Formulas for direct and scattered solar radiation fluxes in a clear sky atmosphere

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The analytic formulas are derived in the single scattering approximation for the incoming (to the Earth's surface) solar radiation fluxes. The paper describes the applicability limits of the formulas and possible fields of their application to studies of the clear sky atmosphere. The use of these formulas shows promise for modeling of processes of solar radiation transfer under conditions of small variations of the content of atmospheric aerosols of natural and anthropogenic origin.

Introduction

The solar radiation transfer in the Earth's atmosphere is one of the decisive factors forming the heat balance at different altitudes in the atmosphere, including the atmospheric boundary layer. The balance of incoming and outgoing solar radiation is described rigorously using the radiation transfer equations. This description allows for such physical processes of solar radiation transformation as absorption by atmospheric gases, thermal radiation, and the atmospheric glow as well as the scattering by atmospheric aerosol, clouds, and the underlying surface.[1]

The solution of the integro-differential equations for the solar radiation transfer through the atmosphere faces severe mathematical difficulties. Only in a very specific cases it is possible to obtain a solution in an analytic form. For practically important cases the solutions are obtained by numerical methods that have been being intensely developed in the last decades,[2] but they provide the possibility of making calculations also for limited variety of atmospheric optics situations. At such calculations it is very difficult to reveal physical regularities of the solar radiation transfer under actually occurring spatiotemporal variations of atmospheric conditions. Therefore the search for the approximate but adaptable to the variations of atmospheric conditions formulas for solar radiation fluxes remains urgent and especially important for solving the problems of global, regional, and local climate and ecological variations.[3]

A wide range of atmospheric optical situations in the Earth's atmosphere regarding its aerosol composition can be subdivided into three basic types of conditions, namely, the clear sky atmosphere, the overcast, and broken clouds. In this case the characteristic feature of a clear sky atmosphere is a relatively weak extinction of solar radiation due to scattering on atmospheric aerosols. Thus, there are good grounds to hope that the formulas obtained in the single scattering approximation will be very accurate. Below we describe the derivation of these formulas for the incoming solar radiation fluxes and the assessment of their applicability limits for the cloudless, clear sky atmosphere. The basis for a new approach is the successive calculation of not only extinction of the direct solar radiation power due to the molecular absorption and scattering by atmospheric aerosols, but also of the single scattered radiation and its extinction due to higher orders of scattering.

Basic considerations

The proposed approach to solution of the problems on solar radiation transfer through the Earth's atmosphere is an attempt to use the results of theoretical and experimental investigations on the radiation transfer through scattering media of optical radiation from different sources (point, collimated, narrow laser beams, and so on) by the methods of physical modeling. These results were generalized in a series of monographs[4–6] and here these results are used for analyzing the incoming solar radiation fluxes propagating through the atmosphere. The principal physical result of the above-mentioned investigations lies in the fact that the optical radiation transfer through scattering media can be considered and described successively:

1) for the direct radiation flux attenuated in a scattered medium following the exponential Bouguer law;

2) for the single scattered radiation flux with the partial account of double scattered radiation, i.e., in the approximation, which is called as single scattering approximation, or as Dr. Ishimaru[7] calls it the first approximation of multiple scattering;

3) for the multiply scattered radiation flux (with the scattering of the second and higher orders of scattering), for which the asymptotic formulas of the so-called "in-depth regime"[8] are derived from the radiation transfer equation, at large optical depths.

Figure 1 shows some results of the experimental investigations of attenuation of each of the above-mentioned optical radiation fluxes under laboratory conditions using a specially developed measurement procedure.[9] Set out on a semilogarithmic scale in Fig. 1

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is the direct radiation flux (curve *1*) denoted by the straight line within the entire range of optical depths in accordance with the Bouguer law. Dashed curve *2* with the maximum at $\tau = 1$ describes the dependence of the measured single scattered radiation flux on the optical depth and corresponds to the formulas in the single scattering approximation. Dashed curve *3* describes the dependence of the measured multiple scattered radiation flux on the optical depth. The continuation of curves *1$3* is extrapolated by dots according to the behavior of curves that ought to be in the absence of background noise in the measurements.

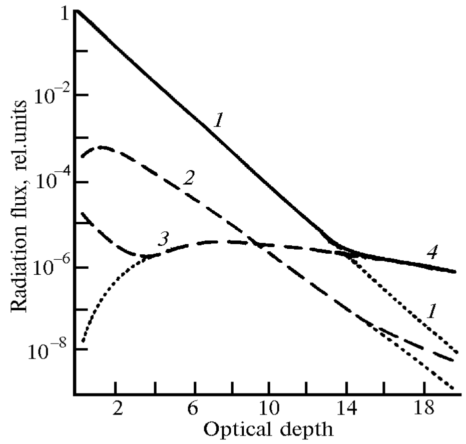

Fig. 1. The dependence of the fluxes of direct (*1*), single scattered (*2*), and multiple scattered (*3*) radiation on the optical depth.

