MODEL OF A GAS AND DUST CLOUD OF ROCKET FLAMES AT HIGH ALTITUDES

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Received November 23, 1992

A model of a cloud of a rocket flame is proposed. The cloud configuration, density distribution of pressure, and some gas dynamics parameters of a cloud along a rocket trace are analyzed analytically.

GENERAL STATEMENTS

The jet flames from geophysical and ballistic rockets at high altitudes are the widespread optical phenomenon in the upper atmosphere. They are the high–power broadening streams of gases and solid particles which are generated when burning the rocket propellant. The chemical composition of flames depends on the type of the used propellant. For example, the *J*–2 liquid rocket engine mounted onboard the Saturn–5 space rocket operates on oxygen–kerosene fuel, and its flame stream contains mainly a mixture of carbon dioxide and water vapor. The jet flames of the solid–propellant rocket engines consist mainly of carbon dioxide, carbon oxide, water vapor, solid particles of aluminum, and carbon oxides heated up to high temperatures. The solid–particle size can reach[1] 13 μm. The spectrograms of rocket flames are a set of sharp spectral lines of water vapor, carbon oxide, and carbon dioxide against the continuous background of the heated particle emission.[1] Both the spatiotemporal characteristics of the inhomogeneity generated by the jet flame and its inner structure are of great interest.

The rate of the gas streams issuing from an exhaust nozzle of rockets is higher than 2 km/s (see Ref. 1), and the dynamic pressure in the near zone of the nozzle is many orders of magnitude (> 10^7) greater than the local dynamic pressure of the air flow[1] since the medium in the ionosphere is very rarefied (for example, at an altitude of 150 km the air density is 10^{-9} of that in the near–ground layer). Therefore the combustion gases of rocket propellant are spreaded over the quite large area of the stream as in vacuum.

The mechanism of interaction of the exhausted combustion gases with an ambient medium changes from a nozzle edge downstream of the jet flame. In the interaction area one can recognize four principal zones with the characteristic gas dynamics peculiarities (Fig. 1).

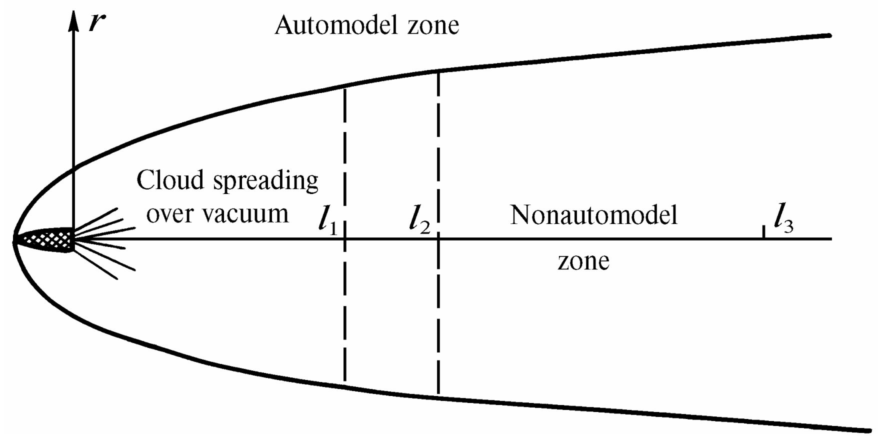

FIG. 1. Diagram of a model of rocket trace at the altitude.

1. The initial zone of the flame ($0 \lesssim z \lesssim l_1$) is described as spreading of a gas and dust cloud in vacuum. The presence of the ambient medium is negligible. In this zone the mass of the combustion gases is much greater than the mass of the disturbed air and the expression

$$\frac{Q}{\upsilon_{\rm roc}} \gg 2\pi \int_0^{r_f} \rho_\infty \, r \, dr$$

is valid. Here Q is the fuel consumption in a second, $\upsilon_{\rm roc}$ is the rocket speed, $r_{\rm f}$ is the coordinate perpendicular to the stream axis and corresponds to the position of the front of an expanding cloud and ρ_∞ is the density of a medium. The exhaust–stream pressure p_0 is many orders of magnitude higher than the pressure p of the ambient medium.

