Energy and polarization characteristics of optical radiation scattered forward by a plane crystal

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Received November 8, 2005

The results are presented of numerical studies of characteristics (extinction cross section, extinction efficiency factor, polarization degree, elements of the scattering phase matrix) of optical radiation scattered forward by an ice plate. It is shown that in the case of a semitransparent oriented plate, different combinations of the quantities determining particle size, its spatial orientation, and refractive index cause variations of the extinction efficiency factor between 0 and 4. Practically monotonic dependence of the extinction in the IR range (at least within the wavelength range of several microns) and fast oscillating one in the visible range have been obtained. It is shown that at variations of microphysical and optical parameters of plates, the highest rate of the change of polarization characteristics of the forward scattered radiation is observed at the angle of a crystal tilt with respect to the sensing direction exceeding 30°.

Introduction

Crystal clouds essentially affect radiative processes in the atmosphere, as well as visibility of different objects through the atmospheric layer. In this connection, the study of the optical characteristics, extinction among them, is an urgent problem. Ice clouds consist of crystals of great variety of shapes and size. The results of investigations of forward scattering by small and large crystals with random orientation are to date widely presented in the literature.[1–4] Because the description of radiation passage through large semi-transparent crystals is too difficult, the problem of extinction of optical radiation by ice crystals still remains open. In particular, as researchers note,[4] no relevant data are practically available for infrared range.

During the study of extinction of light by large crystals, such characteristics as cross section and the extinction efficiency factor are mainly obtained. The total scattered field of some large particles is determined only by the diffraction field, and the cross section of extinction is equal to doubled area of the geometric shadow of a particle in the direction of propagation of radiation. It is possible that the refracted beams either have been attenuated inside the crystal, or have been essentially deviated from the forward direction after leaving the crystal. The extinction efficiency factor of oriented crystals with plane-parallel sides can differ from 2 (Ref. 5). For plane crystals it can change from 0 to 4. In the case of increase of the number of pairs of plane-parallel sides, the range of variations of this characteristics narrows.

Ice plates differ from other types of crystals by the extinction of incident radiation. The greatest variations of the extinction efficiency factor at the change of microphysical and orientation properties are possible for this type of crystals. However, determination of the extinction cross section only is insufficient for complete description of the polarization properties of radiation scattered in the forward direction. Not only energy, but also polarization characteristics of radiation passed through an oriented particle can differ from those of the incident radiation. As a rule, one considers the scattering phase matrix and extinction matrix for complete representation of these characteristics.

Characteristics of optical radiation passed through an oriented plate

The optical model proposed in Ref. 6 enables one to study the energy and polarization characteristics of the optical radiation passed through an oriented ice crystal. To do this, the relations were considered for the scattering cross sections σ_{fj} proportional to the respective parameters of the Stokes vector[7] I_{fj}:

$$\sigma_{fj} = \frac{4\pi r^2}{I_1} I_{fj}, \quad (1)$$

where I_1 is the intensity of electromagnetic field of the incident wave. The Stokes vector parameters of the scattered radiation I_{fj} are represented by the amplitudes of the scattered field

$$I_{f1} = |E_I|^2 + |E_{II}|^2, \quad I_{f2} = |E_I|^2 - |E_{II}|^2,$$
$$I_{f3} = 2\mathrm{Re}(E_I E_{II}), \quad I_{f4} = 2\mathrm{Im}(E_I E_{II}). \quad (2)$$

The diagram of propagation of radiation transformed after the interaction with particles is also presented in Ref. 6. A semitransparent plate of hexagonal shape is considered as an individual particle. A plane wave with an elliptic polarization is incident

0235-6880/06/04 244-05 $02.00

on it at an angle β to the plate axis (which is normal to the plate base). The expressions for the scattering cross sections σ_{fj} at any point of the forward hemisphere are obtained in the frameworks of the method of physical optics:

$$\sigma_{fj} = WM_{ij}\frac{I_i}{I_1},\ i = 1, 2, 3, 4;\ j = 1, 2, 3, 4, \quad (3)$$

where

$$W = \frac{k^2}{\pi}\frac{(1+\cos\vartheta)^2}{2};$$

I_i are the parameters of the Stokes vector of the incident radiation, the elements of the scattering phase matrix M_{ij} are the square functions of the elements of the amplitude matrix; $k = 2\pi/\lambda$ is the wave number, and λ is the wavelength of the incident radiation. The angle ϑ determines the deviation of the reception line from the direction, along which the beams escape from the plate. In this paper we study the characteristics of radiation scattered in the forward direction (i.e., at $\vartheta = 0°$).

