Analysis of amplitude distribution of the acoustic echo signals

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We present the results of statistical processing of the envelope of a narrow-band sonic signal received with an acoustic meteorological radar (sodar). Hypotheses on the distribution law of the envelope at different altitudes under different geophysical conditions were checked. It was found that under convective conditions, experimental histograms in the lower part of the atmospheric boundary layer are best described by the gamma distribution and those in its upper layers – by the lognormal distribution. Under conditions of stable stratification, the envelope most often obeys the gamma distribution. No regularities were found in the distribution of the external (acoustic) noise.

The interest in statistical properties of acoustic signals can be connected not only with the study of basic processes of sound propagation in the atmosphere as a randomly inhomogeneous stratified medium, but also with applied problems oriented at the study of a propagation medium from its effects on various signal parameters. In the classical theory of scattering,[1] it is accepted that in view of the limit theorem of the probability theory, the scattered field is Gaussian, the envelope amplitude is distributed by the Rayleigh law, and the phase is distributed uniformly in the interval $(-\pi, \pi)$. The aim of this work was to check these theses experimentally through remote acoustic diagnostics of the atmospheric boundary layer (ABL).

There are some publications in literature devoted to the study of probability characteristics of signals at acoustic sensing, in particular, to the study of statistics of the amplitude A and its square. A particular attention in these papers was paid to the study of statistical properties of the structure characteristic of temperature C_T^2 that is related to the square amplitude as $A^2 = \gamma C_T^2$, where γ is some coefficient. Thus, yet in Refs. 2–4 it was stated that the structure characteristic of the temperature field C_T^2 is distributed by the lognormal law, and this statement was assumed valid for both the convective conditions of the ABL and for the case of stable stratification. Quite different conclusions were drawn in Ref. 5, where it was shown that the distribution law of $\log C_T^2$ differs from the normal one and is asymmetric. Moreover, bimodal experimental histograms of $\log C_T^2$ were considered in that paper and possible geophysical interpretation was given to this case. The non-Gaussian distribution of C_T^2 or, more exactly, $(C_T^2 - \bar{C}_T^2)/\bar{C}_T^2$ at convection was noticed in Ref. 6. It was mentioned that the asymmetry at the altitude of 39 m is 0.6, and that at the altitude of 95 m is close to 1.7.

The amplitude distributions of acoustic signals scattered by the atmosphere and received by a sodar are studied, in particular, in Refs. 7 and 8. Analyzing the results of sensing in summer in a steppe region, Kallistratova with co-authors[7] concluded that the distribution of $\log A$ at the altitude of 48 m does not obey the normal law. It was also noted that the shapes of the amplitude distribution of a signal are almost indistinguishable for the conditions of convection and stable stratification. The same conclusions follow from Ref. 8, in which the same experimental data as in Ref. 7 are considered, but for different altitudes under convective conditions. Hypotheses on the shape of the $\log A$ distribution were checked neither in Ref. 7 nor in Ref. 8.

The study of noise occupies an important place in analysis of statistical properties of signals recorded in acoustic sensing of the ABL. Thus, in Ref. 3 it was concluded that the distribution of narrow-band noise power obeys the lognormal law. The detailed analysis of experimental data with the check of statistical hypotheses on the shape of noise amplitude distributions at different frequencies in narrow bands was performed in Ref. 9. It is stated that the lognormal distribution takes place in most cases (54%). The gamma distribution occurs rather often (26%). The Maxwell, Rayleigh, Weibull, and Gauss distributions were observed in several percent of realizations. Statistical hypotheses were checked with the use of the Kolmogorov–Smirnov goodness measure with the confidence level of 0.05.

A brief overview of the literature shows that, in spite of certain attention paid to the problem on statistical properties of scattered acoustic signals, it is still an open problem, and the hypotheses call for further check and confirmation. In this paper, our task was to analyze thoroughly the experimental histograms of the amplitude of the envelope of a scattered narrow-band signal, to check the hypotheses of their correspondence to some theoretical probability distribution laws, and to reveal peculiarities in the distributions under various geophysical conditions.