The inflowing stream of the ambient medium is partially scattered by the spreading exhaust gases and partially it diffuses into the jet flame. The head zone of

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the flame is formed. It is characterized by the relatively low concentration of charged particles which is close to its values in the ambient medium and by the high concentration of solid particles.

2. The zone of a strong shock wave ($l_1 \lesssim z \lesssim l_2$) is equivalent to the automodel stage of a point cylindrical blast. Beginning from this zone, the mass of the ionospheric gas involved into the motion is much more than the mass of the combustion gases, i.e.,

$$\frac{Q}{\upsilon_{roc}} \ll 2\pi \int\limits_0^{r_f} \rho_\infty \, r \, dr \, .$$

The power of the exhaust gases is still very high and that provides the great pressure differences at $p_0 \gg p_\infty$.

The maximum concentration of charged particles in condensation areas in this zone is 4–5 times higher than the similar value in the ambient medium.

3. The zone of a weak shock wave ($l_2 \lesssim z \lesssim l_3$) is described analogously to the nonautomodel stage of the point cylindrical blast. Attenuation of a shock wave occurs in this zone due to spreading and cooling. Thus the pressure of the ambient medium cannot be neglected. Here $p_0 > p_\infty$.

4. The zone of acoustic waves ($z > l_3$). The pressure in the flame decreases down to the values corresponding to the pressure of the medium $p_0 \sim p_\infty$. The speed of the particle motion is small in comparison with the sound speed. This zone is described by linear equations of acoustics.

THE INITIAL ZONE OF THE JET FLAME

Let us assume that in the reference coordinate system, whose motion coincides with that of the rocket the flame stream is the steady axisymmetric process, for whose description it is convenient to use the cylindrical coordinate system. At these assumptions the propagation of the stationary stream can be described by the system of differential equations in partial derivatives including the motion equations:

the equations of continuity

$$\upsilon_z \frac{\partial \upsilon_r}{\partial z} + \upsilon_r \frac{\partial \upsilon_r}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial \rho} \, , \tag{1}$$

$$\upsilon_z \frac{\partial \upsilon_z}{\partial z} + \upsilon_r \frac{\partial \upsilon_z}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial z} \, , \tag{2}$$

the equation of energy

$$\frac{\partial(\rho\,\upsilon_z)}{\partial z} + \frac{\partial}{\partial r}(\rho\,\upsilon_r) + \rho\frac{\upsilon_r}{z} = 0 \, , \tag{3}$$

and the equation of state

$$\upsilon_z \frac{\partial \varepsilon}{\partial z} + \upsilon_r \frac{\partial \varepsilon}{\partial r} = -p\left[\upsilon_z \frac{\partial}{\partial z}\left(\frac{1}{\rho}\right) + \upsilon_r \frac{\partial}{\partial r}\left(\frac{1}{\rho}\right)\right] \tag{4}$$

which relates the inner energy ε with the pressure p and the density ρ, $\varepsilon = \varepsilon(p, \rho)$. Here υ_z and υ_r are the longitudinal and radial flow rates in the stream. The boundary conditions at $z = 0$ and $0 \le r \le r_j$ are the following: $\upsilon_z = \upsilon_j \gg a^*$, $\upsilon_r = 0$, $\varepsilon = \varepsilon_j(r)$, and $\rho = \rho_j(r)$ (the values measured at the nozzle edge are marked by the subscript j and a^* is the local speed of sound in the stream). The mass of the gas per unit length is

$$M = 2\pi \int\limits_0^{r_j} \rho_j \, r \, dr = \frac{Q}{\upsilon_{roc}} \, .$$

Integration of this system presents great mathematic difficulties. However, the technique of the nonstationary analogy is appeared to be convenient for statement of principal physical features describing the structure of jet flames of ballistic rockets at altitudes. Really, for the rate of the stationary stream inflowing into vacuum we can derive the following estimates:

$$\Delta\upsilon_z/\upsilon \sim 1/M^2 \, ; \qquad \Delta\upsilon_r/\upsilon \sim 1/M \, , \tag{5}$$

where $M = \upsilon/a^*$ is the Mach number.