The extinction matrix[2] (K_{ij}, $i = 1, 2, 3, 4$; $j = 1, 2, 3, 4$) is used in studying the energy and polarization characteristics of the forward scattering. Its elements are determined as linear combinations of the elements of amplitude matrix. The formulas determining the amplitudes of the field scattered by a plate crystal at any point of the forward hemisphere are presented in Ref. 6. The amplitudes of the forward scattered field are determined by use of coherent summing the diffracted and refracted fields taking into account the phase shifts. It is worthy to note that combinations of the elements of the first row of the extinction matrix define such characteristic as the extinction cross section[2]:

$$S_{ext} = K_{11} + K_{12}\frac{I_2}{I_1} + K_{13}\frac{I_3}{I_1} + K_{14}\frac{I_2}{I_1}. \quad (4)$$

The following formula for the cross section of extinction for polarized radiation is obtained[8] based on the optical theorem for an oriented semitransparent plate:

$$S_{ext} = 2S_{sq} - \mathrm{Re}(Q_{\|} + Q_{\perp}) - \frac{I_2}{I_1}\mathrm{Re}(Q_{\|} - Q_{\perp})\cos 2\gamma + \frac{I_3}{I_1}\mathrm{Re}(Q_{\|} - Q_{\perp})\sin 2\gamma, \quad (5)$$

where S_{sq} is the area of the geometric shadow in the direction of radiation propagation; the angle γ determines orientation of the polarization plane; $Q_{\|}$ and $Q_{\perp}$ are the amplitudes of the scattered field related to the corresponding Fresnel coefficients.

To study the energy characteristics at $\vartheta = 0$, let us concentrate on analysis of the values of the extinction cross section S_{ext} and the extinction efficiency factor

$$\Theta = S_{ext}/S_{sq}. \quad (6)$$

To study the polarization properties, let us analyze the values of the degree of polarization

$$\mathrm{St} = \sqrt{I_{f2}^2 + I_{f3}^2 + I_{f4}^2}\,/\,I_{f1} \quad (7)$$

and the elements of the forward-scattering phase matrix normalized to the first element:

$$N_{ij} = M_{ij}/M_{11}. \quad (8)$$

Discussion of the numerical results

Let us numerically study, using the model of an individual oriented semitransparent plate, the energy (S_{ext}, Θ) and polarization (N_{ij}, St) characteristics of the forward scattered radiation, which are interesting for the study of crystal clouds by optical methods. To do this, let us use the aforementioned formulas (2)–(8). When simulating the process of scattering by a crystal, the absolute coordinate system was introduced, within which the positions of the source, the receiver, and the particle are related by the pair of angles φ_i, θ_i ($i = 1, 2, 3$) of the corresponding spherical coordinate system.[6] It is normally accepted that rotation of a body in space with respect to the rectangular coordinate system is described by use of Euler matrix. The Euler angles α, β, and γ are determined by the combination of the values φ_1, θ_1 and φ_3, θ_3. Physical meaning of the angles β and γ is determined above. The angle α sets the position of a hexagonal plate at rotation about the normal to its base. Obviously, for the hexagonal plate, all values depending on α have the period of 60°. Note that characteristics shown in all figures in this paper are the averages over the angle α.

The dependences of the extinction efficiency factor Θ on the plate thickness d are shown in Fig. 1 for different positions of the particle with respect to the direction of sounding (the angle β) and different values of absorption of the ice plate χ.

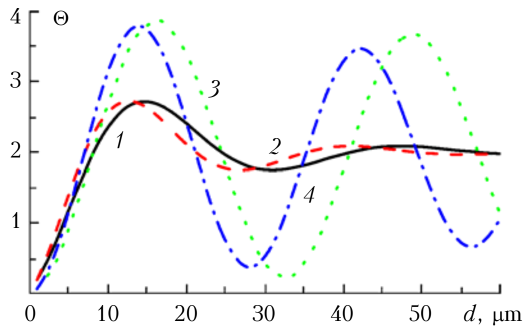

Fig. 1. The extinction efficiency factor as a function of the plate thickness at different values of χ and β: χ = 0.1, β = 20° (*1*); χ = 0.1, β = 40° (*2*); $\chi = 10^{-4}$, β = 20° (*3*); $\chi = 10^{-4}$, β = 40° (*4*); $n = 1.31$, $a = 125$ μm, $\lambda = 10.6$ μm, $\theta_3 = \beta$; $\theta_1 = 0°$, $\varphi_1 = 0°$, $\theta_2 = 0°$, $\varphi_2 = 0°$, $\varphi_3 = 0°$, $I_2/I_1 = 1$, $I_4 = I_3 = 0$.