Figure 1 shows that under these particular experimental conditions (a beam of 4-mm diameter and a wavelength of 0.63 μm, the solution of milk in water with the extinction coefficient 0.85 cm^{-1}) only at optical depths $\tau \geq 15$ the multiple scattered radiation flux exceeds the direct radiation flux and the deviation of attenuation of the net flux from the exponential one (following the Bouguer law). In this case the measured flux of single scattered radiation (curve *2*) is found to be much below the solid curve *4* for the measured net radiation flux in the entire range of the optical depths studied. Such a ratio between the measured flux magnitudes of direct and scattered radiation is the result of these particular experimental conditions. However the separation of the fluxes of direct radiation, single scattered, and multiple scattered radiation in the general description of the optical radiation transfer through scattering media is physically substantiated, and the fundamental dependence of all the three radiation fluxes on the optical depth is a relatively universal and is independent of the conditions of radiation transfer including the conditions of solar radiation transfer through the Earth's atmosphere.

Formulas in the single scattering approximation

Now have look at the scheme of incoming solar radiation flux at the sun zenith angle θ to a horizontal unit area *S* (Fig. 2).

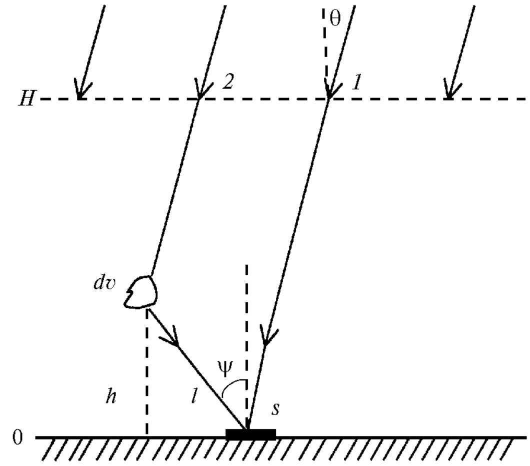

Fig. 2. The scheme for calculation of incoming solar radiation fluxes.

To calculate the incoming flux of direct solar radiation to the area *S*, we should follow the beam path *1* in Fig. 2. Here and below we consider only the case of a weakly selective attenuation due to aerosol attenuation and due to possible continuous absorption by atmospheric gases. In this case we use Bouguer law. Then the irradiance E_d of this area *S*, produced by an attenuated (by the atmospheric layer of the depth *H*) solar radiation flux, is determined by formula

$$E_d(\theta) = E_*(\theta)\ e^{-\tau m(\theta)}, \tag{1}$$

where $E_*(\theta)$ is the irradiance produced by the direct solar radiation flux on the upper atmospheric boundary (at the height *H*); τ is the integral optical depth of the atmospheric column (from 0 to *H*). In this case the optical depth $\tau(h)$ of the atmospheric elementary layer d*h* is determined by the product $k(h)\mathrm{d}h$, where $k(h)$ is the extinction coefficient at the altitude *h*. The function $m(\theta)$ is identified as the Bemporad function and, if neglecting the atmospheric sphericity, we have that $m(\theta) = 1/\cos\theta$.[5]

To calculate the incoming to the area *S* flux of single scattered solar radiation, it is sufficient to follow the beam path *2* in Fig. 1. According to the single scattering theory[5] the radiant emittance of a volume element d*v* equals

$$\mathrm{d}I_{\mathrm{d}v} = \frac{k_s(h)}{4\pi} f(\theta + \psi, h)\ E_*(\theta)\ e^{-k(h)(H-h)m(\theta)}\ \mathrm{d}v, \tag{2}$$

where $k_s(h)$ is the scattering coefficient at the height *h*; $f(\theta + \psi, h)$ is the scattering phase function at the height *h*, and

$$\mathrm{d}v = 2\pi\, l^2 \sin\psi\, \mathrm{d}\psi\, \mathrm{d}l. \tag{3}$$

The radiant emittance of a volume element dv causes a supplementary irradiance dE_s of the area S, which equals

$$\mathrm{d}E_\mathrm{s} = \frac{\mathrm{d}I_{\mathrm{d}v}}{l^2}\, \mathrm{e}^{-k(h)l} \cos\psi, \tag{4}$$

where $l = h/\cos\psi$, and h and ψ have been explained in Fig. 1.