Equations (1), (3), and (4) can be solved for $M > 1$ with respect to υ_r and ρ, independently of motion equation (2) along the axis, and accurated to the terms of an order of $1/M^2$. If we drop the terms of an order of $1/M^2$ in Eqs. (1), (3), and (4) and replace the variable z by $\upsilon_{roc} t$ then (at $\upsilon_{roc} \approx \upsilon$) we get the problem in nonstationary spreading the cylindrical cloud over vacuum. This problem is considered in Refs. 2 and 3

$$\frac{\partial \upsilon_r}{\partial t} + \upsilon_r \frac{\partial \upsilon_r}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} \, ; \quad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}(\rho\,\upsilon_r) + \rho\frac{\upsilon_r}{r} = 0 \, ;$$

$$\frac{\partial \varepsilon}{\partial t} + p\frac{\partial}{\partial t} + \left(\frac{1}{\rho}\right) = 0 \, ; \quad \varepsilon = \varepsilon(p, \rho)$$

with the initial conditions

$$\left.\begin{aligned} \upsilon_r &= 0, \\ \rho &= \rho_j(r), \\ \varepsilon &= \varepsilon_j(r) \end{aligned}\right\} \text{ at } 0 \le r \le r_j \, , \; t = 0 \, .$$

The exhaust gas mass per unit length is $M_0 = Q/\upsilon_{roc}$.

It is possible to derive an exact analytical solution of the problem in spreading of the gas cloud over vacuum only under special boundary conditions which predetermine the spreading in the automodel regime. The numerical calculation is necessary for the other initial conditions. However, the solution with homogeneous initial distribution $p(r)$ and $\rho(r)$ and at $\upsilon_r = 0$ gives the satisfactory accuracy in estimating the structure of the initial stage of the rocket flame at altitudes. The unsteady regime of spreading of the cylindrical cloud over vacuum is described by the following expressions[3]:

$$\rho = \frac{M_0\,(2\alpha)!}{2\pi\, r\, a_0\, t \cdot 2^{2\alpha}(\alpha!)^2}\left[1 - \left(\frac{\gamma_0 - 1}{2}\right)^2 \frac{r^2}{a_0^2\, t^2}\right]^a , \tag{6}$$

where $\alpha = (3 - \gamma_0)/[2(\gamma_0 - 1)]$ ($\alpha = 0, 1, 2, 3, \ldots, \infty$), γ_0 is the index of the isoentropy degree of the spreading gases, and a_0 is the sound speed in the undisturbed medium.

The front boundary of the cloud is spreaded according to the linear law

$$r_f(t) = \frac{2}{\gamma_0 - 1}\, a_0\, t \, . \tag{7}$$

Returning from the nonstationary regime to the stationary one, we can derive that the density of the spreading combustion gases of propellant in the nearest zone of the nozzle is distributed according to the formula

$$\rho(r,\ z) = \frac{(2\alpha)!\ M_0\ \upsilon_{roc}}{2^{2\alpha}(\alpha!)^2\ \pi\ r\ z\ a_0}\left[1 - \left(\frac{\gamma_0 - 1}{2}\right)^2 \left(\frac{r\ \upsilon_{roc}}{z\ a_0}\right)^2\right]^a, \quad (8)$$

where a_0 is the sound speed in the flame at the nozzle edge.

The most characteristic value for the jet flames is $\gamma_0 = 11/9 \simeq 1.2$. Then Eq. (8) takes the form

$$\rho(r,\ z) = \frac{35}{32}\ \frac{Q}{\pi\ r\ z\ a_0}\left[1 - \left(\frac{r\ \upsilon_{roc}}{9\ z\ a_0}\right)^2\right]^4. \quad (9)$$

The boundary of the flame in the initial zone is described by the linear dependence

$$r_f(z) = [2/(\gamma_0 - 1)]\ (a_0\ /\ \upsilon_{roc})\ z\ , \quad (10)$$

and $r_f(z) = (9\ a_0\ /\ \upsilon_{roc})\ z$ at $\gamma_0 = 11/9$.