The plate has the radius a and the complex refractive index $\tilde{n} = n + i\chi$. It is seen from Fig. 1 that the extinction efficiency factor, at weak absorption

($\chi = 10^{-4}$), can change from 0 to 4 depending on the plate thickness d. Obviously, in increasing χ or d, the extinction efficiency factor tends to its asymptotic value equal to 2. Stronger absorption and/or increased thickness of the crystal increase the energy losses of radiation at its propagation in the crystal, due to transformation into heat. In this case the forward scattered field is determined to greater extent by the diffraction field, and the contribution of the refraction field decreases.

The dependences of the factor Θ on the orientation of particle β at different thickness d and different absorption indices χ are shown in Fig. 2. It is seen that, in increasing the angle of incidence β, the phase and the amplitude of the curve $\Theta(\beta)$ change. The frequency of $\Theta(\beta)$ variations increases at the increase of the angle β and the thickness d. The greater is χ, the closer is the extinction efficiency factor to its asymptotic value. In forming the total forward scattered field, the addition of the refraction field to the diffraction field becomes less essential with the increase of the angle β, particle thickness d, or the absorption index χ. It is worth noting that similar dependences are also observed for the extinction cross section, however, the line representing its asymptote and the x axis make some angle (but not parallel to it, as in the case with the extinction efficiency factor). In other words, as β increases, the mean value of extinction for different plate thickness decreases.

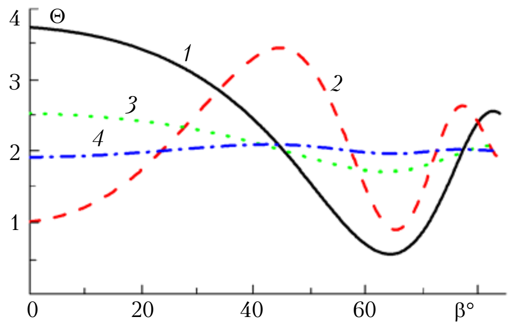

Fig. 2. The extinction efficiency factor as a function of the orientation of the plate at different χ and d: $\chi = 10^{-4}$, $d = 20$ μm (*1*); $\chi = 10^{-4}$, $d = 40$ μm (*2*); $\chi = 0.1$, $d = 20$ μm (*3*); $\chi = 0.1$, $d = 40$ μm (*4*); $n = 1.31$, $a = 125$ μm, $\lambda = 10.6$ μm, $\theta_3 = \beta$, $\theta_1 = 0°$, $\varphi_1 = 0°$, $\theta_2 = 0°$, $\varphi_2 = 0°$, $\varphi_3 = 0°$, $I_2/I_1 = 1$, $I_4 = I_3 = 0$.

The values of the extinction cross section S_{ext} and the extinction efficiency factor Θ, respectively, are shown in Fig. 3 as functions of the plate radius a at its different orientation β, but at a constant thickness ($d = 20$ μm).

It is seen from Fig. 3*a*, that the value of extinction is higher for the particles of larger radius and at smaller angles β. The variation can be of several orders of magnitude. The extinction efficiency factor as a function of the plate radius has relatively stable values. The values of the factor $\Theta(a)$ (Fig. 3*b*) can be considered as constant at $a > 150$ μm. The variation of extinction is better pronounced for the particles of the size $a < 150$ μm. For such values of a, the greater is β, the lower is the rate of the change of S_{ext}, while the rate of the Θ change is higher.

