frac{-ikf}{2f_0'^2}(x_5^2+y_5^2)\right]\times$$
$$\times \exp\left[\frac{i2kx_{04}x_5}{f_0'}\right] \otimes P_0(x_5,y_5)\Bigg|^2. \quad (20)$$

Besides, while varying x_{04}, the interference pattern phase changes by π when displacing the filtering aperture center, e.g., from the interference pattern minimum, located in the hologram plane, to its maximum ("living" fringes).

If the spatial filtration of the diffraction filed is carried out in the Fourier plane (x_5, y_5) (Fig. 3) at the point $(x_{05}, 0)$, then the distribution of the field complex amplitude at the filtering aperture outlet, within diameter of which the phase change $kfax_5/Rf'_0 \le \pi$, takes the form

$$u(x_5,y_5) \sim p_0(x_5,y_5)\Bigg\{\exp\left\{\frac{ikf^2}{2Rf_0'^2}\left[(x_5+x_{05})^2 + y_5^2\right]\right\}\otimes$$
$$\otimes F_1(x_5+x_{05},y_5)\exp\left\{-\frac{ikf^2}{2Rf_0'^2}\left[(x_5+x_{05})^2+y_5^2\right]\right\}+$$
$$+\exp\left(\frac{ika^2}{2R}\right)\exp\left(\frac{ikfax_{05}}{f_0'R}\right)\times$$
$$\times \exp\left\{\frac{ikf^2}{2Rf_0'^2}\left[\left(x_5+x_{05}-\frac{f_0'}{f}a\right)^2+y_5^2\right]\right\}\otimes$$
$$\otimes F_1(x_5+x_{05},y_5)\exp\left\{-\frac{ikf^2}{2Rf_0'^2}\left[(x_5+x_{05})^2+y_5^2\right]\right\}\Bigg\}. \quad (21)$$

In this case, the field complex amplitude distribution, resulting from the Fourier transform, in the plane (x_6, y_6) (see Fig. 3) is defined as

$$u(x_6,y_6) \sim \Bigg\{\exp\left(\frac{ikx_{05}x_6}{f_0'}\right)\exp\left[-\frac{ikR}{2f^2}(x_6^2+y_6^2)\right]\times$$
$$\times\left\{\exp\left(\frac{ikx_{05}x_6}{f_0'}\right)\exp\left[\frac{ikR}{2f^2}(x_6^2+y_6^2)\right]\otimes\right.$$
$$\left.\otimes t\left(\frac{R}{f}x_6,\frac{R}{f}y_6\right)\exp\left(\frac{ikx_{05}x_6}{f_0'}\right)\right\} + \exp\left(\frac{ika^2}{2R}\right)\exp\left(\frac{ikfax_{05}}{f_0'R}\right)\times$$
$$\times\exp\left[\frac{ik}{f_0'}\left(x_{05}-\frac{f_0'}{f}a\right)x_6\right]\exp\left[-\frac{ikR}{2f^2}(x_6^2+y_6^2)\right]\times$$
$$\times\left\{\exp\left(\frac{ikx_{05}x_6}{f_0'}\right)\exp\left[\frac{ikR}{2f^2}(x_6^2+y_6^2)\right]\otimes t\left(\frac{R}{f}x_6,\frac{R}{f}y_6\right)\times\right.$$
$$\left.\times\exp\left(\frac{ikx_{05}x_6}{f_0'}\right)\right\}\Bigg\}\otimes P_0(x_6,y_6). \quad (22)$$

Based on Eq. (22) and with accounting for the known identity[5] and integral convolution representation, the light distribution in the hologram-imaging plane (x_6, y_6) takes the form

$$I(x_6,y_6) \sim \left[1+\cos\left(\frac{ka^2}{2R}-\frac{kax_6}{f}+\frac{kfax_{05}}{f_0'R}\right)\right]\times$$
$$\times\left|\exp\left(\frac{i2kx_{05}x_6}{f_0'}\right)\left\{F(-x_6,-y_6)\otimes\right.\right.$$
$$\left.\left.\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_6^2+y_6^2\right)\right]\right\}\otimes P_0(x_6,y_6)\right|^2. \quad (23)$$

It follows from the comparison of Eqs. (16) and (23), that when the filtering aperture center is displaced on the x-axis in the Fourier plane, the interference pattern displaces due to the fringe parallax, caused by a homogeneous displacement of the above component of the subjective speckle, corresponding to the second exposure. Besides, while varying x_{05}, the interference pattern phase changes by π when displacing the filtering aperture center, e.g., from the interference pattern minimum, located in the Fourier plane, to its maximum.