To calculate the net flux of single scattered radiation, incident on the selected area, it is necessary to integrate Eq. (4) over two independent variables h and ψ of the volume dv. At the same time the integration over azimuth angles related to the selected area S was made in Eq. (2) with the account of the radiation flux from these angles. Having substituted Eq. (2) into Eq. (4) and making some evident transformations we obtain for the net irradiance produced by the single scattered radiation:

$$E_\mathrm{s} = \frac{E_*(\theta)}{2} \int\limits_0^{\pi/2} \sin\psi\, \mathrm{d}\psi \int\limits_0^{H} k_\mathrm{s}(h)\, f(\theta + \psi, h) \times$$
$$\times\, \mathrm{e}^{-k(h)(H-h)m(\theta)}\, \mathrm{e}^{-k(h)h/\cos\psi}\, \mathrm{d}h. \tag{5}$$

The integration in Eq. (5) can be made assuming that in the considered atmospheric layer the scattering phase function $f(\theta + \psi)$ and the scattering coefficients k are independent of the height h. This assumption, being at the first sight unacceptable, is supported by the fact that the formula, derived upon the integration, can be readily extended to the case of a multilayer atmosphere with arbitrary values of these parameters in each layer. Taking into account this assumption, Eq. (5) upon the integration takes the form:

$$E_\mathrm{s} = \frac{E_*(\theta)}{2}\, k_\mathrm{s}\, \mathrm{e}^{-\tau m(\theta)} \int\limits_0^{\pi/2} f(\theta + \psi) \sin\psi\, \mathrm{d}\psi \times$$
$$\times \int\limits_0^{H} \mathrm{e}^{-khg(\theta,\psi)}\, \mathrm{d}h, \tag{6}$$

where $\tau = kH$, and the quantity $[\sec\psi \,\$\, m(\theta)]$, appeared in the exponential function, is expressed as $g(\theta, \psi)$.

In a clear sky atmosphere the fraction of scattered radiation is comparable with that of the direct solar radiation only at optical depths $\tau = 6 \,\$\, 7$,[10] while at small solar zenith angles θ the optical depth is less than unity. Therefore the exponential function derived upon integration over h in Eq. (6) can be expanded in a series and to a high accuracy we have restricted ourselves to two terms of a series $\mathrm{e}^{-x} = 1 \,\$\, x + \dots$. Then for the irradiance E_s we obtain

$$E_\mathrm{s} = E_*(\theta)\, \mathrm{e}^{-\tau m(\theta)}\, \tau_\mathrm{s}\, F(\theta), \tag{7}$$

where $F(\theta)$ denotes the result of integration of the scattering phase function, with the accompanying factors, over ψ in Eq. (6), and $\tau_\mathrm{s} = k_\mathrm{s}H = \Lambda kH = \Lambda\tau$, $\Lambda = k_\mathrm{s}/k$, $k = (k_\mathrm{s} + k_\mathrm{a})$; k_a is the absorption coefficient in the atmospheric layer studied (from 0 to H).

The final result for the scattered solar radiation flux in the single scattering approximation is

$$E_\mathrm{s}(\theta) = E_*(\theta)\Lambda F(\theta)\tau\, \mathrm{e}^{-\tau m(\theta)}, \tag{8}$$

and for the total solar radiation flux in the same approximation

$$E(\theta) = E_\mathrm{d}(\theta) + E_\mathrm{s}(\theta) = E_*\, \mathrm{e}^{-\tau_*}\, [1 + D(\theta)\, \tau_*], \tag{9}$$

where the slant optical depth $\tau_* = \tau m(\theta)$ differs from the vertical one τ by the factor $m(\theta)$ depending only on the position of the sun in the sky. At the same time the quantity $D(\theta) = \Lambda F(\theta)/m(\theta)$ depends not only on the direction of the direct solar radiation θ but also on the scattering properties of the atmospheric layer, i.e., on the extinction coefficient, the scattering phase function and the ratio k_s/k.

The formula (9) seems to be in close agreement with the formula derived previously[5,11] for the optical radiation extinction in scattering media for a point radiation source. Such an agreement is reasonable since the basic limitation of the corresponding formula from Refs. 5 and 11 was associated with the requirement of small size of the radiation cone coming from a source or angular aperture of the radiation detector. When deriving Eq. (9), the area S is assumed to be plane and the incoming solar radiation flux is confined within a narrow cone (no more than 0.5°). Therefore the result of calculations is not an unexpected one and confirms the general regularity of the optical radiation extinction in the single scattering approximation.