To study the statistical properties of acoustic signals received by the Volna-3 sodar (IAO SB RAS), we selected groups of typical observations in Tomsk suburbs obtained in all seasons. The group of spring–

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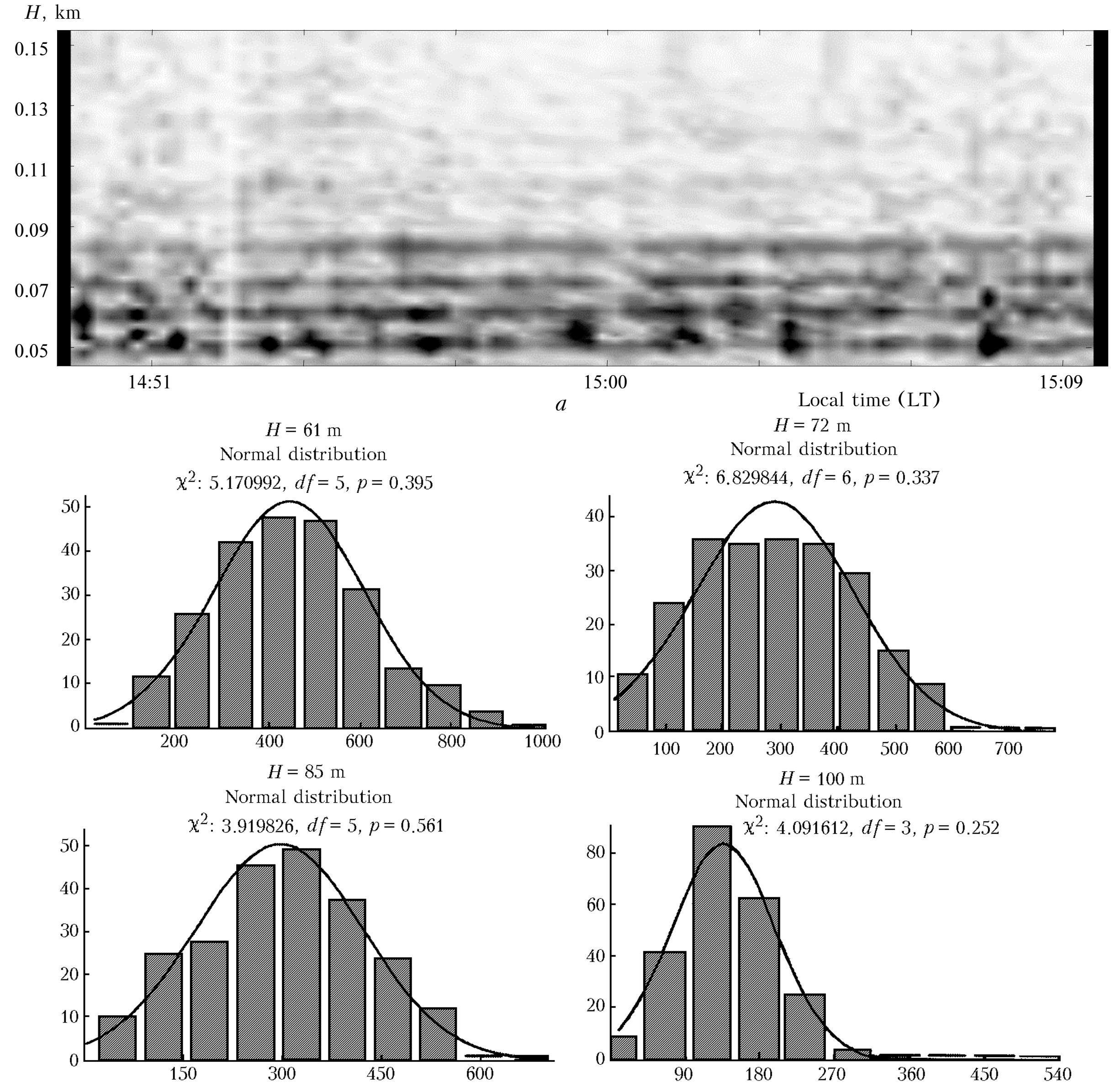
summer records was characterized by developed convection, whereas the group of fall–winter observations was characterized by the presence of temperature inversion. The groups of observations were formed so that the current geophysical situation, in general, was stationary on the whole processing interval. For a comparison with the results on the statistics of signal amplitudes under different geographic conditions, we used the sodar data obtained in a steppe proving ground in the Orenburg Region in 1999. The main characteristics of the Volna-3 sodar are published in Ref. 10. It should only be noted that all measurements processed and presented here were conducted at the sensing pulse duration of 150 ms and the 34-Hz band of a digital filter of the vertical sensing channel (the filter central frequency was 1700 Hz).

