AVAILABLE INFORMATION FOR DETERMINING THE OPTICAL PARAMETERS OF ATMOSPHERIC LAYERS FROM MEASUREMENTS OF SPECTRALRADIATION FLUXES. I. FORMATION OF THE PROBLEM AND COMPUTATIONAL RESULTS FOR A SEPARATE LAYER

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The information content of vertical profiles of the spectral hemispherical fluxes of short-wavelength radiation in the atmosphere for use in reconstructing the corresponding vertical profiles of the optical parameters of the medium is determined the inhomogeneous atmosphere is divided into layers (of the optical density, photon survival probability, and elongation of the scattering phase function) and the problem is solved for all parameters and for all layers simultaneously. The matrices of the partial derivatives of the indicated fluxes with respect to the atmospheric parameters of the atmosphere are studied for the case of clear (τ_0 = 0.09), turbid (τ_0 = 0.9), and highly turbid (τ_0 = 3.5) atmospheric conditions. The information content of measurements is evaluated quantitatively and it is shown that there is enough information to solve the inverse problem (except for the elongation of the scattering phase fund ion in the case of a very clear atmosphere).

The microphysical and optical parameters of the atmosphere can be determined from the characteristics of the attenuated or scattered radiation field by different methods. Most of these methods employ measurements of the direct radiation and angular measurements of the intensity scattered radiation (Bouguer's method for measuring the optical thickness, determination of the scattering phase function from the measured sky-brightness indicatrix at the parallel of altitude of the sun, etc.).

Although there are no great difficulties in performing angular measurements of the scattered radiation at the ground, airborne measurements present considerable technical difficulties. First, the structure of aircraft makes it impossible to point measuring instruments into the upper hemisphere. Second, unfavorable flight conditions – rough air, yawing, vibration, etc. – can sharply reduce the accuracy of such measurements and even make them completely ineffective (for example, measurements performed by Bouger's method). To eliminate these difficulties the measuring part of the instrumentation must be placed on a gyrostabilized platform, which, in its turn, sharply increases the weight and power requirements of the instrumentation, etc. For this reason in performing airborne measurements it is simpler (although not as easy as on the ground) to measure hemispheric radiation fluxes. The spectral instrumentation for performing such measurements has been developed at the Laboratory of Short-Wavelength Radiation of the Scientific-Research Institute of Physics at Leningrad State University and it has been installed on aircraft; the measurements conditions are described in Refs. 1 and 2).

In this connection there arises the question of the information content of flux measurements, i.e., the possibility of obtaining information about the optical and microphysical parameters of the atmosphere and separate layers of the atmosphere from measurements of the angle-integrated characteristics of the radiation.

The first attempt at reconstructing the vertical profiles of the optical thickness τ_λ and the photon survival probability Λ_λ from spectral measurements of upward and downward fluxes was made in Ref. 1. In Ref. 1 the inverse problem is solved with the help of analytical inversion of the two-flux approximation, but neither the random errors in the experimental measurements, nor the systematic error, introduced by the inaccuracy of the two-flux approximation was evaluated. More accurate methods are described in Refs. 3 and 4, but these works are devoted solely to the determination of the vertical profile of the true absorption coefficient for light, though the fluxes carry more information. In the classical book Ref. 5 it is stated on p. 123 that "the spherical albedo depends on the optical thickness of the atmosphere τ_0, the scattering phase function $x(\gamma)$, and the parameter Λ..." and previously"... the spherical albedo is the ratio of the energy reflected by the entire planet to the energy incident on the planet from the sun". It is obvious that this assertion is valid not only for the albedo (upward fluxes normalized to the incident fluxes), but also for the fluxes leaving the atmospheric layer under study.