The spreading of combustion gases of propellant will occur in the regime of cloud spreading over vacuum until the ambient air mass per unit length involved into the motion is equal to the exhausted propellant mass per unit length. By equating the above–indicated masses, we can estimate a limiting length l_1 of the initial zone of a stream. For calculating the flow in this zone we can neglect the presence of the ambient medium. Let the stream be spreaded up to the radius r_{f_1} at the distance l_1 from the nozzle. The mass of the air inside the unit cylinder of radius r_{f_1} is equal to $\pi r^2 f_1 \rho_{\$}$, where $\rho_{\$}$ is the density of the ambient medium. The mass of the exhaust gases in the same volume is equal to Q/υ_{roc}.

By equating these masses we can derive the value of r_{f_1}

$$r_{f_1} = \sqrt{Q/(\pi\ \upsilon_{roc}\ \rho_\infty)}. \quad (11)$$

The stream is spreaded up to this a radius at the distance

$$l_1 = [(\gamma_0 - 1)/2\ a]\sqrt{Q\ \upsilon_{roc}/(\pi\ \rho_\infty)}\ . \quad (12)$$

In the ideal gas approximation Eq. (12) can be written in the form:

$$l_1 = (\gamma_0 - 1)/2\ \sqrt{(Q\ \upsilon_{roc} <m_0>)/(\pi\ \gamma_0\ \tau_0\ \rho_\infty)}\ .$$

Let us present the estimates in the length l_1 of the initial zone of a flame of ballistic rockets. It is characterized by the regime of spreading over vacuum and by the maximum transverse size of this zone is $2\,r_{1max}$. Evidently, a certain ("effective") value of $2\,r_{eff}(l_1)$ rather than a leading edge of spreading gases $2\,r_f$ should be considered as the maximum transverse size. This effective value is less than $2\,r_f(l_1)$ and corresponds to the densities ρ that much more than ρ_∞.

For estimates we can assume that the maximum transverse size of the jet flame agrees with the relative concentrations $\rho(l_1,\ r_{1eff})/\rho_\infty \sim 10^2$, and, therefore, $r_{1eff} = 0.246\ \sqrt{Q/(\upsilon_{roc}\ \rho_\infty)}$.

For example, the combustion gases of propellant of the Saturn–5 launcher, which flies at an altitude of 150 km and a speed of 3 km/s, are spreaded as though over vacuum up to the distance $l_1 = 2.3$ km from the nozzle. The effective width of the flame is 1.38 km. The size l_1 and r_{1eff} increases with altitude and rate of flight.

The value of l_1 equals 16.3 km at an altitude of 300 km and a flight rate of 5 km/s while the maximum transverse size of the initial zone of the stream reaches the values of $2\,r_{1eff} = 5.2$ km. One can obtain $l_1 = 0.74$ km and $r_{1eff} = 0.22$ km for the Titan–2 rocket at an altitude of 150 km and a flight rate of 3 km/s. The initial zone of the Minitmen–3 rocket flame is $l_1 = 0.67$ km in length and 0.4 km in width at an altitude of 150 km and a flight rate of 3 km/s and $l_1 = 8.8$ km, $2\,r_{1eff} = 2.8$ km at an altitude of 300 km and a flight rate of 6 km/s. Rockets with a less propulsive thrust have the dense head zone of the flame a few tens of meters in length.

The medium at the altitudes under consideration is very rarefied, and the free–path length of particles is tens or hundreds of meters. The concentration of the exhaust gases in the outlying zones of the spreading stream is also quite low. But closer to the stream axis its density reaches the values which are many orders of magnitude greater than the ionosphere density. Thus the spreading combustion gases of propellant are the obstacle for the inflowing air stream. The interaction mechanism can be characterized by the immersion and sticking of molecules of inflowing stream in the flame. The distribution of the total concentration depends on a lot of parameters including those which can be determined either only experimentally or numerically (for example, the density distribution function in a real jet flame, free–path length distribution of molecules inside the exhaust streams, etc.). The results of analytical studies based on various idealizations are very far from reality.