It follows from the above analysis of forming the interference patterns, characterizing the transversal scatterer displacement, that they localize in the hologram and Fourier planes as in case of holographic interferometer.[1] However, in the considered holograph interpherometer, the holograph pattern is localized in the scatterer-imaging plane, is formed in the Fourier plane, where identical speckles of two exposures are matched at spatial filtering of the diffraction field in the hologram plane. In this case, the interferometer sensitivity to the transversal displacement of the scatterer depends on the microscope focal length and the curvature of the spherical wave front of the coherent radiation, used for scatterer illumination when recording the hologram. Besides, as in Ref. 1, the fringe parallax effect is characteristic for the interference pattern, located in the scatterer-imaging plane.

The slope angle of the subjective speckle-field, corresponding to the second exposure, in the hologram plane results in formation of an interference pattern at the spatial filtering of the diffraction field in the Fourier plane. In this case, the interferometer sensitivity to transversal displacement depends on the microscope focal length; the fringe parallax takes place due to the displacement of subjective speckles, corresponding to the second exposure, in the hologram plane.

Let opaque screen *1* be z-axial displaced to $\Delta l \ll l_1, R$ before the second exposure of photoplate *2* (see Fig. 1). Then in the used approximation, the distribution of the field complex amplitude, corresponding to the second exposure, in the object channel in the photoplate plane is written as

$$u_2'(x_4,y_4) \sim \exp(ik\Delta l)\iiint\int_{-\infty}^{\infty}\iint t(x_1,y_1)\times$$
$$\times\exp\left[\frac{ik}{2(R-\Delta l)}\left(x_1^2+y_1^2\right)\right]\times$$
$$\times\exp\left\{\frac{ik}{2(l_1+\Delta l)}\left[(x_1-x_2)^2+(y_1-y_2)^2\right]\right\}\times$$
$$\times p_1(x_2,y_2)\exp\left[-\frac{ik}{2f_1}\left(x_2^2+y_2^2\right)\right]\times$$
$$\times\exp\left\{\frac{ik}{2(f_1+\Delta+f_2)}\left[(x_2-x_3)^2+(y_2-y_3)^2\right]\right\}\times$$
$$\times p_2(x_3,y_3)\exp\left[-\frac{ik}{2f_2}\left(x_3^2+y_3^2\right)\right]\times$$
$$\times\exp\left\{\frac{ik}{2l_2}\left[(x_3-x_4)^2+(y_3-y_4)^2\right]\right\}\mathrm{d}x_1\mathrm{d}y_1\mathrm{d}x_2\mathrm{d}y_2\mathrm{d}x_3\mathrm{d}y_3. \quad (24)$$

Equation (24) after transformation takes the form

$$u_2'(x_4,y_4) \sim \exp(ik\Delta l)\exp\left[-\frac{ik\Delta l}{2f^2}\left(x_4^2+y_4^2\right)\right]\times$$
$$\times\left\{F(x_4,y_4)\otimes\exp\left[-\frac{ik(R-\Delta l)}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes\right.$$
$$\left.\otimes P_1(x_4,y_4)\otimes P_2(x_4,y_4)\right\}. \quad (25)$$

According to Eq. (25), the squared exponential phase factor $\exp\left[-ik\Delta l(x_4^2+y_4^2)/2f^2\right]$ characterizes the total slope angle and a slope angle, radially varied from the optical axis of the subjective speckle field, corresponding to the second exposure, relative to the speckle field of the first exposure; an additional radial variation of the slope angle from the optical axis takes place in the photoplate plane of the subjective speckles relative to the similar speckles of the first exposure due to the phase factor $\exp\left[-ik(R-\Delta l)(x_4^2+y_4^2)/2f^2\right]$ under the convolution integral. In contrast to the previous slope angle, this one depends on the curvature of the spherical wave of the coherent radiation, used for scatterer illumination when recording the hologram.