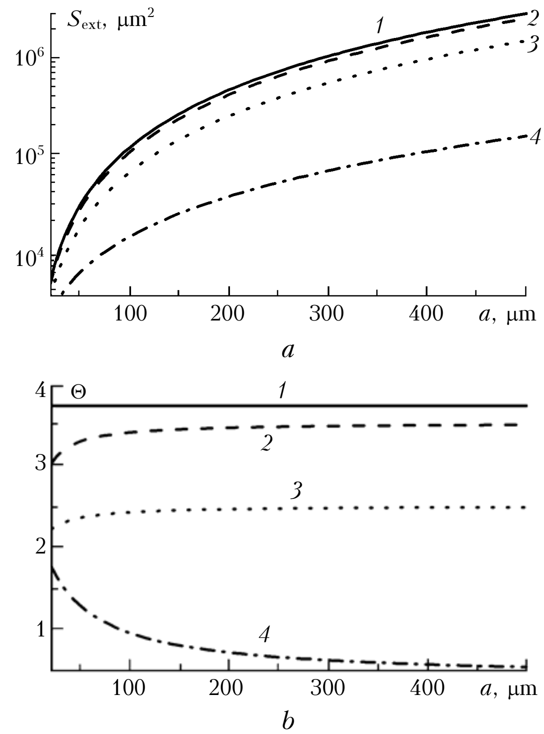

Fig. 3. The extinction cross section as function of the plate radius $S_{ext}(a)$ (*a*) and $\Theta(a)$ (*b*) at different β: $\beta = 0°$ (*1*); 20° (*2*); 40° (*3*); $\beta = 70°$ (*4*); $\tilde{n} = 1.31 + i \cdot 10^{-4}$, $d = 20$ μm, $\lambda = 10.6$ μm, $\theta_3 = \beta$, $\theta_1 = 0°$, $\varphi_1 = 0°$, $\theta_2 = 0°$, $\varphi_2 = 0°$, $\varphi_3 = 0°$, $I_2/I_1 = 1$, $I_4 = I_3 = 0$.

The spectral dependences of the extinction efficiency factor at different β are shown in Fig. 4. To calculate the spectral behavior of the light scattering characteristics as functions of the wavelength, let us use the indices $n = n(\lambda)$ and $\chi = \chi(\lambda)$ constructed using the data presented in Ref. 3. It is seen from Fig. 4*a* that $\Theta(\lambda)$ in the visible and near IR range is a rapidly oscillating function.

Figure 4*b* illustrates the variation of Θ in the IR wavelength range. The behavior of the curve $\Theta(\lambda)$ depends on the angle of incidence, which, in its turn, provides for certain position of minima and maxima. It is seen from Fig. 4*b* that monotonic dependence $\Theta(\lambda)$ is observed at least in the intervals of several micrometers.

In is also seen from Fig. 4 that the extinction efficiency factor in visible and IR range change from 0 to 4. Analyzing the characteristics of the refraction field, one should take into account that the number of beams of electromagnetic radiation in the visible wavelength range within the cross section of the plate is some orders of magnitude greater than the number of beams in the IR range. Even at insignificant changes of the particle size, the phase shift characteristic of visible wavelength range changes some orders of magnitude faster than in the IR range. Therefore, in passing to the integral characteristics, in particular, to calculation of the extinction coefficient of an ensemble of particles, it is necessary to average the extinction cross section (or the extinction efficiency factor) of an individual crystal over the size.