As an illustration, Fig. 3 shows the results of calculations of direct (solid curve) and scattered radiation fluxes in the single scattering approximation by Eq. (9). Assuming that a noticeable deviation from the Bouguer law, describing the direct radiation extinction, occurs when the fluxes of direct and scattered radiation are equal and $\tau = 6$ according to Ref. 10, then under these conditions the quantity D can easily be found from the equality of the fluxes and equals approximately 0.17.

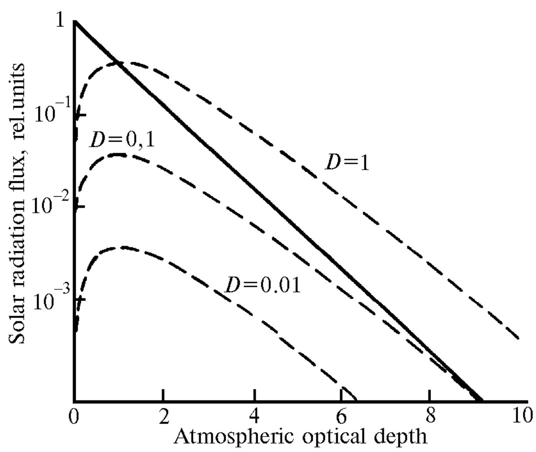

Fig. 3. The dependence of solar radiation fluxes on the slant optical depth of the atmosphere.

Generalization for the multilayer atmosphere

The formulas obtained above for solar radiation fluxes in the single scattering approximation enable a simple generalization for the atmosphere stratified into the aerosol layers. The generalization of this kind can be achieved when the flux of scattered radiation can be neglected as compared with the direct radiation flux in calculating the radiant emittance of a volume element dv in any aerosol layer. This assumption for clear sky atmosphere is well within the framework of the single scattering approximation. In this case the second and higher orders of scattering are considered as a supplementary factor of the radiation extinction along the beam path before and after single scattering by the volume element being studied.

If we consider in Fig. 2 the atmospheric layer from 0 to H as an overlying layer, the formula (9) is written for the irradiance of lower boundary of this layer in the designations with the subscript 1:

$$E_1(\theta) = E_*(\theta)e^{-\tau_1}[1 + D_1(\theta)\tau_1], \qquad (10)$$

where the slant optical depth τ_1 and the parameter $D_1(\theta)$ determine the optical characteristics of the overlying layer.

For the underlying layer we can use the same considerations as for the irradiance produced by the direct and scattered solar radiation fluxes at the lower boundary of the second layer and the formula is written as

$$E_2(\theta) = E_1(\theta)e^{-\tau_2}[1 + D_2(\theta)\tau_2], \qquad (11)$$

where τ_2 and $D_2(\theta)$ determine the optical characteristics of the underlying layer.

We continue the considerations and for the irradiance in the lower boundary of nth aerosol layer the formula is written as

$$E_n(\theta) = E_*(\theta)\ e^{-(\tau_1+\tau_2+\ldots\tau_n)} \prod_l^n (1 + D_i\ \tau_i), \qquad (12)$$

where the symbol $\prod_l^n$ in Eq. (12) denotes the product of the formulas being outside this symbol.

When it is considered that the fraction of scattered radiation in the clear sky atmosphere is small and equals some tenths of the direct radiation flux, the cross products of the type $D_i\tau_i \times D_j\tau_j$ in Eq. (12) are hundredths of the direct radiation flux. Therefore within the framework of the single scattering approximation Eq. (12) can be reduced to a simpler form

$$E_n(\theta) = E_*(\theta)\ e^{-\sum_n \tau_n}\ (1 + \sum_n D_n\ \tau_n), \qquad (13)$$

which makes it possible to perform effective computer simulation of the processes of solar radiation transfer in a multilayer atmosphere.

From formula (13) obtained in the single scattering approximation it follows that the role of aerosol layers is equivalent at different altitudes when forming the solar radiation fluxes. In contrast to the direct solar radiation flux (the first component in the formula), the incoming flux of single scattered solar radiation (the second component in the formula) is formed by each aerosol layer with its weighting factor determined by the quantity D_n. This quantity depends only on the optical characteristics of each aerosol layer and is independent of its height.

Assessment of the applicability limits

The use of the above formulas is very attractive since these formulas make it possible to construct simple models to take into account the effect of atmospheric aerosol of natural and anthropogenic origin on the radiation balance in the atmospheric boundary layer. Thus, a possibility is presented of modeling of one of the most dynamic weather forming component of the climatic system. At the same time the above-mentioned formulas are approximate and their applicability limits are determined by a series of assumptions adopted when deriving these formulas.