In the considered case of controlling the scatterer transversal displacement in the double-exposure recording of the quasi-Fourier hologram on the linear part of the photomaterial blackening curve, the distribution of the complex amplitude of the hologram transmission, corresponding to the (–1)-st diffraction order, on the base of Eqs. (4) and (25) takes the form

$$\tau'(x_4,y_4) \sim \exp(-ikx_4\sin\theta)\left\{F(x_4,y_4)\otimes\right.$$
$$\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes P_1(x_4,y_4)\otimes P_2(x_4,y_4)+$$

$$+\exp(ik\Delta l)\exp\left[-\frac{ik\Delta l}{2f^2}\left(x_4^2+y_4^2\right)\right]\left\{F(x_4,y_4)\otimes\right.$$

$$\left.\left.\otimes\exp\left[-\frac{ik(R-\Delta l)}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes P_1(x_4,y_4)\otimes P_2(x_4,y_4)\right\}\right\}. \quad (26)$$

If spatial filtration of the diffraction field is carried out at the stage of hologram reconstruction (see Fig. 2) in its plane on the optical axis, then assume that the phase change $\left[-k\Delta l(x_4^2+y_4^2)/2f^2\right]$ does not exceed π within the filtering aperture diameter. Then the distribution of the field complex amplitude at the outlet of the spatial filter p_0 is defined by the equation

$$u'(x_4,y_4)\sim p_0(x_4,y_4)\left\{F(x_4,y_4)\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes\right.$$

$$\otimes P_1(x_4,y_4)\otimes P_2(x_4,y_4)+\exp(ik\Delta l)\left\{F(x_4,y_4)\otimes\right.$$

$$\left.\left.\otimes\exp\left[-\frac{ik(R-\Delta l)}{2f^2}\left(x_4^2+y_4^2\right)\right]\otimes P_1(x_4,y_4)\otimes P_2(x_4,y_4)\right\}\right\}. \quad (27)$$

After the Fourier transformation, the distribution of the field complex amplitude in the plane (x_5,y_5) (see Fig. 2) takes the form

$$u'(x_5,y_5)\sim\left\{p_1\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5,\frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]\times\right.$$

$$\times t\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)\exp\left[\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]\times$$

$$\left.\times\left\{1+\exp(ik\Delta l)\exp\left[\frac{ikf^2\Delta l}{2R^2f_0'^2}\left(x_5^2+y_5^2\right)\right]\right\}\right\}\otimes P_0(x_5,y_5). \quad (28)$$

If the variation period of the function $1+\exp(ik\Delta l)\exp\left[ikf^2\Delta l(x_5^2+y_5^2)/2R^2f_0'^2\right]$ is at least one order of magnitude larger than the width of the function $P_0(x_5,y_5)$, then take it out of the convolution integral in Eq. (28). Then the light distribution in recording plane *3* (see Fig. 2) is defined as

$$I'(x_5,y_5)\sim\left\{1+\cos\left[k\Delta l+\frac{kf^2\Delta l}{2R^2f_0'^2}\left(x_5^2+y_5^2\right)\right]\right\}\times$$

$$\times\left|p_1\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5,\frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]\times\right.$$

$$\left.\times\, t\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)\exp\left[\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]P_0(x_5,y_5)\right|^2. \quad (29)$$

It follows from Eq. (29) that a subjective speckle structure in the Fourier plane of the scatterer imaging, restricted by the microscope viewing angle, is modulated by fringes of equal slope – the system of concentric interference rings. Measurement of their radiuses in neighboring fringe orders allows definition of transversal displacement of the scatterer for the known variables λ, R, f, and f_0'. In this case, the interferometer sensitivity to the transversal displacement is independent of sign of the curvature radius R and increases with the decrease in its value due to an increase of the slope angle, radially varying from the optical axis, of subjective speckles, corresponding to the second exposure, with respect to the similar speckles of the first exposure.