Experimental studies make it possible to assume that the area of mixing of the combustion gases with the ambient medium is simply and satisfactory described by the paraboloid equation

$$r_f^2 = \{(\kappa\ T_0\ Q\ )/[(\gamma_0 - 1)\ \upsilon_{roc}^3 <m_0>\ \rho_\infty)]\}\ z,$$

while for distribution of the particle concentration over the stream we can propose the approximation taking into account the conjugation with subsequent zone

$$N_{max}(z) = N_\infty \frac{1 - \gamma_\infty}{\gamma_\infty - 1 + 8\gamma_\infty\ z/\upsilon_{roc}\ t_0}\exp[-\ \delta(l_1 - z)/l_1)], \quad (13)$$

where for each special case the parameter is determined based on the experimental data

$$\delta = -\ln\left(\frac{\gamma_\infty - 1}{\gamma_\infty + 1}\ \frac{\rho_{max}(0)}{\rho_\infty}\right)$$

(κ is the Boltzmann constant, $<m_0>$ is the mean mass of the jet flame particles, and T_0 is the temperature near the nozzle).

THE ZONE OF A STRONG SHOCK WAVE

The exhaust stream becomes progressively wider along the downward flow, and progressively greater amount of ambient gases is involved into the motion. When the disturbed air mass per unit length becomes comparable with the exhaust combustion gas mass per unit length we cannot neglect the presence of the ambient medium. However the power of the exhaust streams is still quite significant in this area, and the stream pressure is much more than the pressure of the ambient medium. One can select the zone $l_1 \le z \le l_2$ in the jet flame which is characterized by the following inequalities:

$$\begin{cases} 2\pi \int\limits_0^{r_f} \rho_\infty \, r \, \mathrm{d}r \gg \dfrac{Q}{\upsilon_{\mathrm{roc}}} , \\ p_\infty \ll p_0 . \end{cases} \qquad (14)$$

For this relation between the parameters and hypersound flight rates ($M_\infty \gg 1$) there is an analogy between the stationary problem on the interaction of high–power jet streams with the ionospheric gas and the nonstationary problem on the cylindrical charge blast. The idea of the so–called shock analogy was used in the papers devoted to the studies of the ballistic wave being near the body moving in the gas medium at the supersonic speed.[4,5,7] The applicability of the shock analogy to the description of the exhaust at $l_1 \le z \le l_2$ is discussed above. For quantitative description of the analogy it is necessary obviously to equate the energy per unit length releasing during a cylindrical blast E_{bl} to the energy per unit length of the exhaust streams E_0, which is transferred to the ambient medium by these streams in spreading. In the ideal gas approximation

$$E_0 = \frac{p_0 \, Q}{(\gamma_0 - 1) \, \rho_0 \, \upsilon_{\mathrm{roc}}} = \frac{\kappa \, T_0 \, Q}{(\gamma_0 - 1) \, \upsilon_{\mathrm{roc}} <m_0>} .$$

Since the transverse size of the flame at $l_1 \le z \le l_2$ is quite large in comparison with the nozzle diameter the energy release can be assumed approximately as point and momentary. The shock analogy makes it possible to use the corresponding dependences of the point blast of the cylindrical charge for description of the distribution of pressure density and particle speed over the zone in which the disturbed air mass per unit length is much more than the mass of the flame gases. To do this we should replace the variable t by $z/\upsilon_{\mathrm{roc}}$ in the solutions of a nonstationary problem.

The problem on the point blast was elaborated by L.I. Sedov. He derived the exact analytical solution[6] for the automodel stage of the blast. Nonautomodel stage was numerically studied by Korobeinikov with colleagues.[3] The pressure of the combustion gases at $l_1 \le z \le l_2$ is much more than the pressure in the ambient medium and the latter can be ignored. The gas motion occurs in the automodel regime, and the relevant system of gas dynamics equations is exactly solved by the similarity technique. The results are the following.

The automodel zone of the trace is a layer of particle condensation whose depth is an order of several free–path lengths of molecules in the undisturbed ionosphere (at the corresponding altitude). The equation of a leading edge of the trace at the automodel stage has the form (in the accompanied cylindrical coordinate system)

$$r_f = \sqrt[4]{\frac{\kappa \, T_0 \, Q}{(\gamma_0 - 1) <m_0> \upsilon_{\mathrm{roc}}^3 \, \rho_\infty}} \sqrt{z} . \qquad (15)$$

The gas density in this layer of particle condensation is almost independent of coordinates and equal to $\rho_f \simeq \rho_\infty(\gamma_\infty + 1)/(\gamma_\infty - 1)$.