We shall apply to the problem of reconstructing the parameters of the atmosphere from flux measurements the well-known methods, described, for example, in Refs. 6 and 7, for solving inverse problems. The measurements are performed at $n + 1$ levels in the atmosphere; this corresponds to the separation of the atmosphere into n layers (Fig. 1). The spectral hemispherical fluxes are measured: the incident fluxes $F_i^{\downarrow}$ (i = 2, ..., n + 1) and the upward fluxes $F_i^{\uparrow}$ (i = 2, ..., n + 1) (there are no measurements at the hop boundary of the atmosphere). The zenith angle of the sun ν and solar constant πS are assumed to be known, and the surface is assumed to be defined as

$$A_0 = \frac{F^{\uparrow}_{n+1}}{F^{\downarrow}_{n+1}} \quad (1)$$

a result of which measurement of $F^{\uparrow}_{n+1}$ is no longer informative (relative to the optical parameters). Here and below, to simplify the formulas λ, which denotes the measured monochromatic quantities, is omitted.

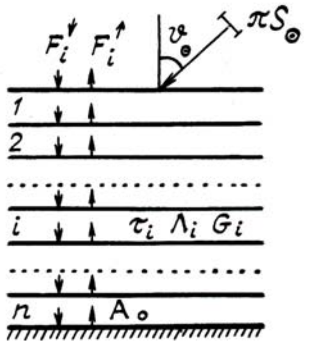

FIG. 1. The model of the atmosphere.

The atmosphere is characterized by the following optical parameters: the optical thickness τ_λ) (the total optical thickness of the atmospher'e is τ_0), the photon survival probability $\Lambda(z)$, and the scattering phase function $x_z(\gamma)$. Since τ is a monotonic function of z, it can be used as the vertical coordinate. Knowing $\nu_\odot$, $\pi S_\odot$, A_0, τ_0, $\tau(z)$ (or $z(\tau)$), $\Lambda(\tau)$, $x_\tau(\gamma)$ the radiation field in the atmosphere can be calculated completely and the values of the measured quantities can be found by solving the transfer equation. To facilitate the solution of this equation parameterization is, as a rule, performed, i.e., τ, $\Lambda(\tau)$ and $x_\tau(\gamma)$ are approximated by piecewise-constant functions in separate atmospheric layers: τ_i, Λ_i $x_i(\gamma)$, where i = 1, ..., n and n is the number of layers (here τ_i is the optical thickness of the i-th layer). The angular behavior of $x(\gamma)$ cannot be determined from flux measurements, so that only the problem of determining one of the basic parameters of the scattering phase function can be formulated. It is natural to choose the elongation of the scattering phase function of this parameter:

$$G = \frac{\int_0^{\pi/2} x(\gamma)\sin\gamma\, d\gamma}{\int_{\pi/2}^{\pi} x(\gamma)\sin\gamma\, d\gamma} \quad (2)$$

To solve this problem we shall employ a one-parameter family of scattering phase functions according to the classification given by O.D. Barteneva[8]. The values of $x(\gamma)$ for arbitrary G (from 1 to 40) are obtained by interpolating between the phase functions of this family.

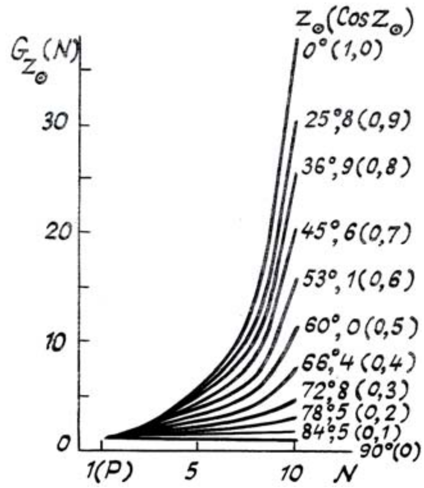

FIG. 2. Asymmetry of different scattering phase function as functions of the zenith angle of the Sun.

Figure 2 shows the calculations of $G_{\nu_\odot}$, performed for different zenith distances of the sun ν_0 as a function of the number of the scattering phase function n in Barteneva's classification[8] (in the graph R is the Rayleigh scattering function).