If spatial filtration of the diffraction filed is carried out in the scatterer-imaging plane (x_5,y_5) (see Fig. 3) at the stage of the double-exposure quasi-Fourier hologram reconstruction, then, neglecting the spatial limitedness of the field, the distribution of its complex amplitude in the above plane takes the form

$$u'(x_5,y_5)\sim p_1\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5,\frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]\times$$

$$\times t\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)\exp\left[\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]+$$

$$+\exp(ik\Delta l)\exp\left[\frac{ikf^2}{2f_0'^2\Delta l}\left(x_5^2+y_5^2\right)\right]\otimes\left\{p_1\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)\times\right.$$

$$\times p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5,\frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]t\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)\times$$

$$\left.\times\exp\left[\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]\exp\left[\frac{ikf^2\Delta l}{2R^2f_0'^2}\left(x_5^2+y_5^2\right)\right]\right\}. \quad (30)$$

If the center of filtering aperture is on the optical axis, and the phase change $\left[kf^2\Delta l(x_4^2+y_4^2)/2R^2f_0'^2\right]\le\pi$, within it, then, accounting for the fact that $\exp\left[ikf^2(x_4^2+y_4^2)/2f_0'^2\Delta l\right]\approx\delta(x_5,y_5)$ in the order of magnitude, the distribution of field complex amplitude at the spatial filter outlet is defined as

$$u'(x_5,y_5)\sim p_0(x_5,y_5)\left\{t\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)\exp\left[\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]+\right.$$

$$+\exp(ik\Delta l)\exp\left[\frac{ikf^2}{2f_0'^2\Delta l}\left(x_5^2+y_5^2\right)\right]\otimes$$

$$\left.\otimes t\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)\exp\left[\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]\right\}. \quad (31)$$

After the Fourier transformation, the distribution of the field complex amplitude in the plane (x_6,y_6) (see Fig. 3) takes the form

$$u'(x_6,y_6)\sim\left\{F(-x_6,-y_6)\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_6^2+y_6^2\right)\right]\right\}\times$$

$$\times\left\{1+\exp(ik\Delta l)\exp\left[-\frac{ik\Delta l}{2f^2}\left(x_6^2+y_6^2\right)\right]\right\}\otimes P_0(x_6,y_6). \quad (32)$$

Based on Eq. (32), when the variation period of the function $1+\exp(ik\Delta l)\exp\left[-ik\Delta l(x_6^2+y_6^2)/2f^2\right]$ is at least one order of magnitude larger then the width of $P_0(x_6,y_6)$, the light distribution in recording plane *3* is defined by the equation

$$I'(x_6,y_6)\sim\left\{1+\cos\left[k\Delta l-\frac{k\Delta l}{2f^2}\left(x_6^2+y_6^2\right)\right]\right\}\times$$
$$\times\left|F(-x_6,-y_6)\otimes\exp\left[-\frac{ikR}{2f^2}\left(x_6^2+y_6^2\right)\right]\otimes P_0(x_6,y_6)\right|^2. \quad (33)$$

According to Eq. (33), a subjective speckle structure in the hologram plane is modulated by fringes of equal slope – the system of concentric interference rings. In this case, the interferometer sensitivity to the longitudinal displacement of the scatterer depends on the microscope focal length and is independent of the curvature of the spherical wave front of the coherent radiation, used for scatterer illumination when recording the hologram. Besides, as in the case of controlling the transversal displacement of the scatterer, taking into account the above conditions of the hologram reconstruction, the spatial extension of the interference pattern is limited to the domain of existence of the Fourier transform of the function $t(-x_1,-y_1)$. When $R=\infty$ and the Fourier transform of the function $t(-x_1,-y_1)$ is formed in the hologram plane, a "frozen" interference pattern is formed there, recording of which does not require spatial filtration of the diffraction field.

To analyze the behavior dynamics of the fringes, characterizing longitudinal displacement of the scatterer, assume that the spatial filtration of the diffraction field in the hologram plane is fulfilled beyond the optical axis, i.e., the filtering aperture in Fig. 2 is centered at $(x_{04},0)$. Hence, the distribution of the field complex amplitude at the spatial filter outlet takes the form

$$u'(x_4,y_4)\sim p_0(x_4,y_4)\left\{F(x_4+x_{04},y_4)\otimes\right.$$
$$\otimes\exp\left\{-\frac{ikR}{2f^2}\left[(x_4+x_{04})^2+y_4^2\right]\right\}\otimes\exp\left(\frac{ikx_{04}x_4}{f}\right)\times$$
$$\times P_1(x_4,y_4)\otimes\exp\left(\frac{ikx_{04}x_4}{f}\right)P_2(x_4,y_4)+\exp(ik\Delta l)\times$$
$$\times\exp\left(-\frac{ik\Delta l x_{04}^2}{2f^2}\right)F(x_4+x_{04},y_4)\otimes$$
$$\otimes\exp\left\{-\frac{ik(R-\Delta l)}{2f^2}\left[(x_4+x_{04})^2+y_4^2\right]\right\}\otimes\exp\left(\frac{ikx_{04}x_4}{f}\right)\times$$
$$\left.\times P_1(x_4,y_4)\otimes\exp\left(\frac{ikx_{04}x_4}{f}\right)P_2(x_4,y_4)\right\}. \quad (34)$$