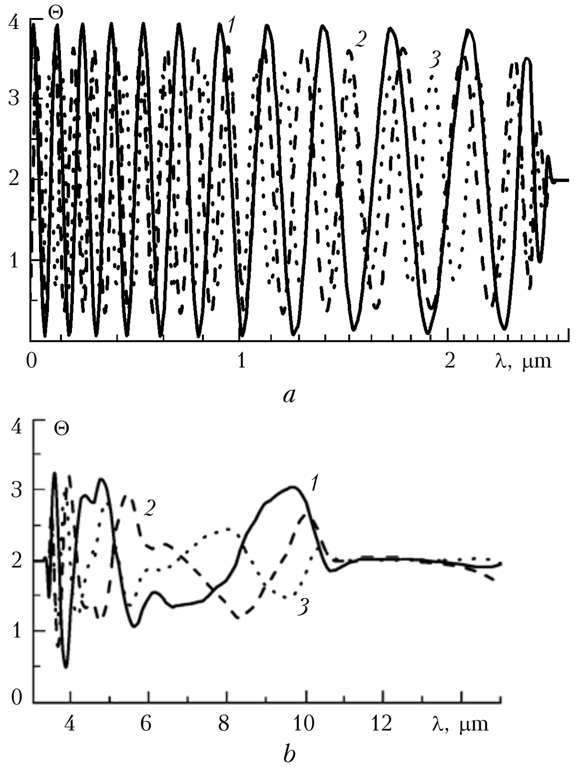

Fig. 4. The extinction efficiency factor as a function of wavelength $\Theta(\lambda)$ in the visible and near IR ranges (*a*) and in the IR range (*b*) at different β: β = 10° (*1*); 50° (*2*); 70° (*3*); d = 20 μm, λ = 10.6 μm, $\theta_3 = \beta$, $\theta_1 = 0°$, $\varphi_1 = 0°$, $\theta_2 = 0°$, $\varphi_2 = 0°$, $\varphi_3 = 0°$, $I_2/I_1 = 1$, $I_4 = I_3 = 0$.

Obviously, the high frequency of the extinction efficiency factor oscillation in the visible and IR wavelength ranges predetermines the neutral behavior of the extinction coefficient, while its practically monotonic dependence determines its spectral behavior in the IR range.

The dependences of S_{ext} on β at different λ are shown in Fig. 5. It is seen that the frequency of oscillations is caused, first of all, by the values λ. But, as the angle of incidence of radiation β increases, the frequency of the dependence $S_{ext}(\beta)$ increases, and its amplitude decreases.

The dependences of the degree of polarization St on orientation of the plate β at different thickness d and different values of the refractive index are shown in Fig. 6.

The degree of polarization was determined by Eq. (7) as a combination of the parameters of the Stokes vector. In its turn, the forward scattering phase matrix was used in calculating these parameters. It is seen from Fig. 6 that in the case of unpolarized radiation incident on the plate the radiation passed through the plate is partially polarized. The essential change of the values St occurs at β greater than 30°. The size and the refractive index of the crystal affect the behavior of the curve St(β). One can expect the greatest changes of the degree of polarization at smaller thickness and greater values of the refractive index (its real part, i.e., $n = \mathrm{Re}(\tilde{n})$).

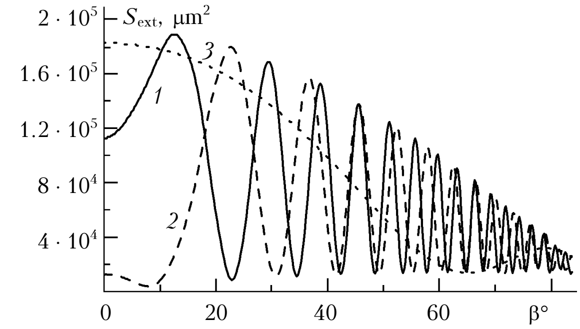

Fig. 5. The extinction cross section as a function of the orientation of the plate at different λ: λ = 0.55 μm (*1*); 0.694 μm (*2*); 10.6 μm (*3*); d = 40 μm; $\tilde{n} = 1.31 + i \cdot 10^{-4}$, a = 125 μm, $\theta_3 = \beta$, $\theta_1 = 0°$, $\varphi_1 = 0°$, $\theta_2 = 0°$, $\varphi_2 = 0°$, $\varphi_3 = 0°$, $I_2/I_1 = 1$, $I_4 = I_3 = 0$.

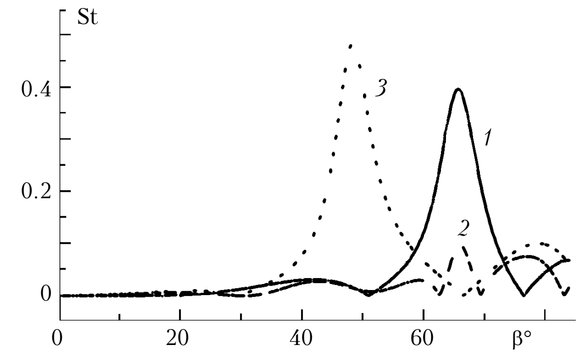

Fig. 6. The polarization degree as a function of orientation of the plate at different d and n: n = 1.31, d = 20 μm (*1*); n = 1.31, d = 40 μm (*2*); n = 1.41, d = 20 μm (*3*); a = 125 μm, $\chi = 10^{-4}$, λ = 10.6 μm, $\theta_3 = \beta$, $\theta_1 = 0°$, $\varphi_1 = 0°$, $\theta_2 = 0°$, $\varphi_2 = 0°$, $\varphi_3 = 0°$, $I_4 = I_3 = I_2 = 0$.