The transfer equation was solved by the Monte-Carlo method. This method is convenient for problems of this type because it gives both the quantity of interest as well as its variance. We employed the standard method of weighted modeling "without absorption" and "without leaving the medium"[9,10] truncating the trajectories when the photon weight drops below 0.1 of the required computational accuracy.

Thus the collection of optical parameters A_0, τ_i, Λ_i, τ_i (i = 1, ..., n) permits determining completely the measured quantities $F_i^{\downarrow}$ and $F_i^{\uparrow}$ (i = 2 ... n+1). We shall formally write the measured fluxes as f_i (I = 1,...,n): f_i is $F^{\downarrow}_{i+1}$ for i = 1,...,n and $F^{\uparrow}_{i-n-1}$ for i = n+1,...,2n–1; x_i (i_i = 1,...,3n) are the optical parameters of the atmosphere: x_i is τ_I for i = 1,...,n, Λ_{i-n} for i = n+1,...,2n, and G_{i-2n} for i = 2n+1,...,3n. Then the collection of numbers x_1 can be interpreted as the coordinates of the point x in a 3n-dimensional space, while the collection of numbers f_1 can be interpreted as the coordinates of the point F in a (2n–1)-dimensional space. Solving the transfer equation (the direct problem) is an operation that to each point X associates a point X a point F; this can be written as

$$F = KX \quad (3)$$

where K is the solution operator of the direct problem. We note some properties of the operator K: a) an exact analytical expression does not exist for K and b) K is nonlinear over all coordinates x_1. We expand the operator K in a Taylor series about the point $F^0 = KX$

$$F=F^0+\frac{dK(X^0)}{dX}(X-X^0)+...,$$

where the symbol $\frac{dK(X^0)}{dx}$ is the operator for differentiating K, evaluated at the point problem X^0. Writing $F - F^0 = \Delta F$; and $X - X^0 = \Delta X$ and truncating the series at the first derivative gives

$$\Delta F=\frac{dK(X^0)}{dX}\Delta X \tag{4}$$

Equation (4) is linear the unknowns ΔX. It is expressed in terms of the coordinates by the matrix of the partial derivatives

$$\left(\frac{\partial f_1}{\partial x_i}\right)_{\substack{i=1,...,2n-1;\\ j=1;...;3n}}$$

Then Eq. (4) in the coordinate form is:

$$F=F^0+\frac{dK(X^0)}{dX}(X-X^0)+...,$$

The foregoing discussion shows that the optical parameters of the atmosphere can be found from flux measurements by an iteration method, namely, a zeroth-order approximation X^O is chosen (for example, simply the average model of a clear atmosphere) and the direct problem is solved for it - the fluxes F^O are determined. Once the difference ΔF between the observed and computed fluxes and the matrix of the partial derivatives $\frac{dK(X^0)}{dx}$ (see below) have been calculated Eq. (4) can be solved, the corrections ΔX to the zeroth-order approximation can be determined, and the next approximation can be obtained: $X^1 = X^0 + \Delta X$, etc.

Observations at the boundaries of each layer (top and bottom) give two quantities that depend on the optical parameters of the given layer, namely, the outgoing fluxes. In addition, because the divergence of the total flux (influx of radiant energy into the layer) is nonzero the photon survival probability Λ_1 can be found directly from it.

Equation (4) can be solved by Tikhonov's method of regularization[5]. This method is preferable because the regularization takes into account correctly the measurement error and also because the number of unknowns in Eq. (4) is formally greater than the number of equations. One should not be daunted by this last fact, since the method of regularization was developed precisely for choosing from an infinite set of possible solutions (and inverse problems always have an infinite number of solutions because of the presence of measurements error), the physically meaningful solution, and the error in the solution can be determined.