The density distribution behind the shock wave front cannot be derived analytically even at the automodel stage. It is calculated numerically (Fig. 2).

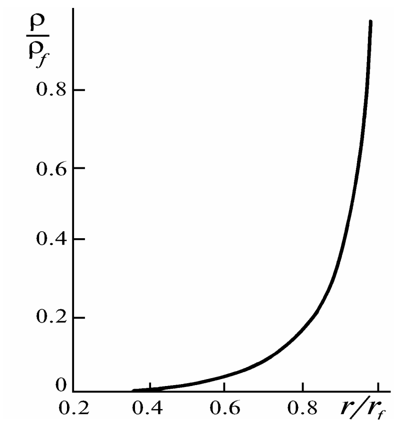

FIG. 2. Density distribution of combustion gases of propellant behind the shock wave front.

The variation in pressure of the trace front downward the flow is determined by the formula

$$p_f = \left(\upsilon_{\mathrm{roc}} \sqrt{E_0 \, \rho_\infty}\right) / \left[2(\gamma_\infty + 1) \, z\right] .$$

The length l_2 of the automodel zone of the trace depends to a certain extent on the requirements for the description accuracy. The automodel solution is based on the assumption $p_\infty = 0$. Therefore the value of the ratio p_∞/p_f is the criterion for application of the similarity technique. For example, when $(\rho_\infty/\rho_f) \sim 10^{-2}$ the error in results is 5% while when $(p_\infty/p_f) \sim 10^{-1}$ the error is as high as 40%.

If we assume the accuracy $p_\infty/p_f \sim 1/\nu$, then the length of the automodel zone is equal to $l_2 - l_1$, where

$$l_2 = \frac{<m_\infty>}{2 \, T_\infty \, (\gamma_\infty + 1) \, \nu} \sqrt{\frac{T_0 \, Q \, \upsilon_{\mathrm{roc}}}{\mathrm{k} \, \rho_\infty <m_0> (\gamma_0 - 1)}} \qquad (16)$$

(angular brackets denote averaging).

For example, at $\nu = 10$, at the altitude 150 km, and flight rate 3 km/s the automodel zone l_2 is 8.1 km in length for the Saturn–5 and Saturn–2, 2.6 km for Minitmen–3, and 2 km for Polaris rockets.

This zone increases up to 29 km for the Saturn–5 and up to 15.6 km for Minitmen–3 rockets at an altitude of 300 km and a flight rate of 6 km/s, while at $\nu = 100$ it decreases essentially.

LINEAR ZONE

In the course of further spreading of combustion gases of propellant (the zone $z > l_2$) the difference in the pressures of the stream and ambient medium decreases, and the shock wave is attenuated. We cannot neglect the counter pressure p_∞. However the energy of the exhaust gases is still high. It is also possible to apply the shock analogy at this stage of the disturbance caused by the rocket. But the problem of the blast with regard to the counterpressure is not automodel and cannot be solved analytically. Therefore it is necessary to use the numerical results for description of this zone of the rocket trace (they are given in dimensionless variables in Ref. 3). For description of the trace these data are transformed by the formulas

$$z = \upsilon_{roc}\, t_0\, \tau\ ;\quad r_f = r_0\, R_f\ ,$$

where τ and R_f are the dimensionless variables[2]

$$r_0 = \sqrt{\frac{T_0\, Q <m_\infty>}{(\gamma_0 - 1)<m_0>\, \upsilon_{roc}\, \rho_\infty\, T_\infty}}\ , \qquad (17)$$

$$t_0 = \frac{<m_\infty>}{T_\infty} \sqrt{\frac{T_0\, Q}{(\gamma_0 - 1)\, \kappa <m_0>\, \upsilon_{roc}\, \rho_\infty}}\ .$$

The form of the front surface and other trace characteristics are obtained based on the numerical results of the problem of the point blast of the cylindrical charge at the nonautomodel stage.