After the Fourier transformation, the distribution of the field complex amplitude in the scatterer-imaging plane (x_5,y_5) (see Fig. 2) is defined by the equation

$$u'(x_5,y_5)\sim\left\{p_1\left(\frac{f}{f_0'}x_5-x_{04},\frac{f}{f_0'}y_5\right)\times\right.$$
$$\times p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5-\frac{f_2(f_2+\Delta)}{f\Delta}x_{04},\frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]\times$$
$$\times t\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)\exp\left[\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]\exp\left(\frac{i2kx_{04}x_5}{f_0'}\right)\times$$
$$\times\left\{1+\exp(ik\Delta l)\exp\left(-\frac{ik\Delta l x_{04}^2}{2f^2}\right)\times\right.$$
$$\left.\left.\times\exp\left[\frac{ikf^2\Delta l}{2R^2f_0'^2}\left(x_5^2+y_5^2\right)\right]\right\}\right\}\otimes P_0(x_5,y_5), \quad (35)$$

on the base of which the light distribution in recording plane *3* (see Fig. 2) takes the form

$$I'(x_5,y_5)\sim\left\{1+\cos\left[k\Delta l+\frac{kf^2\Delta l}{2R^2f_0'^2}\left(x_5^2+y_5^2\right)-\frac{k\Delta l x_{04}^2}{2f^2}\right]\right\}\times$$
$$\times\left|p_1\left(\frac{f}{f_0'}x_5-x_{04},\frac{f}{f_0'}y_5\right)\times\right.$$
$$\times p_2\left[\frac{f_2(f_2+\Delta)}{f_0'\Delta}x_5-\frac{f_2(f_2+\Delta)}{f\Delta}x_{04},\frac{f_2(f_2+\Delta)}{f_0'\Delta}y_5\right]\times$$
$$\times t\left(\frac{f}{f_0'}x_5,\frac{f}{f_0'}y_5\right)\exp\left[\frac{ikf^2}{2Rf_0'^2}\left(x_5^2+y_5^2\right)\right]\times$$
$$\left.\times\exp\left(\frac{i2kx_{04}x_5}{f_0'}\right)\otimes P_0(x_5,y_5)\right|^2. \quad (36)$$

It follows from the comparison of Eqs. (29) and (36), that in case of filtering aperture center displacement in the hologram plane, the interference pattern center is fixed relative to the immovable scatterer image, i.e., the fringe parallax is absent. In this case, while varying x_{04}, the interference pattern phase changes by π when displacing the filtering aperture center, e.g., from the interference pattern maximum, located in the hologram plane, to its minimum.

If, according to Fig. 3, the spatial filtration of the diffraction field at the stage of double-exposure quasi-Fourier hologram reconstruction is fulfilled beyond the optical axis, e.g., the filtering aperture is centered at the point $(x_{05},0)$, then the distribution of the field complex amplitude at the spatial filter outlet is defined by the equation

$$u'(x_5,y_5)\sim p_0(x_5,y_5)\left\{t\left[\frac{f}{f_0'}(x_5+x_{05}),\frac{f}{f_0'}y_5\right]\times\right.$$
$$\times\exp\left\{\frac{ikf^2}{2Rf_0'^2}\left[(x_5+x_{05})^2+y_5^2\right]\right\}+\exp(ik\Delta l)\times$$
$$\times\exp\left(\frac{ikf^2\Delta l x_{05}^2}{2R^2f_0'^2}\right)\exp\left[\frac{ikf^2}{2f_0'^2\Delta l}\left(x_5^2+y_5^2\right)\right]\otimes$$
$$\left.\otimes t\left[\frac{f}{f_0'}(x_5+x_{05}),\frac{f}{f_0'}y_5\right]\exp\left\{\frac{ikf^2}{2Rf_0'^2}\left[(x_5+x_{05})^2+y_5^2\right]\right\}\right\}. \quad (37)$$

