THEORETICAL ANALYSIS OF AN EYE–SAFE LIDAR

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In this paper we present a theoretical analysis of eye–safe semiconductor laser–based lidar with a photon counting signal recording system. We have derived basic expressions to be used for assessing some physical parameters of the medium under study for the case of extremely low (micro–Joules) energy of sounding radiation per pulse that is several times lower than that of the daytime sky background. In this paper we also present a technique for transforming the function of photon distribution over intervals to the form convenient for linear presentation of signals. Analytical expressions for estimating limiting parameters of the lidar under study are derived.

Wide application of lidars for investigating the environment, the atmosphere and water bodies is limited by the level of the optical radiation energy density at the object under study. Thus, in the visible spectral range the maximum value of laser energy density must not exceed 0.5 μJ/cm^2 in accordance with the recommendations of the American National Standard Institute (ANSI) and requirements of the World Health Organization.[1] In November, 1992 this problem and the other problems of standard eye–safe level of laser radiation were discussed at the special symposium in Boston, the USA.[2]

Substantial progress in developing highly efficient quantum counters based on avalanche photodiodes has made it possible to design the lidars with infinitely low level of transmitter radiation comparable with natural background in intensity, when the probability of signal recording from every laser shot is always much below unit.[3–5] Use of the quantum counting technique in such lidars,[6] when sounding the atmosphere, has its specific features typical of sampling mode of acquiring and statistical processing of data as opposed to traditional approaches with the use of analog signals.[6] Creation of such instruments with the eye–safe radiation can strongly expand the range of use of optical remote means for environmental monitoring.[5]

The subject of this paper is a theoretical analysis of operation of a microjoule aerosol lidar with a single–photon recorder to derive the basic relationships for measuring physical parameters of the medium under study. In this case, much attention was paid to a detailed investigation of the photocount distribution function considering the finite duration of the detector dead time and stochastic sources of different noise determining the maximum value of the signal–to–noise ratio when the noise exceeds the signal several times.

1. FORMULATION OF THE PROBLEM

A characteristic feature of any photon counter[7] is the dead time, during which the counter is out of operation. In general, dead time and time for restoring the counter efficiency can be different, however, in this paper both above–mentioned times will be considered equal. In practical applications it is desirable that this time should be as small as possible, because it results in miscounts thus decreasing the efficiency of the detector. Let us analyze operation of a pulsed radar with a single–photon detector as a quantum counter.

Let the pulsed source of photons switch on periodically and irradiate a medium under study within a certain sector determined by the radiation divergence. The photons scattered by an obstacle or along a sounding path arrive at the detector aperture which opens synchronously with a certain delay with respect to the transmitter pulse and remains open till the moment of a photon detection or during a certain period T, normally called a strobe or time gate. The strobe T is divided into cells of τ duration, determining the digitization step and spatiotemporal resolution along the sounding path.

At each switching of the transmitter in one of the strobe cells, being erased prior to recycling, the unity can be added if during the period of this cell duration the photodetector records either an external photon or an internal photoelectron. If such an event does not happen during the strobe, the contents of the cells remain in the previous state. With multiple switching on of the transmitter, the photocounts are accumulated in the strobe cells and the histograms of events distribution are formed.

Such an operation mode of recording events corresponds to operation of the counter of discrete signals with large dead time (LDT) t_d, when $T < t_d < T_0$, where T_0 is the period of the strobe or photon pulsed source switching on, and T is the strobe duration. The characteristic view of the distribution function of the number of counts of such a counter when recording a signal and the background is shown in Fig. 1*a*. Here the first peak is due to light scattering by atmospheric aerosol along a sounding path in a homogeneous atmosphere and the second peak is due to scattering by an obstacle or an aerosol atmospheric layer.

It should be noted that an actual counter with LDT can detect only one event during a strobe T, while an ideal counter with a zero dead time (ZDT) can record any number of events occurring during the strobe T. The histogram of counts for a detector with ZDT and large number of laser shots could characterize real dynamics of photon arrival and the number of counts in the histogram cell could be proportional to the intensity of a signal received. So the histogram of counts for a counter with ZDT (ZDT–histogram) contains all information about the lidar return in the most suitable for processing form. However, the counting mode with the ZDT counter cannot be performed in practice because of the finite time of the detector relaxation. Note, that the possibility exists of coming from LDT–histogram, see Fig. 1*a*, to ZDT–histogram. An example of such a transform is shown in Fig. 1*b*.

