DISTRIBUTION OF FLUCTUATIONS IN A RECEIVED POWER DUE TO LASER BEAMWIDTH FLUCTUATIONS IN THE TURBULENT ATMOSPHERE

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Received February 2, 1994

Experimental results are presented from the study of the effect of laser beamwidth fluctuations in the turbulent atmosphere on the received power fluctuations. It is shown that the laser beamwidth fluctuations and the fluctuations of a received power follow the lognormal distribution.

Random variations in the refractive index of a propagation medium engender the fluctuations in a received power primarily due to energy re–distribution over the beam cross section (scintillations), random wanderings (displacements), and beam broadening. Such processes can be considered as multiplicative noise of atmospheric optical information systems (AOIS's).

To calculate the basic AOIS characteristics, the probability distribution of the received power fluctuations due to each noise component must be known.[1] Such distribution laws have been established for scintillations[2] and random wanderings,[3] whereas for pure beam broadening such data are unavailable.

In this case the determination of the probability density function (pdf) of the received power fluctuations requires the knowledge of the pdf of the instantaneous beam width as well as the relation between the two pdf's. To find such relationships, we consider the most general case of a laser beam with the Gaussian distribution of the intensity over the beam cross section, which, like a receiving antenna, represents circles

$$I(x, y) = P_t(2\pi R_m)^{-1} \exp\left(-\frac{x^2+y^2}{2R_m}\right), \tag{1}$$

where P_t is the total power in the beam; R_m is the instantaneous beam radius at exp(–1/2) energy level; x, y are the current coordinates in the beam cross-sectional plane.

Then the power received by an aperture of radius r is (see Ref. 4):

$$P_r = \iint\limits_{x^2+y^2 \le r^2} I(x, y)\,dx\,dy = P_t\left[1 - \exp\left(\frac{1}{2}\left(\frac{r}{R_m}\right)^2\right)\right]. \tag{2}$$

For $r/\langle R_m\rangle < 1$, with $\langle\rangle$ standing for averages, we have

$$P_r \simeq \frac{P_t}{2}\left(\frac{r}{R_m}\right)^2 . \tag{3}$$

Since R_m is random, its mean value $\langle R_m\rangle$ is of interest and can be determined by averaging either in a coordinate system affixed to the beam center of gravity or in a moving one. Correspondingly, two beam radii, namely, stationary R_s and instantaneous R_i, related by the expression

$$R_s^2 = R_i^2 + \langle R_m^2\rangle, \tag{4}$$

with R_i measurement time being much shorter than the characteristic time of beam wandering.

The stationary radius R_s accounts for the entire spectrum of perturbations in the turbulent atmosphere. It has been investigated in ample detail both theoretically and experimentally (e.g., see Refs. 6, 7, and 8), unlike the instantaneous radius, whose pdf was studied experimentally only in a few papers.

The form of the $W(R_m)$ distribution was first determined theoretically and related to the atmospheric turbulence in Refs. 9 and 10, where it was found to be well approximated by the lognormal distribution. However, those works made no clear distinction between the effects of beam broadening and beam random wandering occurring simultaneously in most practical cases. Such a distinction has required more thorough experimental investigation of the laws of beamwidth fluctuations considering the two effects simultaneously and separately.

The experimental setup was that used in Refs. 9 and 10 for instantaneous measurements of the beam width by the "knife" method, but with a slight modification; its block diagram is shown in Fig. 1.

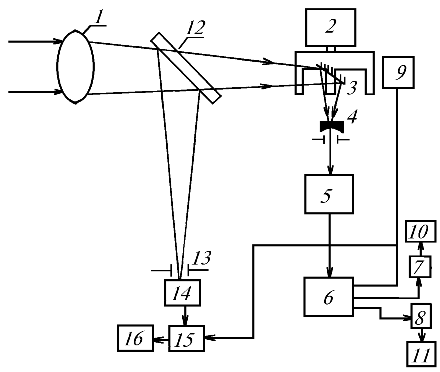

FIG. 1. Block diagram of the experimental setup

The lens *1* diminishes the incident beam diameter to reduce the overall size of the drum *2*. The factor of diminishment depends on the lens–drum distance, and its maximum value is limited by the magnitude of permissible intensity fluctuations engendered by the phase fluctuations.

For the given setup, the beam diameter was diminished threefold. The drum has eight rectangular notches. Its rotational speed (f = 50 Hz) was chosen to be large enough to match a wide spectrum of the light beam intensity fluctuations in the atmosphere calculated in Ref. 11. For such a regime, intensity distribution over the beam cross section can be considered to be constant during measurement period ($\tau_i \le i_k \sim 1$ ms).