After the Fourier transformation, the distribution of the field complex amplitude in the plane (x_6, y_6) (see Fig. 3) takes the form

$$u'(x_6,y_6) \sim \left\{\exp\left(\frac{ikf}{f_0'^2}x_{05}x_6\right)F(-x_6,-y_6)\otimes\exp\left(\frac{ikx_{05}x_6}{f_0'}\right)\times\right.$$
$$\times\exp\left[-\frac{ikR}{2f^2}\left(x_6^2+y_6^2\right)\right]\Bigg\}\left\{1+\exp(ik\Delta l)\exp\left(\frac{ikf^2\Delta l x_{05}^2}{2Rf_0'^2}\right)\times\right.$$
$$\left.\times\exp\left[-\frac{ik\Delta l}{2f^2}\left(x_6^2+y_6^2\right)\right]\right\}\otimes P_0(x_6,y_6), \qquad (38)$$

on the base of which the light distribution in recording plane *3* (see Fig. 3) is defined as

$$I'(x_6,y_6) \sim \left\{1+\cos\left[k\Delta l-\frac{k\Delta l\left(x_6^2+y_6^2\right)}{2f^2}+\frac{kf^2\Delta l x_{05}^2}{2R^2f_0'^2}\right]\right\}\times$$
$$\times\left|\exp\left(\frac{ikf}{f_0'^2}x_{05}x_6\right)F(-x_6,-y_6)\otimes\exp\left(\frac{ikx_{05}x_6}{f_0'}\right)\times\right.$$
$$\left.\times\exp\left[-\frac{ikR}{2f^2}\left(x_6^2+y_6^2\right)\right]\otimes P_0(x_6,y_6)\right|^2. \qquad (39)$$

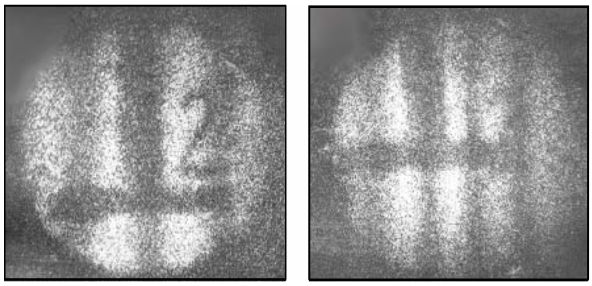
It follows from the comparison of Eqs. (33) and (39), that in case of filtering aperture center displacement in the Fourier plane, the interference pattern center is fixed, i.e., the fringe parallax is absent. In this case, while varying x_{05}, the interference pattern phase changes by π when displacing the filtering aperture center, e.g., from the interference pattern maximum, located in the hologram plane, to its minimum.

It follows from the above analysis that the formed interference patterns, characterizing longitudinal displacement of the scatterer, localize in two planes, i.e., the hologram and Fourier ones, like in the holographic interferometer.[1] However, in the considered holographic interferometer, an interference pattern localized in the scatterer-imaging plane, is formed in the Fourier plane, where subjective speckles of two exposures are identical due to spatial filtration of the diffraction field in the hologram plane. In this case, the interferometer sensitivity to the longitudinal displacement of the scatterer depends on the microscope focal length and the curvature of the spherical wave of the coherent radiation, used for scatterer illumination while recording the hologram. In its turn, the interferometer sensitivity to the interference pattern, localized in the hologram plane and recorded in the plane of its imaging, where identical speckles of two exposures are formed while spatial filtering in the Fourier plane, depends on the microscope focal length.

In addition, similarly to the holographic interferometer,[1] the absence of the fringe parallax is characteristic for the longitudinal displacement control in the considered interferometer. Here the mechanism of interference pattern formation is connected with the slope angle of the subjective speckles, corresponding to the second exposure, radially varying from the optical axis, in the hologram plane, while in Ref. 1 it is also connected with speckle extension in the hologram plane.

In our experiment, double-exposure quasi-Fourier and Fourier holograms were recorded on Mikrat VRL photoplates by means of 0.6328 μm He–Ne laser radiation; microscope parameters were the following: $f_1 = 50$ mm, $f_2 = 90$ mm, $\Delta = 100$ mm, $d_1 = 15$ mm, $d_2 = 20$ mm. The angle $\theta = 11°$ for a plane reference beam of 35 mm in diameter. The curvature range was $30 \le |R| \le \infty$ for divergent and convergent spherical waves of the coherent radiation, used for illumination of the opaque screen; the diameter of illuminated area was 20 mm. The experimental technique consisted in comparison of hologram records for fixed values of transversal $a = (0.02 \pm 0.002)$ mm and longitudinal $\Delta l = (0.5 \pm 0.002)$ mm displacements of the scatterer.