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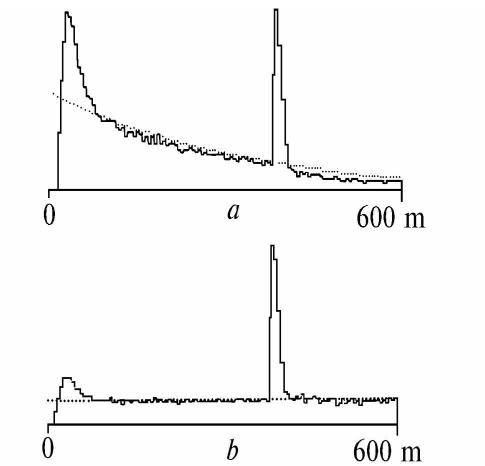


Fig. 1. Typical view of the photocount number distribution histograms for counters with large (a) and zero (b) dead time obtained at identical parameters of the input event flux.

As is seen from Figs. 1*a* and 1*b*, the histograms are strongly different. So, in the histogram shown in Fig. 1*a* the monotonic decrease of the number of noise photocounts with increasing time corresponds to the constant flux of an external noise. In this case the rate of the fall off and the area of the signal peak depend on the level of the external noise. Besides, the decrease of the number of counts in channels, following the signal peak, depends on its intensity. In the second histogram (see Fig. 1*b*) these peculiarities are not observed.

Let us consider in more detail the problem on how the formation of a distribution and statistics of the number of counts over cells both for the counters with LDT and with ZDT while recording an arbitrary stationary flux of events in order to select a method of processing histograms obtained with an actual counter.

It is assumed that during the measurement time and collection of data for a histogram the optical properties of the obstacle surface and the atmosphere between the object sounded and the detector change slowly and insignificantly, that is, the case of stationary flux of independent random events is considered.

We denote the number of cells into which the strobe T is divided by k, then for both types of counters with ZDT and LDT, the space of random events Ω_1 and Ω_2 can be built up in the following way:

$$t_{\rm d} = 0; \quad \Omega_1 = \{P_i\},\ i = 1, \dots, k; \quad T < t_{\rm d} < T_0 ;$$

$$\Omega_2 = \{\theta_i\}, \quad i = 1, \dots, k ;$$

$$P_i;\ t \in [(i-1)\tau;\ i\tau]; \quad \theta_i = \prod_{j=1}^{i-1} (1 - \tilde{P}_j)\, \tilde{P}_i; \quad t \in [(i-1)\tau;\ i\tau] .$$

We choose the cell size so that $P_{\rm i} < 1$ in Ω_1 and $\theta_{\rm i} < 1$ in Ω_2 for any $i = 1, \dots, k$. Here $P_{\rm i}$ characterizes the frequency of events at input of the ith cell of the histogram in the space Ω_1. It should be noted that because of a single–response of the counter during the time T the occurrence of an event at the ith cell of the strobe in the space Ω_2 assumes the absence of responses in the preceding time for the cells $i = 1, \dots, i-1$, what makes up corresponding probability distribution $\{\theta_{\rm i}\}$ over cells $i = 1, \dots, k$ for the counter with LDT: if $P_i = P$, $i = 1, \dots, k$, then the obtained distribution of counts over cells $\theta_i = (1 - \tilde{P})^{i-1}\tilde{P}$ is described by the Pascal distribution law.

The case when a stationary flux of events is to be recorded with a ZDT counter falls into the specific class of stochastic processes called point processes.[7] To describe these processes one can use both the distribution functions over moments of recording $Q_s(\tau_1, \dots, \tau_2)$ and over the frequencies of the events occurrence. The relationship between the above functions is presented in Ref. 7. Note that the function Q_s should be used in the cases when the number of events occurring during any time interval is well–defined, that is, at each measurement the m–number of events is recorded, m being constant. If the value m varies randomly over different intervals and different quantity of events can be recorded, one can use the function f_n, namely, the frequency distribution function. Since an actual counter with LDT can record no more than a single event during the strobe T, to describe its operation we must use the function $f_1(t)$, which actually presents the frequency of events at time t.

A very important conclusion following from the analysis of the point processes[7] is that the total number of events recorded during a time interval is described by the Poisson distribution. In this case, using functions $f_n(t_1, \dots, t_n)$ we obtain an expression for the basic moments of the count number distribution over a time interval $[t_a, t_b]$

$$\langle N\rangle = \int_{t_a}^{t_b} f_1(t)\, {\rm d}t; \quad \langle N^2\rangle = \langle N\rangle + \int_{t_a}^{t_b} f_2(t_1, t_2)\, {\rm d}t_1\, {\rm d}t_2 , \qquad (1)$$

where $f_2(t_1, t_2) = f_1(t_1)\, f_2(t_2)$, which is valid in the case of recording a stationary flux of independent events.