After reflection from the flat immovable *12* and rotatable *3* semi–transparent mirrors, the beam is transmitted through the filter *4* designed for background suppressing, and is focused onto the photodiode receiver *5*. The photodiode output voltage is processed with the special electronic device *6*, which generated pulses with amplitudes proportional to either the width or the displacement of the beam. The pulses are then statistically processed by the AI–128 pulse analyzers (*7* and *8*) whose outputs were connected with the digital printers *10* and *11* recording histograms.

To measure displacements, a synchronizing pulse triggering the integrator of the module *6* was generated in the module *9* when the drum notch edge covered the IR radiation from a photodiode located ahead of a special photodetector. Simultaneously with the beamwidth and displacement measurements, the fluctuations in the received power were measured by the photodetector *14*, whose aperture was formed by the diaphragm *13*, were then converted into a voltage for subsequent statistical processing by the AI–256 pulse analyzer *15*, and were recorded by the digital printer *16*.

Thus, the measurement results represent three histograms of pdf of desired quantities that were recorded by digital printers for their subsequent processing on a computer.

The 4-month measurements were made at the polygon of the Institute of Physical Research (IFR) (Ashtarak, Armenia) on two parallel horizontal paths (separated by 3 m) 950 m long located at altitudes of 15 m above the ground. The main path was used to measure random wanderings, instantaneous beam radius, and received power, and the auxiliary path was used to measure the parameter C_n^2 with a device for measuring the diameter of a laser spot at the focus of the receiving lens.[12] The device ensured C_n^2 measurements in the range from $5\cdot10^{-16}$ to $5\cdot10^{-13}$ m$^{-2/3}$. One measurement run lasted 240 s (sufficient to process 96 000 pulses).

Each measurement run was preceded by instrument calibration with the help of slits illuminated by a source with uniformly distributed radiation intensity. The device for measuring beam displacement was calibrated by successive displacements of slits (0.5– mm wide), while the device for measuring beam radius – by the use of a set of slits with fixed widths.

Examples of the measured instantenous beam radii are shown in Figs. 2 and 3. Figure 2 shows the probability density histograms of the fluctuations with $2a = 7\cdot10^{-2}$ m (a is the radius of the receiving aperture) for different values of C_n^2 (cf. Table I), and Fig. 3 shows the probability distribution for the same C_n^2 values. Table I presents the first, second, and second normalized moments calculated for the same histograms. From Fig. 2 and the measurement results we conclude that the instantaneous beam radius fluctuations do follow the lognormal distribution.

As seen from Fig. 2 and Table I, the mean radius of a beam and its variance grow with increasing C_n^2, but at the same time the relative magnitude of fluctuations decreases. Although the second normalized moment is only within the limits from 0.052 to 0.078, relative beamwidth fluctuations may be as large as 23–28%, and since the intensity at the beam center is inversely proportional to the square of the beam radius, the relative intensity fluctuations engendered solely by the beamwidth fluctuations may be as large as 45–56%.

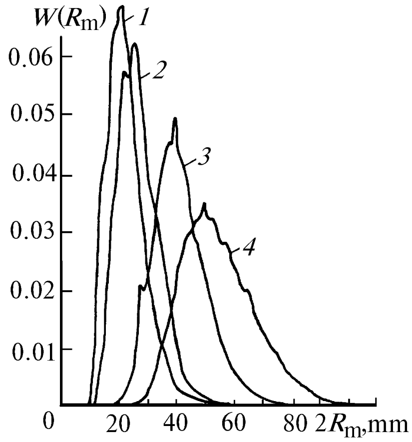

FIG. 2 Histograms of probability density of fluctuations of the instantaneous beam radius R_m *for* $C_n^2 = 3.6\cdot10^{-15}$ *(1)*, $5.1\cdot10^{-15}$ *(2)*, $1.1\cdot10^{-14}$ *(3)*, and $2.2\cdot10^{-14}$ m$^{-2/3}$ *(4)*.

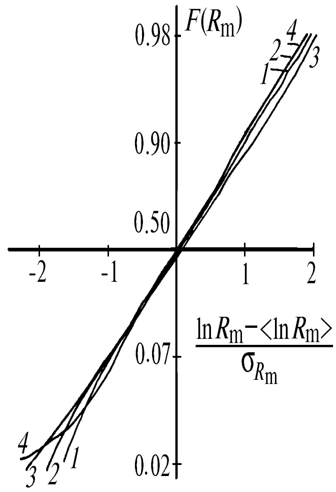

FIG. 3. Probability distribution corresponding to the histograms shown in Fig. 2..

TABLE I.