The fundamental restriction of the formulas obtained is associated with the basic assumption written in the paper title and forming a series of the considered atmospheric-optical situations as the “clear sky atmosphere.B This assumption means that the approximate formulas describe the solar radiation transfer under atmospheric conditions when the direct radiation flux exceeds the scattered radiation flux.

The so-called days of sunshine are related to such conditions. The number of these days, e.g., in Tomsk for one year exceeds 140.[12] For a comparison, the number of cloudy days in Tomsk is about 90. In other words, the atmospheric optical situations, included into calculation by the formulas in the single scattering approximation, are about 40% even in Tomsk, which is referred to the mid-latitude zone of Russia according to this criterion.

The quantitative estimate of the applicability limits of the above approximate formulas may be based on the consideration of a basic assumption made when deriving these formulas. This assumption means that in Eq. (6) the exponential function was substituted by two first terms of its series. If the fraction of scattered radiation flux is 30% of the direct radiation flux, then for the maximum admissible error we can obtain the values presented by the curves in Fig. 4. The dependence of the calculated error on the solar zenith angle in this figure is a result of the dependence of the slant atmospheric optical depth on the position of the sun at a fixed vertical profile of the optical depth τ (shown in Fig. 3).

From the estimates presented in Fig. 3 of the maximum errors it follows that the application of the approximate formulas is valid only at small values of the vertical optical depth (at $\tau \le 0.3$). According to the available statistical data on the atmospheric spectral transmittance,[5] these conditions are typical for the cloudless clear sky atmosphere in the spectral range $\lambda > 0.5$ μm. These applicability limits of the approximate formulas follow from the rigid estimations without the account of a series of additional circumstances (strongly forward-peaked scattering phase function, smaller fraction of scattered radiation in absorbing atmosphere, and so on). The account of these circumstances and more precise evaluation of the applicability limits of the formulas obtained are out of the scope of this paper and require additional research. It should be mentioned that in real atmosphere the applicability limits of the formulas of single scattering approximation will be extended. In particular, such theoretical estimates for the optical radiation transfer in scattering media from a point source resulted in the applicability limits of analogous formulas up to $\tau \approx 3$, and the experimental investigations in artificial fogs and hazes supported their applicability up to $\tau \approx 9$.[5]

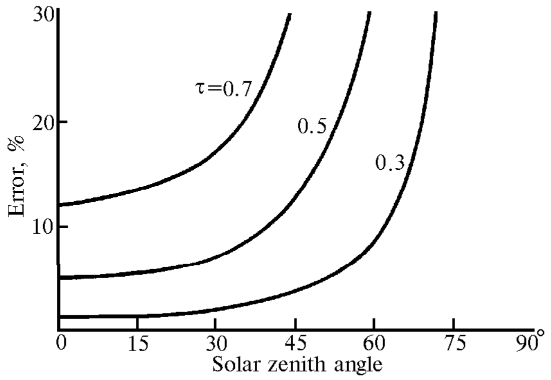

Fig. 4. The errors of calculations by the approximate formulas at different vertical optical depths τ of the atmosphere.

A supplementary argument in favor of a more extended applicability of the approximate formulas is connected with the primary goal of obtaining these formulas. Analytical form of these formulas is intended not only for calculations of the fraction of solar scattered radiation but also for modeling the variations of this fraction due to possible changes in the atmospheric aerosol content conditioned by natural and anthropogenic processes. To solve these problems, the applicability limits of approximate formulas may appear to be more wide.

Conclusion

The above formulas obtained in the single scattering approximation have a set of parameters with a simple physical meaning and their applicability limits are accessible for an experimental check. When investigating the atmospheric processes, including the processes of interaction of the atmosphere with other components of the environment, the above-mentioned conditions are of primary importance. In particular, a simple form of these formulas makes it possible to obtain the first derivatives with respect to the atmospheric optical depth that allows one to assess both the velocity of atmospheric radiation processes and the sensitivity of these processes to weak perturbations of natural and anthropogenic origin.

When deriving the approximate formulas, the scattering polarization effects were not considered but this approach can be used to solve this problem in the nearest future. Moreover, the necessary account of the beam spherical geometry in calculation of the sky polarization for a cloudless, clear sky atmosphere in the single scattering approximation may be useful for refinement of the above formulas and for extending their applicability limits.

The present work was performed after publication of the monograph,[13] which summarized the current state of the problems in studying the basic processes and the interaction of the atmosphere with environmental factors as well as presented the concepts of the atmospheric regional monitoring. Therefore the author has the right to refer a reader to the above-mentioned monograph or the paper[3] for detailed explanations of the problems and fields of application of the formulas obtained.

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