For the average value and the variance of the number N with the Poisson distribution

$$P_N = \frac{\langle N\rangle^N}{N!} \exp(-\langle N\rangle) \qquad (2)$$

we have

$$M(N) = \langle N\rangle,\ D(N) = \langle N\rangle . \qquad (3)$$

Since ZDT–histograms are linearly related to the physical parameters of an object under study, it is interesting to consider a possibility of transforming data recorded with an actual LDT counter to the histogram of such a type.

2. TRANSITION BETWEEN THE PROBABILITY SPACES Ω_1 AND Ω_2

As noted above, for the stationary and independent random flux of events over the interval $t_a < t < t_b$ the distribution of the sum of counts is of the Poisson type with the parameter

$$\langle N\rangle = \int_{t_a}^{t_b} f_1(t)\, {\rm d}t = f(t_b - t_a) . \qquad (4)$$

Then the probability of no event during a time interval is determined as $P = \exp(-\langle N\rangle)$, and it is identical for both types of counters in Ω_1 and Ω_2. In this case the probability of recording an event in the strobe is $1 - \exp(-\langle N\rangle)$ and is always less than unity, while $\langle N\rangle$ can take any positive values. For weak quantum fluxes, when $< N > < 1$, the value $< N >$ can also be used for determining the probability of a ZDT counter response. In this case $P = < N > = f\tau < 1$, where f is the average frequency of occurence over the time interval τ, for example, the strobe cell. As a result the relationship between P_i in Ω_1 and P_i in Ω_2 can be presented as

$$\tilde{P}_i = 1 - \exp(-P_i) = 1 - \exp(-f_i\,\tau)\,, \quad i = 1, \dots, k\,. \qquad (5)$$

In the case of τ satisfying the equation $\max f_i\,\tau = P_i \ll 1$,

$$P_i = \tilde{P}_i = f_i\,\tau \ll 1\,,\ i = 1, \dots, k\,. \qquad (6)$$

In our below discussions we shall assume the condition (6) to be fulfilled. The cases, when this condition is not fulfilled, will be considered separately.

Consider now in more detail how the probability of an event occurrence in the ith cell of space Ω_2 is determined. Its structure represents a successive series of the probability products. The formula (5) for θ_i can be written as

$$\theta_i = \prod_{j=1}^{i-1}(1 - \tilde{P}_j)\,\tilde{P}_i = \exp\left(-\int_0^{(i-1)t} f_1(t)\,\mathrm{d}t\right) =$$

$$= \exp\left(-\sum_{j=1}^{i-1} f_j\,t\right)P_i\,. \qquad (7)$$

Multiplication of both parts of this equality by N yields the relationship between the average values of counts in the ith channels from Ω_1 and Ω_2 spaces.

In a limiting case when the cell size vanishes, one can obtain the differential form of the probability in Ω_2 in terms of the distribution function

$$d\theta(t) = \exp\left(-\int_0^t f_1(t')\,\mathrm{d}t'\right)f_1(t)\,\mathrm{d}t\,. \qquad (8)$$

The expression in the right–hand side is the most general view of the event distribution function over time intervals.

Let us write basic expressions relating the probabilities of photocount recording in a count channel with a ZDT and LDT counters

$$\theta_i = P_i \exp\left(-\sum_{j=1}^{i-1} f_j\,t\right) = P_i \exp(-\mu(1;\,i-1)),$$

$$\exp(-\mu(1;\,i-1)) = 1 - \left(\sum_{j=1}^{i-1} N_j(\Omega_2)\,/\,N\right), \qquad (9)$$

where $N_j(\Omega_2)$ is the number of photocounts in the ith channel of a ZDT–histogram and N is the number of the counter operation cycles.

Let us consider the question on the precision to determine $\{P_i\}$, $i = 1, \dots, k$ in Ω_1 if we know $\{\theta_i\}$, $i = 1, \dots, k$ in Ω_2.

For a stationary flux of random independent events the number of photocounts recorded in a time gate after N recyclings of a ZDT counter obeys the Poisson statistics. However, in the case of an LDT counter the distribution over channels will change. Below we shall derive, based on qualitative considerations, the probability distribution law, and write the basic moments of the photocounts distribution function in a channel for this case.

Let us assume that $f\tau$ equals w for the counter with ZDT, then for LDT $P = 1 - \exp(-w)$, $P < 1$ at any w.

Consider the case when $w \gg 1$

$$\langle N\rangle = f\tau\, N\,.\quad D(N) = \langle N\rangle = f\tau\, N\,,$$

$$\langle N\rangle = N(1 - \exp(-w)) = N(1 - \varepsilon)\,,$$

$$D(N) = N\varepsilon(1 - \varepsilon)\,,$$

there exist such values N and ε that $N\varepsilon \ll f\tau \ll N$, that is, the rms deviation of the count number in the interval is overestimated. Actual variance of the count number distribution in a channel from Ω_2 is less than the corresponding value resulting from the Poisson distribution. From Eq. (2) it follows that at $< N > \ll 1$ the probability of recording a single event during a time gate is $P = < N > \exp(- < N >)$, corresponding value for two events equals $(< N >^2/2) \exp(- < N >)$, that is, $< N >/2$ times less than for a single event. Thus, at a small probability of occurrence of events during a time interval of duration τ, the parameters of the sought and the Poisson distributions are approximately the same. At a large probability of occurrence of events during a time interval the rms deviation of the sought distribution becomes less than the corresponding value for the Poisson distribution. The distribution with such characteristics is binomial.

