AVAILABLE INFORMATION FOR DETERMINING THE OPTICAL PARAMETERS OF ATMOSPHERIC LAYERS FROM MEASUREMENTS OF SPECTRAL RADIATION FLUXES. II. EVALUATION OF THE INFORMATION CONTENT OF MEASUREMENT IN A MULTILAYERED ATMOSPHERE

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A procedure for linearizing the solution of the inverse problem in the analysis of experimental measurements of spectral fluxes of short-wavelength radiation in the atmosphere for the purpose of obtaining vertical profiles of the optical parameters of the atmosphere – the optical thickness, the photon survival probability, and the scattering phase function – is described. As an example the optical parameters of a separate layer of an inhomogeneous multilayered atmosphere are determined by the method of successive approximations. It is shown that the solution converges rapidly and the required accuracy can be achieved.

In Part I of this work we formulated the problem of reconstructing the optical parameters of atmospheric layers from measurements of the vertical profiles of radiation fluxes and we gave an example of the convergence of the iteration process in determining the optical parameters of one layer in a four-layer atmosphere.

We shall now examine the possibility of determining the optical parameters of all layers of the atmosphere at the same time. The most important stage in the solution of the inverse problem is the calculation of the matrix of partial derivatives of the fluxes with respect to the optical parameters of the atmosphere. Before solving the inverse problem for specific experimental data, however it is useful to determine the information content of the solution based on the obtained matrix of partial derivatives. This will show which parameters of the atmosphere can be determined from measurements (with variance less than the a priori value).

To evaluate the information in the case when several layers are studied simultaneously we calculated the derivatives for a three-layer atmosphere in three cases: "thin" atmosphere $\tau_0 = 0.09$, average atmosphere $\tau_0 = 0.9$, and "thick" atmosphere $\tau_0 = 3.5$. The layers had the same thickness, and the values of the remaining parameters were as follows: $\Lambda_i = 0.5$: $G_i = 15$; $\pi S_\odot = 100$ (arbitrary units); $A_0 = 30\%$ and $\vartheta_0 = 45°$. Before analyzing the results we note the characteristic values of the variations $\Delta\tau \sim 0.1$, $\Delta\Lambda \sim 0.1$, and $\Delta G \sim 10$. For this reason, if the partial derivative with respect to G_i is two orders of magnitude less than the derivative with respect to τ_i or Λ, then their information content is identical. This difference is, obviously, less pronounced for the values of the logarithmic derivatives (Table 1, no indication of the variances). We shall examine all three cases in greater detail.

a) "Thin" atmosphere, $\tau_0 = 0.09$. The derivatives of the upward and downward fluxes with respect to τ have the largest values. This is as expected, since only scattered radiation is dependent on Λ and G, and its relative contribution at the boundaries of the layers is 4.8 and 12%, respectively. As regards derivatives with respect to Λ_i and G_i, probably only Λ_i can be reconstructed from them, since the derivatives with respect to G_i are smaller contribution is very small in this case. It can be expected that for such small values of τ_0 the information content of the derivatives with respect to Λ and G will increase as the scattering increases, i.e., as $\vartheta_\odot$ and A_0 increase.

b) "Average" atmosphere, $\tau_0 = 0.9$. The derivatives of the incident flux with respect to τ, Λ, and G are approximately equally informative), i.e., the downward flux leaving a layer is approximately equally dependent on each of the optical characteristics. We note that in all three cases the matrices of the derivatives of the incident flux (to within order of magnitude) triangular; this is natural, since the incident flux contains very little information about the layers through which the flux has not yet passed (this information is formed by backscattering of the upward flux, i.e., scattering of second and higher orders). As regards the upward flux, its dependence on Λ is strongest. The weak (as compared with Λ) dependence of the upward flux on τ and G can be explained by the existence of opposing tendencies: as τ increases the decrease of this flux reduces the upward flux at the surface, but as the backscattering of the incident flux increases the upward flux increases; analogously, as G increases the incident flux also increases, while the backscattering decreases.

TABLE 1.

Matrix of logarithmic derivatives of the fluxes with respect to the optical parameters of the layers