Consider the calculation of the matrix of partial derivatives $\left(\frac{\partial f_1}{\partial x_j}\right)$. To find this matrix we employed the Monte-Carlo method[9]. In this method photon transfer is modeled by identical sequences of random numbers but for different models of the atmosphere differing by a change in one of the parameters ΔX (ΔX_1 is quite small). The computational scheme of the algorithm is analogous to the solution of the direct problem, except that local values are employed to calculating the fluxes.

Analysis of a concrete form of the matrices of partial derivatives for given models of the atmosphere may give a preliminary answer to the question of the information content of the measurements. For this it is convenient to employ logarithmic derivatives (see Ref. 7), which show the relative change in the flux owing to a change in the optical parameters. It is obvious that only those parameters for which the logarithmic derivatives exceed the measurement error can be determined (more precisely, they can be determined with a variance that is less than the *a priori* value).

We shall first study as an example the calculations performed for a four-layer model of the atmosphere, but the parameters of only one layer are determined. The parameters of the starting optical model of the atmosphere were taken from observations performed on October 25, 1970 at λ = 500 nm (Table 1)[1].

TABLE 1.
The starting optical model of the atmosphere.

LAYER No.	TOP AND BOTTOM LEVEL (mBar)	τ^0	Q^0	Λ^0
1	0-400	0.26	10	0.84
2	400-700	0.23	10	0.85
3	700-900	0.18	10	0.87
4	900-1000	0.10	10	0.89

For the indicated layer between the 700 and 900 mbar levels the matrix of values of the logarithmic derivatives has the form

$$\left\|\frac{\partial(\lg F^{\downarrow}, \lg F^{\uparrow}, \lg\beta}{\partial(\lg\tau, \lg G, \lg(1-\Lambda))}\right\| \approx (0.3)^3 \left\|\begin{matrix} -1 & 0.816 & -0.558 \\ 0.301 & -1 & -0.598 \\ 0.965 & -0.096 & 1 \end{matrix}\right\|$$

Examination of this matrix shows that the downward outgoing flux from this layer is the most informative parameter for determining τ, the upward outgoing flux is most informative for determining G, and the relative radiant influx is most informative for determining Λ.

TABLE 2.

The results of the iterative process for single layer.

PARAMETER	EXPERIMENT	CALCULATION of "ZERO" APPROXIMATION	DIFFERENCES of "ZERO" APPROXIMATION		CALCULATION of "PRELIMINARY" APPROXIMATION	DIFFERENCES of "PRELIMINARY" APPROXIMATION	
			absolute	relative (%)		absolute	relative (%)
$F_\lambda^\downarrow$	0.9042	0.9123	−0.0081	−0.9	0.9048	−0.0006	−0.1
$F_\lambda^\uparrow$	0.1926	0.1785	+0.0141	+7.3	0.1900	+0.0026	+1.3
β_λ	0.0546	0.0614	−0.0068	−12.4	0.0507	+0.0039	+7.1

The data in Table 1 were calculated based on calculations of the indicated quantities in the two-flux approximation[1] (except for the values of $G^{(0)}$). The experimental data and the Monte-Carlo results (in units of $F_\lambda^\downarrow(H_1)$ are presented in Table 2.

It follows from Table 2 that, first, the iteration process converges and, second, even the first approximation gives in our example values of $\{\tau_\lambda, \Lambda_\lambda, G_\lambda\}$ with whose help $\{F_\lambda^\downarrow, F_\lambda^\uparrow, \beta\}$ can be calculated with an error less than the error of made in measuring them. To simplify the analysis we study an example in which the optical parameters of one layer were determined more accurately. But even in this example varying $\{\tau_\lambda^{(0)}, G_\lambda^{(0)}, \Lambda_\lambda\}$ over a range of ±50% changed the flux $F_\lambda^\downarrow(H_1)$ incident of the top boundary of the layer by ±2%. This indicates that the corrections to the starting parameters of the atmospheric model must be found simultaneously for all layers into which the atmosphere is separated. The information content of flux measurements for this case will be studied in Part II of this work.

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