Let us multiply Eq. (9) by N and rewrite it in the form:

$$N_i(\Omega_1) = N_i(\Omega_2) \exp(\mu(1;\,i-1))\,.$$

From this equation it follows that the number of counts in a channel of the reconstructed histogram is determined by the product of two random values. The contribution into the error of $N_i(\Omega_1)$ coming from the exponential factor can be neglected here because of the integral nature of the latter, see Eq. (9). Thus, relative rms deviation in a channel of the reconstructed histogram is determined by the distribution function in the corresponding channel of the initial histogram. In this case, for the histogram reconstructed in Ω_1 we have

$$N_i(\Omega_1) = N_i(\Omega_2) \exp(\mu(1;\,i-1))\,;$$

$$D(N_i(\Omega_1)) = N_i(\Omega_2) \exp(2\mu(1;\,i-1))\,,\quad i = 1, \dots, k\,;$$

$$\exp(-\mu(1;\,i-1)) = 1 - \left(\sum_{j=1}^{i-1} N_j(\Omega_2)\,/\,N\right). \qquad (10)$$

In cases when the condition (6) is not fulfilled, one must use binomial distribution for the number of photocounts in a time interval. Calculations made using expressions for differentiating a function of a random variable result in the following mean value and variance of the number of counts in a ZDT–histogram recalculated from a LDT–histogram:

$$N_i(\Omega_1) = N \ln \left[\frac{1}{1 - \frac{N_i(\Omega_2)}{N}} \right] \exp(\mu(1;\, i-1)) ;$$

$$D(N_i(\Omega_1)) = \left[\frac{N\sqrt{N_i(\Omega_2)}}{N - N_i(\Omega_2)} \right]^2 \exp(2\mu(1;\, i-1)) ; \qquad (11)$$

$$\exp(-\mu(1;\, i-1)) = 1 - \left(\sum_{j=1}^{i-1} N_j(\Omega_2) / N \right), \quad i = 1, \dots, k .$$

Figures 1*a* and *b* give an example of such a reconstruction made using this technique.

3. APPROXIMATION OF THE DISTRIBUTION OF COUNTS OVER A RECONSTRUCTED HISTOGRAM

Let us consider here the problem on convergence of the obtained distribution functions for number of counts to the Gaussian distribution. We shall use the following designations:

$$P_N(K) = C_N^K\, P^K\, q^{N-K}, \quad C_N^K = \frac{N!}{K!\,(N-K)!},$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{i^2}{2} \right), \quad P_N(x) = P_N(Np + x\sqrt{Npq}) .$$

$$F_N(x) = P_N(-\infty;\, x) .$$

Omitting here strict derivation, we shall give the final result which is a special case of the Berry–Esseen theorem[9,10]

$$\sup_{-\infty \le x \le +\infty} |F_N(x) - \Phi(x)| \le \frac{p^2 + q^2}{\sqrt{Npq}} . \qquad (12)$$

From this inequality it follows that the approximation with a Gaussian distribution of the number of counts in a channel is the more precise the larger is the number of the count in it. As an example, one can use the criterion $N_i(\Omega_2) > 100$ (see Ref. 8).

The rate of convergence of the Poisson distribution to the Gaussian one may be assessed by the asymmetry factor γ (see Ref. 11)

$$\gamma = \frac{M(\xi - M(\xi))^3}{\sqrt{D(\xi)^3}} = \frac{1}{\sqrt{N_i(\Omega_2)}} . \qquad (13)$$

It should be noted that for the Gaussian distribution $\gamma = 0$. These relationships will be used for developing the criterions of signal detection against a preset background noise.

4. GROUNDS FOR MAKING INDEPENDENT ESTIMATIONS OF NOISE

In the preceding sections we have considered the methods for estimating the error of count numbers in a separate channel and in a gate.

Let us consider the case when in addition to background counts there is a signal peak in the histogram, the width of the peak being on the order of several channels (see Fig. 2*a*). The area covered with this peak on a plot is the sought value.