	C_n^2, m$^{-2/3}$	$\langle 2R_m\rangle$, mm	$\langle (2R_m)^2\rangle$, mm	$\langle R_m^2\rangle / \langle R_m\rangle^2$
1	$3.6\cdot10^{-15}$	22.6	40.2	0.078
2	$5.1\cdot10^{-15}$	27.5	49.1	0.069
3	$1.1\cdot10^{-14}$	41.6	89.3	0.052
4	$2.2\cdot10^{-14}$	54.7	175.3	0.058

Since the instantaneous beam radius fluctuations and hence the received power fluctuations are engendered by cooperative effects of beam wandering and broadening, to eliminate the effect of wandering the beam was defocused in the receiving plane. As an example, Fig. 4 illustrates individual realization of the probability distribution of the received power fluctuations for a 9.4 mm receiving aperture, $C_n^2 = 1.4\cdot10^{-4}$ m$^{-2/3}$, and $D_s(a) = 17$. Both curves are seen to deviate from the lognormal distribution, with much more pronounced deviation for wandering beam in the deep fading range. Averaged measurements show that the probability distribution of fluctuations of nonwandering beam more closely approaches the lognormal distribution.

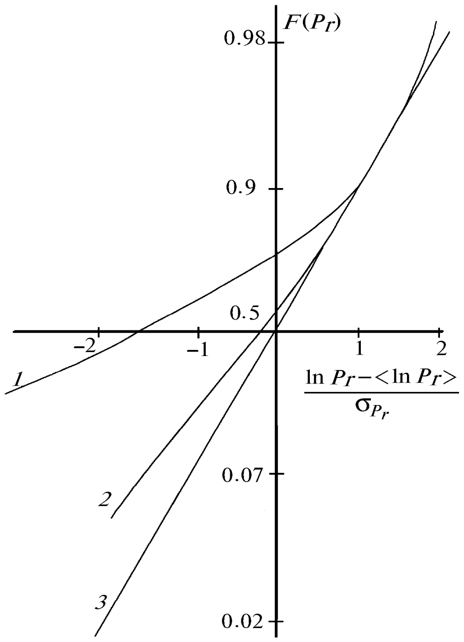

FIG. 4. Probability distribution of the received power fluctuations for wandering (1) and nonwandering (2) beams and the lognormal distribution (3).

The obtained experimental results are readily explained. Indeed, for the fluctuations of the instantaneous beam radius following the lognormal law

$$W(R_m)=\frac{1}{\sqrt{2\pi}R_m\sigma_{R_m}}\exp\left[-\frac{\left(\ln R_m-\ln<R_m>+\frac{\sigma^2_{R_m}}{2}\right)^2}{2\sigma^2_{R_m}}\right], \quad (5)$$

where $\sigma^2_{R_m}$ is the variance of the log – radius fluctuations, from Eq. (2) and the well–known formula for the nonlinear transformation of random variables[13]

$$W(P_r) = W(R_m)\left|\frac{\partial R_m}{\partial P_r}\right| = W(R_m(P_r))\left|\frac{\partial (R_m(P_r))}{\partial P_r}\right|, \quad (6)$$

where $R_m(P_r)$ is the function inverse to $P(R_m)$ and $\left|\frac{\partial R_m}{\partial P_r}\right|$ is the Jacobian of transformation, we finally obtain the probability density of the received power fluctuations in the form

$$W(P_r) = \frac{P_t}{\sqrt{2\pi}\,\sigma^2_{R_m}\left[\ln\left(1-\frac{P_r}{P_t}\right)\left(1-\frac{P_r}{P_t}\right)\right]}\times$$

$$\times\exp\left\{-\frac{\ln 2\left[-\ln\left(1-\frac{P_r}{P_t}\right)\right]-\ln P_t\frac{r^2}{<R_m>^2}-\sigma^2_{R_m}}{8\,\sigma^2_{R_m}}\right\}. \quad (7)$$

For a point aperture ($r/<R_m> < 1$) using Eq. (3) we derive

$$W(P_r)=\frac{1}{\sqrt{2\pi}P_r\sigma_{R_m}}\exp\left[-\frac{\ln 2\,P_r-\ln P_t\frac{r^2}{<R_m>^2}-\sigma^2_{R_m}}{8\,\sigma^2_{R_m}}\right].(8)$$

Thus, we see that the lognormal fluctuations of the instantaneous beam radius engender the received power fluctuations that are lognormal as well. From formally mathematical viewpoint this fact is due to peculiar inertia of the lognormal law through multiplication procedures defined by Eq. (3) and division as was pointed out in Ref. 14, analogous to inertia of the normal distribution law through summation and subtraction. As to the incomplete coincidence of the experimental distribution of the received power fluctuations for a defocused beam with theoretical straight line showing the logarithmic distribution (Fig. 4), it is obvious that this is due to fast scintillations within the beam.

ACKNOWLEDGMENT

The authors would line to acknowledge the staff of the IFR Department headed by Prof. R.A. Kazaryan, whose assistance enabled us to submit the present paper.

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