Figure 4 shows the interference patterns, localized in the Fourier plane, where the image of opaque screen is formed, and characterizing transversal displacement of the screen. The interference patterns were recorded in the lens focal plane with $f_0' = 50$ mm when spatial filtering the diffraction field in the hologram plane by means of its reconstruction with a small-aperture (≈ 1 mm) laser beam. The opaque screen was illuminated by the coherent radiation with convergent ($R = 70$ mm, Fig. 4*a*) and divergent ($R = 45$ mm, Fig. 4*b*) spherical waves. The letter "T" (blowup and flip image in the Fourier plane) was preliminary drawn on the opaque screen and the mark "2" – on the L_2 lens side (see Fig. 1).



a *b*

Fig. 4. Interference patterns localized in the plane of scatterer imaging and characterizing its transversal displacement: the scatterer is illuminated by radiation with convergent (*a*) and divergent (*b*) spherical waves.

The interference pattern, localized in the hologram plane and characterizing the transversal displacement of the scatterer, is shown in Fig. 5*a*. It was recorded at illuminating the hologram (see Fig. 3) by a collimated beam of 30 mm in diameter; spatial filtration of the diffraction field was carried out in the focal plane of the lens L_0' (see Fig. 3) of 40 mm in diameter and 200 mm in focal length. In this case, the spatial extension of the interference pattern, localized in the hologram plane, was 9 mm and corresponded to the calculated value.

The periods Δx_5 and Δx_6 of the fringes, localized both in the Fourier and hologram planes, were calculated for the known variables λ, a, R, f, f'_0 and compared with measurement results. They agree to each other up to 10% error, allowable in the experiment.

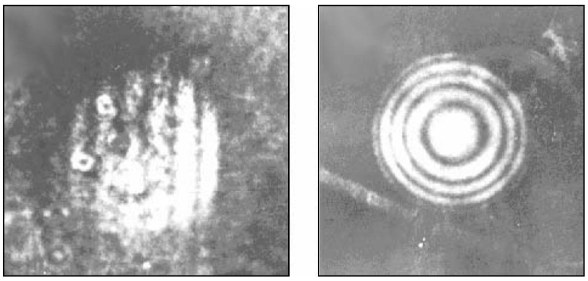


Fig. 5. Interference patterns localized in the hologram plane and characterizing transversal (a) and longitudinal (b) displacement of the scatterer.

The interference patterns in Fig. 6 are localized in the plane of opaque screen imaging and characterize its longitudinal displacement, when the scatterer is illuminated by the coherent radiation with convergent ($R = 50$ mm, Fig. 6a) and divergent ($R = 40$ mm, Fig. 6b) spherical waves at the stage of hologram recording. The pattern was recorded similarly to the interference patterns characterizing the transversal displacement of the scatterer and localized in the Fourier plane.

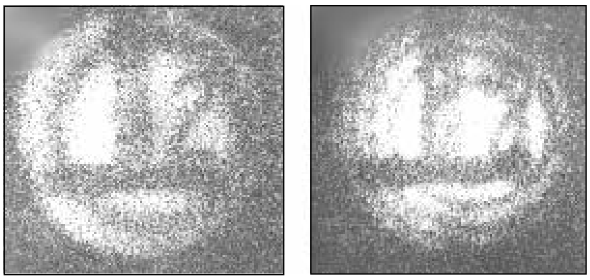


Fig. 6. Interference patterns localized in the Fourier plane and characterizing transversal displacement of the opaque screen in its illuminating by coherent radiation with convergent (a) and divergent (b) spherical waves.

The interference pattern, localized in the hologram plane and characterizing the longitudinal scatterer displacement, corresponds to Fig. 5b; it was recorded similarly to the pattern, characterizing the transversal scatterer displacement and localized in the hologram plane.