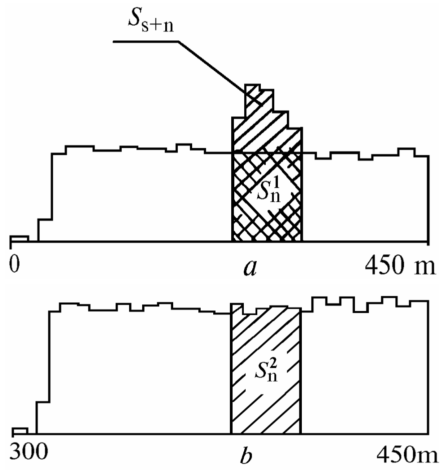


Fig. 2. Illustration of the method of measuring the signal level with the background: a) the reconstructed histogram of the count number when recording the signal and background noise and b) the reconstructed histogram of the count number when recording only the background noise.

Having restored the histogram, obtained for a counter with LDT, we write the expression for the signal peak area

$$S_{s+n} = \sum_{i=k_1}^{k_2} N_i , \qquad S_{s+n} = \sum_{i=k_1}^{k_2} D(N_i) \qquad \text{in } \Omega_1 . \qquad (14)$$

Based on thus reconstructed ZDT–histogram, one can estimate the background noise level by a straight line drawn through the points that represent boundary channels of a given peak (see Fig. 2*b*). In this case the background characteristics can be determined as follows:

$$S_n = \sum_{i=k_1}^{k_2} N_{in}(\Omega_1) ,$$

$$D(S_n) = \sum_{i=k_1}^{k_2} N_{in}(\Omega_1) \exp(\mu(1;\, i-1)) . \qquad (15)$$

To calculate $\exp(-\mu(1;\, i-1)) = 1 - \left(\sum_{j=1}^{i-1} N_j(\Omega_2) / N \right)$ the value of $N_i(\Omega_2)$ should be determined for the channels $k_1 < j < k_2$ in the absence of the peak. It should be noted that $N_{jn}(\Omega_1)$ is the number of counts in the cells of reconstructed histogram when recording the noise solely. The numbers in these channels correspond to the region of the signal peak on the "signal + noise" histogram.

It is evident that the determination of S_n is connected with the curve approximation using count numbers in the channels on both sides of the peak. Note that such a procedure of separating out the background is not always justifiable statistically from the standpoint of estimating the accuracy of the results obtained. To improve the accuracy of measurements as well as to simplify the algorithms of extracting the signal from noise it is proposed to subtract from signal + noise counts the number of noise counts only that are counted in a strobe of the same duration as that of a signal + noise strobe.

Thus, the sought values are calculated based on the histograms available. Note that sometimes we can do this without recording number of counts in the background strobe. Such cases will be considered below.

In a real experiment, when evaluating physical parameters, the additional errors must be considered. These errors can be caused by instabilities in the recording electronics. Let us consider the case of equally probable fluxes of events at the counter input. It is assumed that the mean value and the variance of the count number in a channel of the histogram for the LDT counter are estimated using data in all channels with the subsequent calculation of a sample mean and variance. In this case, for the probability of recording a count in a channel of the histogram we use the following randomized expression (the analog of the Mandel formula):

$$P_n = \int\limits_0^\infty \Phi(z) \frac{z^n}{n!} \exp(-z)\, dz . \qquad (16)$$

It is not difficult to obtain the expressions for the mean value and the variance

$$M(n) = \bar{z} = \int\limits_0^\infty \Phi(z)\, z\, dz , \qquad (17)$$

$$D(n) = \overline{z^2} - (\bar{z})^2 + \bar{z} . \qquad (18)$$

Let us involve generalized functions of the type $\Phi(z) = \delta(z - z_0)$ into the range of definition $\Phi(z)$. As a result for a ZDT counter we have

$$M(n_i) = z_0 . \qquad (19)$$

Then for a LDT counter the following expression is valid

$$M(n_i) = (1 - \exp(-z_0)) \exp(-(i-1) z_0) . \qquad (20)$$

It is evident that with the increase of the count numbers in the histogram channels one can make the relative deviations from z_0 negligible for all the channels.

Let us assume that a ZDT counter recorded a histogram of counts, and there are small deviations from z_0 in its histogram channels because of different efficiencies of recording in the channels. This circumstance can be taken into account by introducing a set of values $\{\Delta z_i\}$, $i = 1, \ldots, k$. In this case

$$\Phi_i(z) = \delta(z - z_0 - \Delta z_{0i}) = \delta(z - z_i) \text{ in } \Omega_1 . \qquad (21)$$

Consider now a technique following which we can compare the errors of two ways of estimating the signal peak area. The first way implies the use of the approximation by a straight line through a separated out portion of the reconstructed "signal + background" histogram for determining the background level (see Fig. 2*a*). In the second way the background level is characterized by the count numbers in the relevant cells of a separate background histogram.