	$\partial \ln F_2^{\downarrow}$	$\partial \ln F_3^{\downarrow}$	$\partial \ln F_4^{\downarrow}$	$\partial \ln F_1^{\uparrow}$	$\partial \ln F_2^{\uparrow}$	$\partial \ln F_3^{\uparrow}$
			$\tau_0=0.09$			
$\partial\tau_1$	-0.042	-0.039	-0.042	-0.018	-0.040	-0.044
$\partial\tau_2$	-0.002	-0.039	-0.042	-0.018	-0.006	-0.044
$\partial\tau_3$	-0.002	-0.006	-0.042	-0.018	-0.006	-0.025
$\partial\Lambda_1$	0.046	0.046	0.046	0.099	0.044	0.042
$\partial\Lambda_2$	0.001	0.049	0.050	0.103	0.097	0.042
$\partial\Lambda_3$	0.001	0.001	0.056	0.121	0.114	0.106
∂G_1	0.004	0.006	0.006	0.007	-0.002	0.003
∂G_2	0.001	0.004	0.004	0.004	-0.002	-0.003
∂G_3	0.001	0.002	0.004	0.000	-0.007	-0.002
			$\tau_0=0.9$			
$\partial\tau_1$	-0.031	-0.511	-0.444	-0.062	-0.125	-0.111
$\partial\tau_2$	0.012	-0.511	-0.444	-0.062	-0.103	-0.111
$\partial\tau_3$	0.012	0.020	-0.444	-0.062	-0.103	-0.000
$\partial\Lambda_1$	0.308	0.323	0.273	1.125	0.206	0.128
$\partial\Lambda_2$	0.040	0.485	0.433	1.076	0.563	0.174
$\partial\Lambda_3$	0.015	0.044	0.353	0.832	0.469	0.317
∂G_1	0.089	0.147	0.107	-0.114	-0.055	0.009
∂G_2	0.045	0.073	0.107	-0.016	-0.102	-0.009
∂G_3	0.000	0.034	0.134	-0.122	-0.117	-0.085
		$\tau_0=3.5$				
$\partial\tau_1$	-2.334	-1.776	-0.708	-0.108	-0.231	-0.045
$\partial\tau_2$	-0.093	-1.776	-0.708	-0.108	-0.038	-0.045
$\partial\tau_3$	-0.093	0.148	-0.708	-0.108	-0.038	-0.029
$\partial\Lambda_1$	2.271	2.043	1.486	4.941	0.192	0.034
$\partial\Lambda_2$	0.149	2.102	1.163	1.765	0.323	0.037
$\partial\Lambda_3$	0.049	0.207	1.571	0.706	0.162	0.062
∂G_1	0.311	0.148	0.283	-0.490	-0.256	- 0.021
∂G_2	0.000	0.345	0.354	-0.127	-0.385	-0.062
∂G_3	0.016	-0.205	0.283	-0.000	-0.077	-0.492

3) "Thick" atmosphere, $\tau_0 = 3.5$. The information content of derivatives of the incident flux with respect to τ, Λ and G is also approximately the same. For the upward flux, however, as compared with the preceding case, the information content of the derivatives with respect to τ is lower, while the information content of the derivatives with respect to Λ and G is higher (the increase for derivatives with respect to is especially sharp). All matrices of the derivatives of both the incident and upward fluxes are nearly triangular, i.e., for such large values of τ_i the fluxes carry information primarily about the optical characteristics of the layers which the flux has already passed. The derivatives of the scattering phase function with respect to the parameter G_i are unexpectedly large, though theoretically the dependence on this function should gradually vanish as the optical thickness increases. The absence of this effect can probably be explained by the fact that for a strongly elongated scattering phase function ($G_i = 15$) $\tau = 3.5$ is still a quite small optical thickness.

We shall now evaluated the information content of each optical parameter of the atmospheric layers relative to the entire complex of measurements. We shall evaluate the information content quantitatively with the help of the matrix of a posteriori variance - Fisher's matrix (see, for example, (Ref. 1)).

By definition, the amount of information is given by the logarithm of the ratio of the a posteriori and a priori variances:

$$J=-\frac{1}{2}\log_2\frac{\sigma^2_{i,apost}}{\sigma^2_{i,apr}}.$$

The a posteriori variance is given by the diagonal elements of Fisher's matrix:

$$\mathbf{F}=(\mathbf{A}+\Sigma^{-1}\mathbf{A}+\mathbf{D}^{-1})^{-1},$$

where A is the matrix of partial derivatives and Σ is a diagonal matrix of a priori variances. Then the information content of the i-th component of a vector relative to the entire complex of measurements is given by

$$J_i=-\frac{1}{2}\log_2\frac{f_{ii}}{d_{ii}}.$$

where d_{ii} are the diagonal elements of the matrix D, while f_{ii} are the diagonal elements of the matrix F. The amount of information is measured in bits.

We shall now evaluate the amount of information in the cases 1, 2, and 3 above. We shall assume that the fluxes are measured with an accuracy of 1.5%, and for the a priori variances we choose τ_0 for τ, 0.25 for Λ, and 15 for G, which covers the range of possible variations in these quantities in a clear atmosphere.

TABLE 2.

The amount of information (in bits) contained in the entire complex of flux measurements about the parameters of the atmospheric layers

	"THIN" ATMOSPHERE	"MEAN" ATMOSPHERE	"THICK" ATMOSPHERE
τ_1	4.05	3.88	3.96
τ_2	3.02	4.32	3.91
τ_3	3.51	4.32	4.49
Λ_1	0.51	1.12	0.70
Λ_2	0.53	1.01	0.82
Λ_3	0.74	1.11	1.14
G_1	0.42	0.15	0.34
G_2	0.02	0.19	0.33
G_3	0.18	0.21	0.13

The results of the calculations are given in Table 2. It is obvious that the obtained estimates of the information content confirm the conclusions drawn based on "visual" analysis of the matrices of partial derivatives.

Thus analysis of the partial derivatives of the fluxes with respect to the optical parameters of the atmospheric layers shows that the optical parameters of the atmosphere can be determined from flux measurements. In Part 3 of this work we shall examine the question of finding the parameters of the atmosphere from concrete experimental data.

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REFERENCES

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