For the interference patterns in Fig. 6, the magnitude of transversal displacement is

$$\Delta l = 2\lambda R^2 f_0'^2 / f^2 \left(r_2^2 - r_1^2\right),$$

where r_1 and r_2 are the radii of interference rings in the neighboring interference orders. Hence, the longitudinal displacement of the opaque screen was determined for the known variables: λ, R, f, f'_0 and measured r_1 and r_2 and compared with value of $\Delta l = (0.5 \pm 0.002)$ mm. For the interference pattern in Fig. 5b, the longitudinal displacement $\Delta l = 2\lambda f^2 / (r_2'^2 - r_1'^2)$. In this case, the longitudinal displacement of the opaque screen was determined for the known λ, f, and the value of spatial extension of the interference pattern, allowing measurements of the ring radii in the neighboring interference orders, and compared with value of $\Delta l = (0.2 \pm 0.002)$ mm. They agree to each other up to 10% error, allowable in the experiment.

It is evident from the above analysis of formation of the interference patterns, characterizing transversal or longitudinal displacement of the scatterer, that "frozen" fringes are localized in the Fourier hologram plane at its double-exposure recording with a microscope in case of combining longitudinal and transversal scatterer displacements. This is explained by the combination of only homogeneous and axisymmetric inhomogeneous (radially varying from the optical axis) slopes of the subjective speckle-field, corresponding to the second exposure, in the hologram plane relative to the speckle-field of the first exposure in case of the double-exposure recording of the Fourier hologram with a positive lens, when a photoplates is in its back focal plane.

The interference pattern, localized in the Fourier hologram plane and characterizing the transversal scatterer displacement to $a = (0.02 \pm 0.002)$ mm and longitudinal one to $\Delta l = (0.5 \pm 0.002)$ mm, is shown in Fig. 7. It was recorded without spatial filtration of the diffraction field.

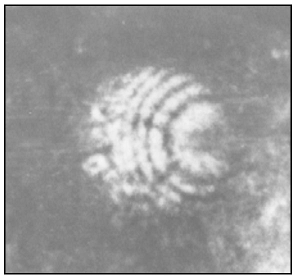


Fig. 7. Interference patterns localized in the Fourier hologram plane and characterizing transversal and longitudinal displacement of a flat diffusively scattering surface.

Thus, results of the analysis of formation of the interference patterns, characterizing longitudinal or transversal displacements of a flat diffusively scattering surface, at double-exposure recording of quasi-Fourier and Fourier holograms using a collimating microscope, and the experimental study show the following.

At the stage of double-exposure quasi-Fourier hologram reconstruction, the interference patterns, characterizing both transversal and longitudinal displacement of the scatterer, are localized in the hologram plane and the Fourier one, where the scatterer is imaged. In this case, the interferometer sensitivity to the interference pattern, localized in the Fourier plane, depends on the curvature of a spherical wave of the coherent radiation, used for the scatterer illumination while recording the hologram, and the microscope focal length. For the interference pattern, localized in the hologram plane, the interferometer sensitivity depends on the microscope focal length. Besides, in case of recording in the plane of localization of the interference pattern, characterizing transversal scatterer displacement, the fringe parallax takes place due to the homogeneous displacement of a component of the subjective speckle, corresponding to the second exposure, in the hologram plane. The parallax is absent in recording the interference patterns, characterizing the longitudinal scatterer displacement.

Double-exposure recording of the Fourier holograms using a microscope was accompanied by formation of "frozen" interference patterns in the hologram plane, which were recorded without spatial filtration of the diffraction field.

References

1. V.G. Gusev, Atmos. Oceanic Opt. **20**, No. 4, 325–339 (2007)
2. D. Goodman, *Introduction to Fourier Optics* (McGraw Hill, New York, 1968), 360 pp.
3. V.G. Gusev, Opt. Spekstrosk. **77**, No. 5, 847–850 (1994).
4. M. Franson, *Speckle Optics* (Mir, Moscow, 1980), 165 pp.
5. V.G. Gusev, Atmos. Oceanic Opt. **4**, No. 8, 558–564 (1991).
6. M. Born and E. Wolf, *Principles of Optics* (Pergamon, New York, 1959).
7. V.G. Gusev, Opt. Spekstrosk. **69**, Is. 5, 1125–1128 (1990).
8. R. Jones and C. Wykes, *Holographic and Speckle Interferometry* (Cambridge University Press, 1986), 320 pp.
9. V.G. Gusev, Izv. Vyssh. Uchebn. Zaved. Ser. Fizika, No. 2, 62–70 (2006).