Thus, following the expression (16) for the probability, the spread $\{\Delta z_i\}$, $i = 1, \ldots, k$ in the channels of a reconstructed histogram can be characterized by the distribution function $\Phi^1(z)$. The width $\Phi^1(z)$ characterizes the additional contribution coming from stochastic sources into the Poisson part of the error (see Eq. (18)).

Assume that in the measurement process two strobes are formed alternately for obtaining two data sets. The signal and background photocounts can be accumulated in the first data set and in the second strobe only background events are accumulated (the photons of external radiation and the detector noise). In both cases stationary random fluxes of equally probable events arrive at the receiver input. After reconstruction of histograms the part, limited by channels k_1 and k_2, is separated out (Fig. 3).

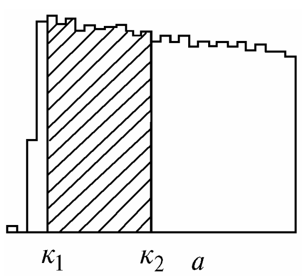
Let us write the expression for the average number of counts in a channel for the first histogram:

$$\bar{N}^{(1)} = \frac{1}{k_2 - k_1} \sum_{j=k_1}^{k_2} N_j^{(1)} \text{ in } \Omega_1 , \qquad (22)$$

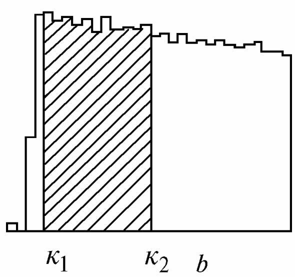
and for the second histogram

$$\bar{N}^{(2)} = \frac{1}{k_2 - k_1} \sum_{j=k_1}^{k_2} N_j^{(2)} \text{ in } \Omega_1 , \qquad (23)$$

where $N_j^{(1)}$ and $N_j^{(2)}$ are the numbers of counts in the jth channel for the first and the second histogram, respectively.



224	5896
0	6108
2873	5936
6486	6112
6699	5766
6444	5902
6649	5787
6233	5939
6410	5795
6340	5940
6371	5627
6424	5868
6247	5638
6105	5686
6271	5479



249	5996
0	6069
2861	5805
6652	6055
6739	5823
6471	5889
6607	5770
6360	5918
6563	5739
6152	5689
6577	5757
6195	5835
6289	5559
6194	5588
6353	5443

Fig. 3. Typical view of histograms to illustrate the method of estimating the contribution of sources of stochastic noise.

Let us write the expression for the variance of counts in a channel in the presence of additional sources of stochastic noise

$$D_a = \sigma_a^2 = \sigma_P^2 + \sigma_{a.s.}^2 , \qquad (24)$$

where σ_P is the rms deviation of the count number in the channel assuming Poisson distribution; $\sigma_{a.s.}$ is the rms deviation due to additional stochastic noise; D_a is actual variance of the count number in the channel; and, σ_a is actual rms deviation.

In the first case, for characterizing the contribution from additional sources of noise without the use of the background histogram one can write the following expression:

$$\xi_1 = \frac{1}{k_2 - k_1} \sum_{j=k_1}^{k_2} \frac{(\bar{N}^{(1)}(\Omega_1) - N_j^{(1)}(\Omega_1))^2}{\bar{N}^{(1)}(\Omega_1)\,\exp^{(1)}(\mu\,(1; j-1))} - 1. \qquad (25)$$

In the second case, when the background histogram used, the same characteristic is

$$\xi_2 = \frac{1}{k_2 - k_1} \times$$

$$\times \sum_{j=k_1}^{k_2} \frac{(N_1^{(1)}(\Omega_1) - N_1^{(2)}(\Omega_1))^2}{\bar{N}^{(1)}(\Omega_1)\exp^{(1)}(\mu(1; j-1)) + \bar{N}^{(2)}(\Omega_1)\exp^{(2)}(\mu(1; j-1))} - 1. \qquad (26)$$

In the general case the expressions under the sum sign for ξ_1 and ξ_2 present the ratio of the squared absolute deviation from the average count number in an actual channel of the reconstructed histogram to the theoretical value of the variance in the channel, resulting from the procedure of reconstruction. In the absence of additional sources of noise the values ξ_1 and ξ_2 must be close to zero, since the corresponding expressions are written assuming Poisson distribution of counts in the channels of initial histograms.

When comparing the values ξ_1 and ξ_2 we may state the following:

1. If $\xi_1 > \xi_2$ the use of the background histogram is justified from the standpoint of an increase in the precision of the results obtained.