All realizations were formed as sets of data at different altitudes for intervals from 4 to 7 h (the number of points in them varied from 1000 to 3000). Under stationary conditions, this allowed the conclusions on the character of distribution of these samples to be thought statistically confident. When analyzing the altitude–time distributions of the envelopes of narrow-band signals, we separated three characteristic parts corresponding to (1) the area of strong echoes from buildings around the sodar, (2) area of useful signal mixed with random external noise, (3) area of noise in the absence of useful signal. Figures 1*a*–4*a* depict some examples of facsimile records of the analyzed groups under conditions of temperature inversion (Figs. 1*a* and 4*a*) and developed convection (Figs. 2*a* and 3*a*).



a

b

Fig. 1. Facsimile record (*a*) and four histograms of the envelope for one-hour interval (sample of 232 readings) with approximating curves (*b*). The gap between sensing pulses was roughly 16 s. Measurements were conducted on January 6 of 2000 in Tomsk under nearly calm conditions and stable stratification.

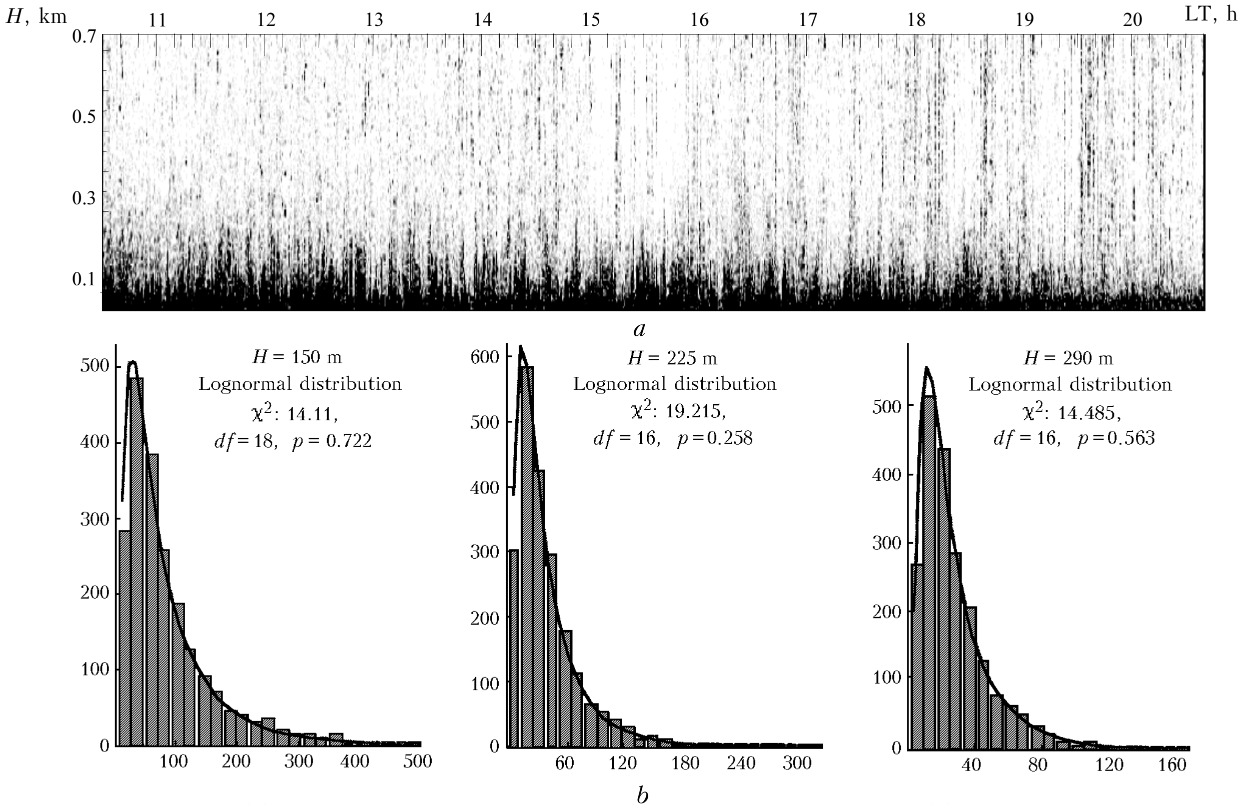


Fig. 2. Facsimile record (*a*) and three histograms of the envelope with approximating curves (*b*). The gap between sensing pulses was roughly 17 s. Measurements were conducted on May 25 of 2000 in Tomsk. The sample size was 2139 readouts.

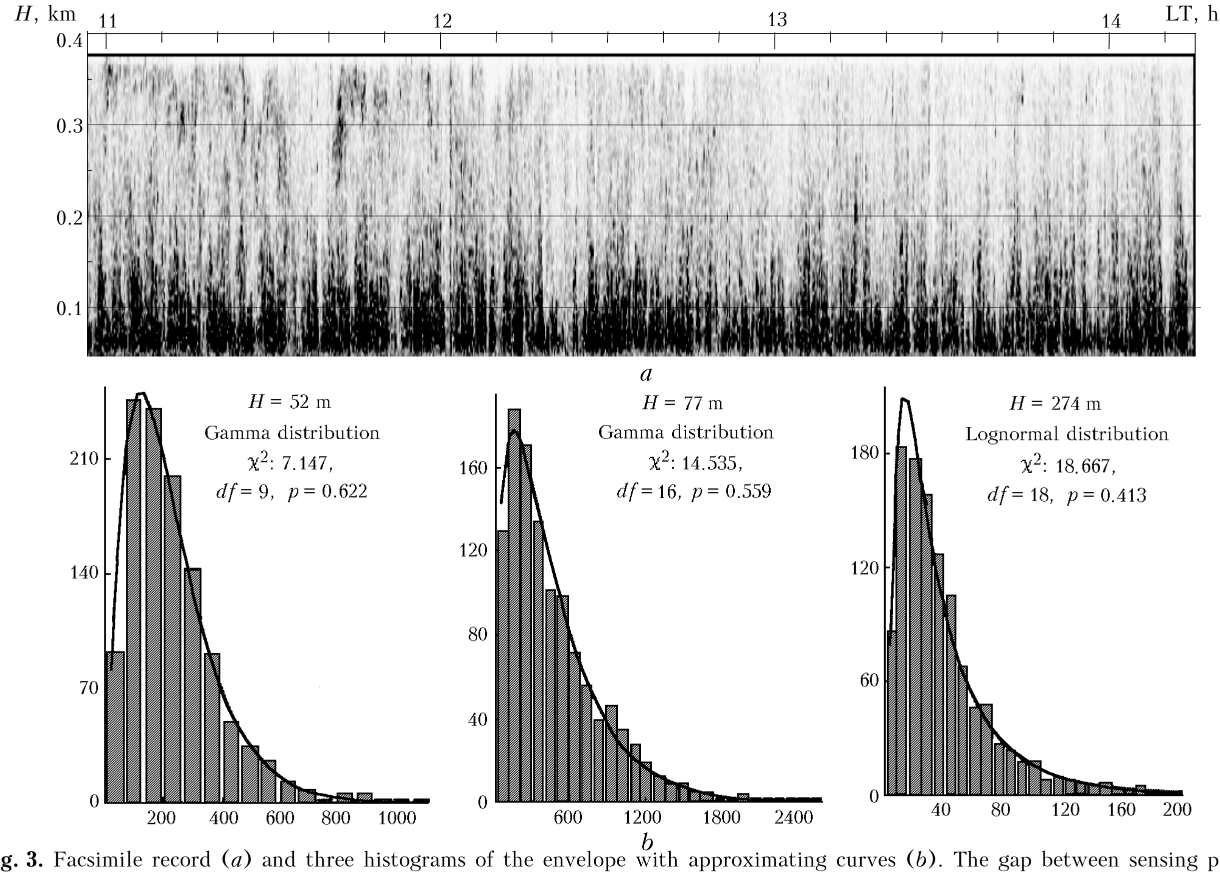


Fig. 3. Facsimile record (*a*) and three histograms of the envelope with approximating curves (*b*). The gap between sensing pulses was a little longer than 10 s. Measurements were conducted in a steppe region on June 1 of 1999. The sample size was 1154 readings.