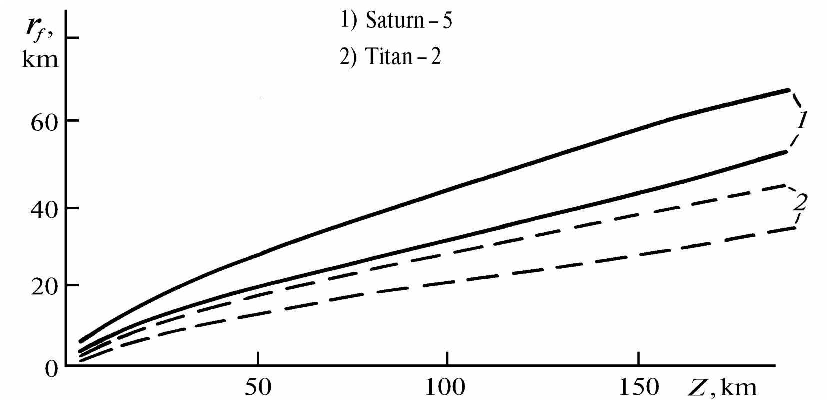

FIG. 3. Characteristic size of the jet flame $r_f(z)$ in a segment of the trace typical of the nonautomodel regime at υ_{roc} = 3 (solid line) and 5 km/s (dashed line).

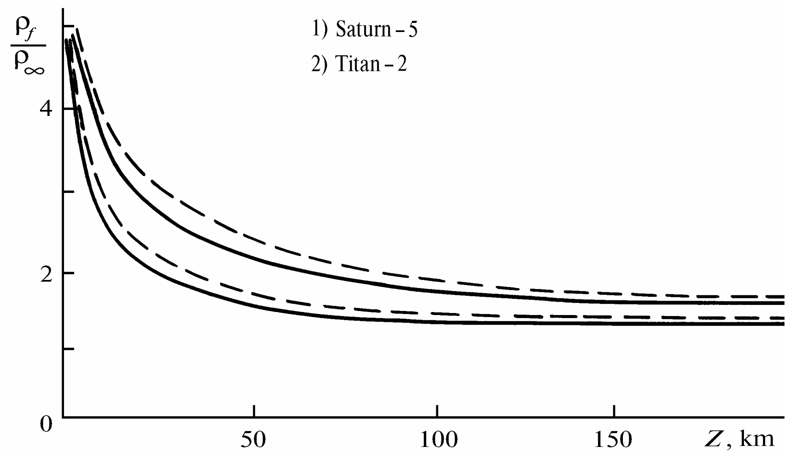

FIG. 4. Densities of particle condensation along the rocket flame axis at an altitude of 150 km at υ_{roc} = 3 (solid line) and 5 km/s (dashed line).

The dependences $r_f(z)$ in a segment of the trace typical of nonautomodel regime are given in Fig. 3 for some kinds of foreign ballistic rockets at an altitude of 150 km and different flight rates.

It is evident that the gas dynamics parameters can be calculated on the basis of the shock analogy practically along the entire length of the trace. The nonautomodel zone is the most extended in comparison with the automodel zone and area of spreading over vacuum. Figure 4 shows the distribution of particle condensation density (including the aerosol) at the maximum of particle condensation along the flame axis of the ballistic rockets of the Saturn and Titan types. The density distribution inside the trace (over radius) is shown in Fig. 5 (the numerical results are for two values of the parameter $q = a_\infty^2/[\mathrm{d}r_f/\mathrm{d}t)]^2$.

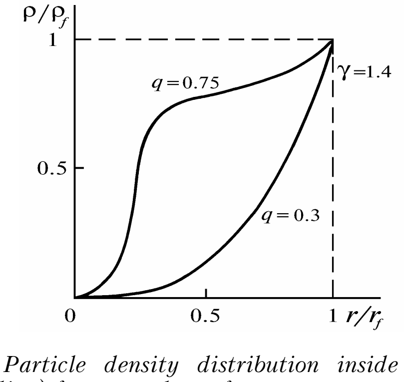

FIG. 5. Particle density distribution inside the trace (over radius) for two values of q.