2. If ξ_1 is essentially less than ξ_2, the use of the background histogram is not always justified. From values of ξ_1 and ξ_2 one can evaluate the relationship between the actual variance and the variance corresponding to a purely Poisson process

$$D_{\text{a}\,i} = (\xi_i + 1)\,D_{\text{P}}\,, \qquad i = 1,\,2\,. \qquad (27)$$

Then the estimate of additional contributions from sources of stochastic noise is written as a simple relation

$$D_{\text{a.r}\,i} = \xi_i\,D_{\text{P}}\,, \qquad i = 1,\,2\,. \qquad (28)$$

Here $D_{\text{a.r}i}$ is the variance of counts caused by additional sources of random noise.

Besides, to check the hypothesis on the validity of the Poisson distribution we use here the χ^2 criterion

$$\xi_i = \frac{1}{n}\chi_i^2 - 1\,, \qquad i = 1,\,2\,. \qquad (29)$$

The analogy with χ^2 becomes more complete if the sums for ξ_1 and ξ_2 are expressed in terms of $N_i(\Omega_2)$.

This criterion should be used when analyzing actual histograms since in this case we can compare the performance characteristics of real devices in values of contributions from additional noise sources into the distribution (16). Note that at small number of counts in the channels of a LDT–histogram the relative contribution of additional factors into the Poisson part of errors is, as a rule, insignificant. If we should measure the intensity of the signal–photons flux with a high precision, the contribution from additional sources of stochastic noise into the error of the measured value should necessary be accounted for.

Recording of the background should be also performed for simplifying the algorithms of signal search and processing.

5. ASSESSMENT OF CONTRIBUTION FROM SOURCES OF STOCHASTIC NOISE TO THE ERROR OF SIGNAL MEASUREMENTS

A possibility of using the developed procedure for assessment of a contribution from sources of stochastic noise to the error of signal measurements was considered as an example when analyzing histograms obtained with an operating lidar model. The histograms of the numbers of counts (see Fig. 3*a* and *b*) were formed in the alternating mode when recording only the noise in the absence of laser pulses and signal + noise. The number of measurement cycles, in both cases, was equal to 1 024 000. To calculate the values ξ_1 and ξ_2, arbitrary portions of the histograms including more than 10 channels were selected. The results of calculation, necessary for revealing the presence of sources of stochastic noise, and for estimating the noise level are summarized in Table I.

TABLE I.

No.	Average number of counts	ξ_1	ξ_2	Channel in Ω_1
1	N = 6622	1.23	– 0.12	5 – 30
2	N = 6685	1.015	0.042	5 – 15
3	N = 6576	1.38	– 0.23	16 – 30

Note. Here N is the average number of counts in the histogram channels (see Fig. 3*a*) after reducing it to a linear form. The values ξ_1 and ξ_2 are connected with the distribution χ^2 (29) and are calculated using only the values from the first histogram (25) and the differences of actual counts in the current channels of both histograms (26) after their reconstruction, respectively.

An example of calculation of the contribution from sources of stochastic noise to the measurement error is done below using the data of the third line of Table I. Let us verify the hypothesis that the distribution of numbers of counts in channels of the histogram (Fig. 3*a*) is Poissonian. Having assessed only one parameter of $N_i(\Omega_2)$ of the distribution function, we obtain 13 degrees of freedom for χ^2 (see Ref. 11).

As seen from Table I, the value ξ_1, calculated on the basis of the experimental data of the first histogram, is close to unity that corresponds to the value $\chi^2 = 13 \times 2 = 26$. For the purpose of revealing insignificant deviations of the process statistics under study from the proposed hypothesis we select, from a standard set of values, the maximum level of significance at $\alpha = 0.05$ (see Ref. 11). For this value of α we find $\chi^2 = 22.4$ using the tables from Ref. 11. As is evident from the comparison, a tabulated value of χ^2 is smaller than a real one, therefore our hypothesis about the Poisson distribution of the counts in a cell of the analyzed histogram is inadequate to the process under study,[11] since the variance of the number of counts, calculated for a selected number of channels, significantly exceeds the mean value.

This inadequacy can be explained by the presence of sources of stochastic noise, making an additional contribution to the variation of the parameters of signal measurements. These errors can be due to different efficiencies of recording in different cells, though the difference being a small value $\{\Delta z_i\}$, $i = 1, \ldots , k$. This is well seen from a visual comparison of counts in the channels of histograms for different measurement cycles (see Fig. 3). The observed periodic modulation of the number of counts along the histogram results in the value ξ_1 close to unity, i.e., real variance of counts in cells exceeds the Poisson one. Hence, there are some additional sources of stochastic noise in the device. The contribution coming from them into the variance of the parameters may be assessed by formula (27). The calculation shows that, in the considered example, the actual variance of the parameters is twice as large as the calculated one. The account of the contribution from sources of stochastic noise, producing periodic modulation of the numbers of counts in channels (Fig. 3*a*), is possible with the use of the second histogram and the parameter ξ_2.