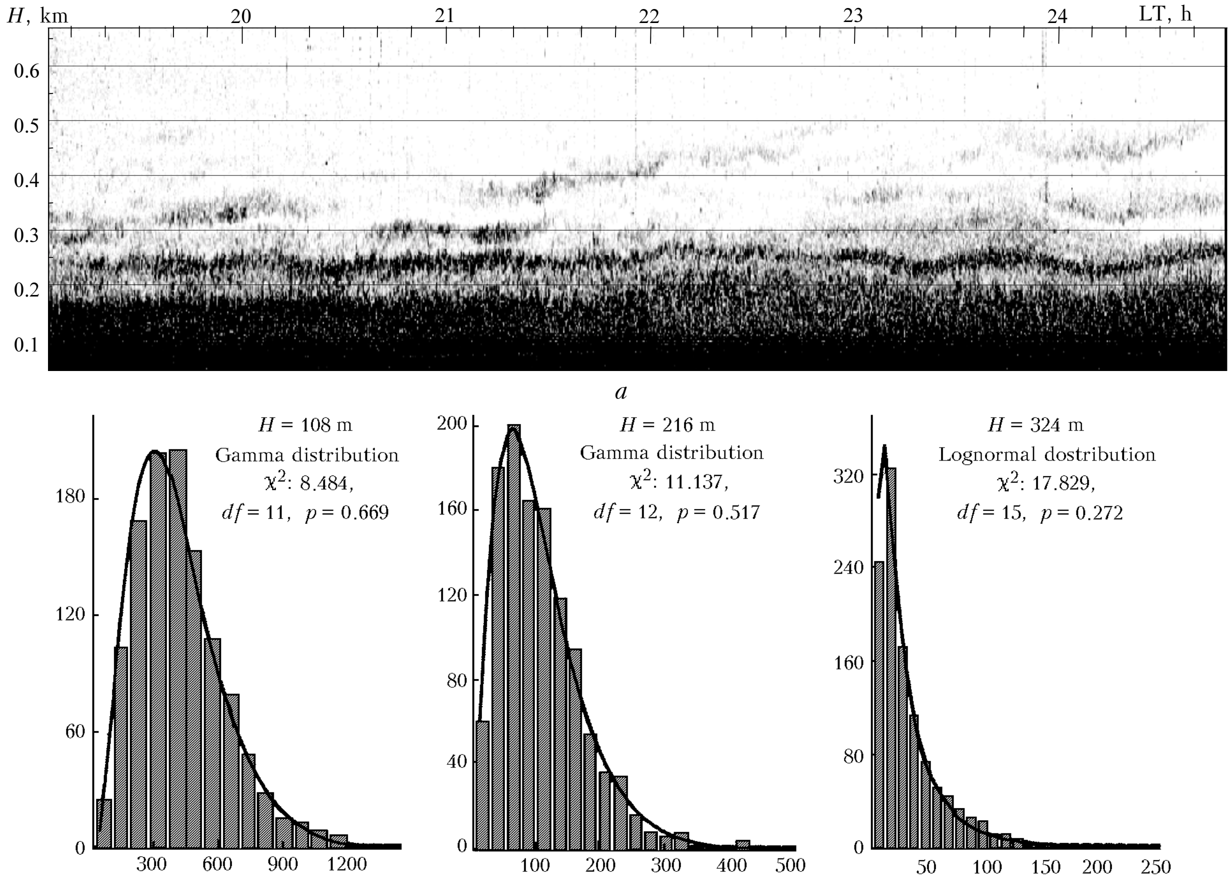


Fig. 4. Facsimile record (*a*) and three histograms of the envelope with approximating curves (*b*). The gap between sensing pulses was roughly 18 s. Measurements were conducted in Tomsk on January 4 of 2000. The sample size was 1158 readings.