When the shock–wave velocity is still high ($q \ll 1$), the particle density ρ decreases very quickly from the front to the trace center. In the course of the shock–wave attenuation, its velocity approaches to the sonic speed in the medium a_∞ and $q \to 1$, while the front is strongly "smeared out". It is appropriate already to the next zone, i.e., the zone described by laws of linear acoustics. The asymptotic analytical expressions for the trace parameters in the nonautomodel zone are of great interest for the applications. The equation of the trace surface (front) in the nonautomodel zone (according to the shock analogy technique) has the form

$$\begin{cases} l_1 + \dfrac{\upsilon_{roc} t_0}{u \upsilon_{roc} \sqrt{\alpha}} \left[\sqrt{\psi} - \sqrt{\psi_1} - \ln \dfrac{1+\sqrt{\psi}}{1+\sqrt{\psi_1}} \right] & \text{at } l_1 \le z \le 1.2 \upsilon_{roc} t_0, \\ \dfrac{\upsilon_{roc} t_0}{u} + M_{roc} r_f - \Gamma r_f^{1/4} & \text{at } z \gg 1.2 \upsilon_{roc} t_0. \end{cases} \tag{18}$$

Here the following designations are used:

$$\psi = 1 + 16\,\alpha\,\gamma_\infty \frac{r_f^2}{r_0^2};$$

$$\psi_1 = 1 + 16\,\alpha\,\gamma_\infty \frac{l_1}{\upsilon_{roc} t_0} = 1 + 16\,\alpha\,(\gamma_0 - 1)\sqrt{\frac{\gamma_\infty(\gamma_\infty - 1)}{2\pi}\,\frac{T_\infty <m_0>}{T_0 <m_\infty>}};$$

$$M_{roc} = \frac{\upsilon_{roc}}{a_\infty};\quad \alpha_\infty = \sqrt{\gamma_\infty \frac{p_\infty}{\rho_\infty}};\quad l_3 = 1.2\,\upsilon_{roc}\,t_0;\quad \Gamma = \frac{\upsilon_{roc}\,t_0}{\gamma_\infty \sqrt[4]{2\alpha^2 r_0}}.$$

The approximate dependences for the density distribution over the trace front

$$\frac{\rho_f}{\rho_\infty} = \begin{cases} \left[1 - \dfrac{4}{(1+\gamma_\infty)(1+\sqrt{\psi})} \right]^{-1} & \text{at } l_1 \le z \le l_3, \\ \left[1 - \dfrac{4}{(1+\gamma_\infty)(1+\sqrt{\chi})} \right]^{-1} & \text{at } z > l_3, \end{cases} \tag{19}$$

where $\chi = 1 + 16\sqrt{2}\alpha\,\gamma_\infty (r_f / r_0)^{3/2}$.

The maximum excess pressure changes along the trace according to the law

$$\frac{\rho_f}{p_\infty} = \begin{cases} 1 + \dfrac{u\,\gamma_\infty}{(1+\gamma_\infty)(\sqrt{\psi} - 1)} & \text{at } l_1 \le z \le l_3, \\ 1 + \dfrac{u\,\gamma_\infty}{(1+\gamma_\infty)(\sqrt{\chi} - 1)} & \text{at } z > l_3. \end{cases} \tag{20}$$

Relations (18)–(20) make it possible to calculate all the necessary parameters in the $z \ge l_2$ trace zone, if the fuel consumption, trace speed, flight altitude, and ionosphere state are known. In spite of simplicity, the formulas have the sufficient accuracy which is in fact determined only by the conditions of applicability of the shock analogy technique.

CONCLUSION

The total gas dynamics model of the rocket trace at the altitude along the active zone of the ballistic missile motion is described by Eqs. (9) and (13) in the zone of broadening of combustion gases as though over vacuum ($0 < z < l_1$), by Eqs. (15) and (17) in the zone of automodel stage ($l_1 \le z \le l_2$), and by Eqs. (18)–(20) in the entire zone $z > l_1$. The proposed model describes analytically the variations in the gas dynamics pressure and the particle density distribution over the trace in the zone of the maximum reconcentration.

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