The characteristic value of ξ_2 calculated using the difference of two histograms (26) is close to zero. In this case,

$$\chi^2 = n\xi_2 + n \text{ for } n = 15 \,,\ \xi_2 \cong 0 \,,\ \chi^2_{1-\alpha} = 15 \,.$$

Since real value of χ^2, obtained from two histograms, is less than the tabulated one, our assumption is in agreement with the hypothesis on the Poisson distribution of the numbers of counts in the channels of the initial histogram. This is the result of decreasing the contribution from the modulation of numbers of counts in channels to the error of signal measurement when subtracting the noise histogram, since there is a correlation between counts in relevant channels of the histograms.

Thus, when considering the criterion of signal detection with a LDT counter, the signal being above the background level, using two histograms with a separate noise recording, one can use the Poisson distribution law for the number of counts in a channel. The negative values of ξ_2 can be due to insufficient precision of the approximation (10) used when calculating the parameters of reconstructed histograms, as well as the characteristics of χ^2 distribution used.

The discussion of correctness of use of χ^2 distribution for investigating the contribution from sources of stochastic noise requires further investigation.

Here we shall pay special attention to the sensitivity of the statistical approach discussed to the search of a signal. Let the signal be a 4% variation in the number of counts in isolated channel, from 5787 to 6037 (see the 22nd channel at the first histogram and the table of numbers of counts in Fig. 3). Then we obtain the value of the parameter ξ_2, calculated over 15 channels, from 16 to 30, in the absence and in the presence of a signal in the 22nd channel of the histogram. A comparison shows that the value of the parameter $(\xi_2 + 1)$ varies from 0.77 to 1.16, respectively, that exceeds essentially (more than by 4%) the value of the parameter $(\xi_2 + 1)$, calculated for different portions of the histogram (see Table I). Thus, the considered approach can be used for searching the strobe with signal counts, distributed over some histogram channels and for detecting the signal localized in one or in several channels at high level of noise.

As an example we consider a question on detecting extremely weak signal–photons fluxes by a lidar with a single–photon detector. It is well known that the criterion of reliable recording is given by the probability of false count. Let us assume that the distribution of counts for an individual channel of a histogram can be approximated, highly accurate, with normal distribution.[12,13] For detecting a signal with the probability of false count of 0.3%, the excess of more than 3σ of the average value of signal amplitude in one channel is needful.[11]

Let us assume that the noise level measured with a lidar during a single cycle is 5479 counts in the last channels of histogram for a LDT counter. According to the previous discussion one can obtain two values of the rms deviation resulting from the distribution function $\Phi^{\text{I}}(z)$ without the background histogram and with it, respectively.

In the first case we obtain $\sigma_1 = \sqrt{5479 \cdot 2} = 105$, and in the second case $\sigma_2 = \sqrt{5479} = 74$.

From this result the limiting values of the signal and the signal–to–noise ratio necessary for providing a given reliability of detection are: in the first case not less than $3\sigma_1 = 315$, $(\text{S/N})_1 \cong \dfrac{315}{5479} = 5.7 \cdot 10^{-2}$, and in the second case it is not less than $3\sigma_2 = 222$, $(\text{S/N})_2 \cong \dfrac{222}{5479} = 4 \cdot 10^{-2}$.

Thus, these estimates show that the lidar with a single–photon detector can measure the signal in a real time scale at the level of some per cent of the background noise with the recording reliability not less than 0.997. This conclusion is especially important for the problems of sounding the environment or the living tissue since it shows that it is possible to use a sounding beam with the tens times lower than that of natural background from solar radiation.[12]

Besides, the implementation of the technique of a separate evaluation of noise level with the subsequent subtraction from signal plus noise counts can essentially increase the detection quality. The above technique enables one to increase the sensitivity and detectability of the receiving channel, which is of a particular importance when studying nonstationary processes with limited time for signal recording.

CONCLUSIONS

1. The model of lidar performance with a single–photon detector is proposed and justified theoretically. The paper analyzes the operation of the detectors with dead time of different types. On the basis of this analysis the authors developed a method of transition from actual histograms, characterizing the distribution function of photon recording in the intervals for the counter with large dead time, to linearized histograms for the counter with zero dead time. Such histograms in a linear form show the physical characteristics of the medium sounded.

2. The paper presents the analysis of the factors determining the maximum value of the signal–to–noise ratio in the lidar with a quantum pulse counter. The influence of stochastic noise sources on this ratio is considered, and the technique for assessment of contribution from such sources into the measurement error is developed. It has been experimentally demonstrates that the signal measurement with subsequent subtraction of noise makes it possible to increase about 1.4 times the maximum value of signal–to–noise ratio when recording the signal being tens times low than the background intensity.

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