For processing of the experimental data we used the standard STATISTICA program, first, because its convenient interface allows very efficient processing of large arrays of experimental data and, second, using the program available for all other investigators, we can hope for adequate check of our conclusions by all interested persons.

To check the hypothesis on the correspondence of envelopes to that or another theoretical distribution law, we selected the goodness measure – chi-squared, since it is the strictest criterion in the STATISTICA menu. For approximation of experimental histograms we used the following distributions: Rayleigh, gamma, lognormal, normal, Weibull, chi-squared, and exponential ones.

The sodar amplitude records obtained in Tomsk in the altitude range up to 100 m always include reflections from neighboring buildings. Figure 1*b* exemplifies experimental histograms of signals reflected from buildings, as well as the best-fit theoretical curves. Hereinafter, histograms are accompanied by STATISTICA plots. The digital analog of signal amplitudes in relative units is plotted as an abscissa, and the rate of amplitude fall off within given intervals is plotted as an ordinate. Every histogram is marked with the altitude H, for which the check was performed, as well as with the type of the distribution and the parameters of fit by the chi-squared criterion: the chi-squared value, the number of degrees of freedom (df), and the probability (p) that the measure of discrepancy between the theoretical and experimental distributions, which obeys the chi-squared law, exceeds the actually observed value of chi-squared in this sample due to purely random causes.[11] It should be noted that the value of p exceeding 0.1 is considered in practice as high enough for the hypothesis to be accepted. At the same time, the level $p > 0.9$ may indicate the nonrandom character of the sample.

According to the results presented in Fig. 1*b*, as well as the data of analysis of a great number of other similar situations, we can conclude that the envelopes of signals reflected from fixed objects have the close-to-normal distribution. Excesses and asymmetry coefficients in this case tend to zero, and this also indicates that samples are close to the normal distribution law. The check for correspondence to other distribution laws gives the values $p \ll 0.1$.

Quite different pattern is observed in the parts corresponding to “mixed” areas, where both the signal and noise are present. Consider the records obtained under conditions of developed convection (see Figs. 2 and 3). The asymmetry coefficients here are positive and usually varying from 0 to 4. The excesses are also positive and widely spread. Based on the results of analysis of a great number of records for spring and summer seasons (under convective conditions), we can conclude the following: in the overwhelming majority of cases (up to 90%) at the altitudes higher than 100 m the experimental data are best described by the lognormal distribution law. Up to 5% of the considered records obeyed the gamma distribution. The Rayleigh and Weibull distributions were observed very rarely. In some cases, no of the distributions fitted the experimental data. Figure 2*b* shows typical examples of processing for convective conditions in Tomsk suburbs.

Signals received by the sodar in daytime in the steppe region at rather weak external noise can be interpreted with confidence as a classical example of developed convection. At lower altitudes (below 100 m), where temperature turbulence scattering the sound is almost always present, experimental histograms were well described by the gamma distribution. The amplitude statistics changed with height – the histograms became to be best approximated by the lognormal law. As an example, Fig. 3*b* depicts the results of comparison of the distribution laws for samples at different altitudes.

Comparing our results with the conclusions of Refs. 7 and 8 for the convective conditions, we can conclude that the distribution of $\log A$ in the atmospheric surface layer actually does not obey the normal law. However, at high altitudes, according to our data, in contrast to the results from Ref. 8, $\log A$ has the close-to-normal distribution. This contradiction demands further investigations.

We also would like to notice the possible problem arising at high repetition frequency of sensing pulses. If the pulse repetition frequency is not connected with the length of the ABL scattering region, then the amplitude of the recorded signal at every instant includes an addition due to scattering of the previous sensing pulse at higher altitudes. This effect possibly took place in Refs. 7 and 8, where the gap between pulses was 1 s in some daytime experiments (the limit sensing altitude of about 170 m), and the length of the scattering regions achieved 500 m and more under these conditions, as judged from the facsimile records. This problem also could manifest itself in Ref. 5, where the gap between pulses was also about 1 s. Naturally, this effect distorts the statistics of $\log A$ and $\log C_T^2$.