The most pronounced dependences of the elements of the forward scattering phase matrix, namely N_{34}, N_{43}, and N_{12} on β at different d values are shown in Fig. 7. Obviously, the normalized diagonal elements of the scattering phase matrix (8) of an individual plate are equal to 1, and, besides, $N_{12} = N_{21}$. It is seen from comparison of curves presented in Figs. 7*a* and *b* that the change of the particle size leads to a change in the values of the elements of the scattering phase matrix. These changes are better seen at large β angles. In the case of normal incidence of the wave on the plate (at β = 0), all elements of the matrix, except for the diagonal ones, are equal to 0.

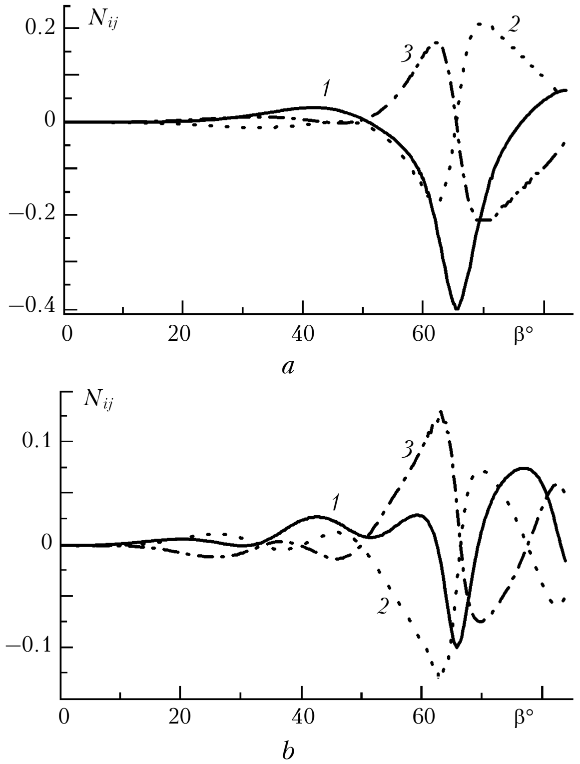

Fig. 7. The elements of the scattering phase matrix as functions of orientation of the plate of the thickness d = 20 μm (*a*) and d = 40 μm (*b*): N_{12} (*1*); N_{34} (*2*); N_{43} (*3*) at a = 125 μm, n = 1.31, $\chi = 10^{-4}$, λ = 10.6 μm, $\theta_3 = \beta$, $\theta_1 = 0°$, $\varphi_1 = 0°$, $\theta_2 = 0°$, $\varphi_2 = 0°$, $\varphi_3 = 0°$.

Conclusions

The solution of the multi-parameter problem of light scattering by an ensemble of particles is sought through solving the problem for the case of scattering by an individual particle. It is necessary to reveal the most essential relations between the scattering characteristics with the parameters of the crystal and the incident radiation.

The total forward scattered field for an oriented plate is determined as a coherent sum of the diffraction and refraction fields of comparable strengths. For an ice oriented plate, a combination of the values setting the particle size a, d, its refractive index $\tilde{n}$, and orientation angle β, as well as the wavelength of incident radiation λ sets the variability of its extinction efficiency factor from 0 to 4.

The boundaries of this interval can narrow at increasing absorption coefficient or the plate thickness. Increase of the angle of radiation incidence on the plate also narrows the boundaries of the variation of the extinction efficiency factor. Passing a longer distance in the semitransparent plate, the radiation loses more energy that transforms into heat.

Obviously, the addition of the refraction field to the diffraction one decreases in this case, that finally results in the fact that the value of the extinction efficiency factor approaches its asymptotic value equal to two.

In spite of the fact that the extinction efficiency factor can take the values from 0 to 4, its high-frequency dependence in the visible and near IR ranges causes the neutral behavior of the extinction of an ensemble of particles. The frequency of oscillations of the extinction efficiency factor in the IR range decreases many times that determines the spectral dependence of the extinction in this range.

The rate of variation of the polarization characteristics, in particular, the degree of polarization and the elements of the forward scattering phase matrix, increases with the increase of the plate tilt angle relative to the direction of the radiation incidence. Polarization characteristics at the angle β larger than 30° are most sensitive to the changes in microphysical and optical properties of the plates.

Acknowledgments

The work was supported in part by the Russian Foundation for Basic Research (grant No. 05–08–18150).

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