Consider now the statistics of signals under conditions of stable temperature stratification. Voluminous experimental material, especially, for the winter period allowed us to process many similar situations and to draw certain conclusions. In particular, the asymmetry coefficients usually range from 1 to 4, the excesses are positive, but not so widely spread, as in the case of convection. Selecting the conditions with relatively stationary amplitude at the given altitudes, we approximated experimental distributions by the above-mentioned theoretical laws.

Based on the analysis of obtained results, we can draw the following conclusions. In the regions with high turbulence intensity, the envelope histograms are well described by the gamma distribution. In the transition zones, that is, at the record parts including a boundary between useful signal and noise regions, we can see the agreement with the lognormal distribution law. We can assert that in 90% of considered realizations (at temperature inversions) the signal amplitude has the gamma distribution.

The rest 10% are the cases that the signal obeyed the lognormal law (about 5%) or no one of considered laws. It should also be noted that the Rayleigh distribution was observed only in single realizations. Figure 4*b* demonstrates the correspondence of the lognormal and gamma distributions to experimental histograms of the envelope of a narrow-band signal received from different altitudes under conditions of stable temperature stratification.

As was already mentioned, it follows from the theory[1] that the envelope of the scattered narrow-band acoustic signal is distributed by the Rayleigh law. However, our experimental results and the results of our colleagues (see, for example, Refs. 7 and 8) do not agree with this. Now we have no acceptable hypothesis explaining different distribution laws of the envelope of a sodar echo signal. Remind that the gamma distribution usually takes place in the situations, when the analyzed characteristic is a sum of independent random parameters, each distributed by the exponential law. The lognormal probability distribution is often used in statistical radio physics when studying signals in communication channels with scattering and statistics near the lower physical limit of parameters, as well as when approximating noise of various origin.

In addition, it should be noted that the parameters of the gamma distribution obtained from fitting of the theoretical curve to the experimental histogram did not reduce this distribution to some other law (chi-squared, exponential, etc.) at all types of stratification.

In statistical analysis of signals under convective conditions, we undertook an attempt to exclude "nonstationarity" of the signal amplitude at a certain altitude. The point is that at relatively short intervals the signal behavior at some altitude under convective conditions can be thought nonstationary, since "convective feathers" alternate with "hollows" (see Figs. 2*a* and 3*a*). To exclude the effect of this alternation, the technique of signal processing was changed. "Hollows" were excluded by a computer program through filtering according to a threshold connected with the current noise level. The obtained "pure" signal was then approximated by the same theoretical laws. As a result, if a sample signal initially obeyed, for instance, the lognormal law, after such filtering, it stopped to obey this law, as well as any other of the studied laws, including the Rayleigh one.

It follows from here that this filtering significantly destroys the statistical properties of the envelope and therefore it cannot be applied to its study.

Consider the parts of records corresponding to the end of the sensing path. The useful signal is, as a rule, absent here and noise dominates. After analysis of a large number of realizations, we concluded that no certain law is observed in the distribution of noise recorded by the sodar. This conclusion differs from the results of Ref. 9, where it was stated that the distribution of the noise amplitude largely obeys the lognormal law. This is likely connected with application of the stricter criterion – chi-squared, in contrast to the Kolmogorov–Smirnov criterion at the confidence level of 0.05 used in Ref. 9.

As a result of this analysis, we can draw the following conclusions:

1. Regardless of stratification of the atmospheric boundary layer, the envelope of the signal reflected from a fixed object has the close-to-normal distribution law. This is explained by the fact that the signal includes regular components prevailing over random ones, and in this case the distribution law tends to the normal one.[12]

2. In the absence of useful signal, that is, when analyzing the noise, no certain distribution law was found in all considered situations.

3. The distribution law of the signal envelope depends on the atmospheric stratification:

– under convective conditions, the amplitude of the envelope of a sodar echo signal coming from regions above the surface layer in the most cases is distributed by the lognormal law, while in the surface layer it has the gamma distribution;

– under conditions of temperature inversion, the amplitude of the received signal mostly obeys the gamma law, and near the boundaries of scattering regions it agrees with the lognormal law.

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