SIMPLE METHOD FOR DETERMINING CORRECTIONS TO THE SLANT RANGE

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A fast and simple method for determining range corrections was developed based on the model of a homogeneous atmosphere. The accuracy of the formulas derived for calculating the range corrections is evaluated and the limits of applicabi1ity of the formulas is studied.

It is well known that in order to make an accurate determination of the atmospheric correction ΔS to the measured range it is necessary to known the profile of the refractive index along the measuring path and a ΔS must be calculated by numerical integration. All this raises great obstacles in using accurate methods. In this connection the purpose of this work was to develop a simple but accurate method for determining ΔS using as little measured meteorological data as possible. Methods based on the use of theoretical models of the atmospheric partially meet these conditions. Among such methods the homogeneous-atmosphere model is of practical interest; in this model the integrals appearing in the exact formulas can be calculated analytically.

For a homogeneous atmosphere ΔS is calculated from the formula[1]:

$$\Delta S_e = \eta_0^g\left[\sqrt{(R_0 + H_e)^2 - A_1^2} - R_0\cos\xi\right] + \sqrt{(R_0 + H)^2 - A^2} - \sqrt{(R_0 + H_e)^2 - A^2} - \sqrt{(R_0 + H)^2 + R_0^2 - 2R_0(R_0 + H)\cos\theta} \tag{1}$$

where

$$\theta = \xi - \arcsin\frac{A_1}{R_0 + H_e} + \arcsin\frac{A}{R_0 + H} - \arcsin\frac{A}{R_0 + H} \tag{2}$$

H_e is the scale height of the atmosphere; $A = R_0 n_0 \sin\xi$, $A_1 = R_0 \sin\xi$; H and ξ are the height and zenith angle of the observed object; n_0 and n_0^g are the phase and group refractive index of air at the observation point; R_0 is the radius of curvature of the normal section of the earth's ellipsoid and. is given by

$$R_0 = \frac{a\sqrt{1-e^2}}{1 - e^2\sin^2\varphi}\left[1 - \frac{e^2}{2}\cos^2\varphi\cos 2A_0\right] \tag{3}$$

In .the formula (3) $a = 6378.245$ km is the long semiaxis of the earth's ellipsoid; $e^2 = 0.006693422$; φ is the latitude of the observation point; and, A_0 the geodesic azimuth of the direction of observation.

If in the exact formulas for calculating ΔS the refraction extension of the ray trajectory is neglected, then the following simple formula is obtained for a homogeneous atmosphere[1,3]:

$$\Delta S_e^* = (n_0^g - 1)\left[\sqrt{(R_0 + H_e)^2 - A_1^2} - R_0\cos\xi\right]. \tag{4}$$

The main parameter of the homogeneous atmosphere is its height H_e (Ref. 4):

$$H_e = \left[H_e^0 + \frac{k_2}{2k_1}(H_e^0)^2\right] / k_1, \tag{5}$$

where

$$H_e^0 = \frac{R_c T_0^v}{g_0}\left[1 - \frac{P(H)}{P_0}\right]. \tag{6}$$

In the formulas (5) and (6) P_0 and $P(H)$ are the pressure at the point of observation and at the altitude H; $R_e = 287.05$ m^2/(deg × s^2) is the specific gas constant of dry air; $K_1 = 1–0.0026 \cos2\varphi$; $K_2 = 3.14\times10^{-7}$m^{-1}; g_0 is the acceleration of gravity at the point of observation located at altitude H_0 (Ref. 5):

$$g_0 = g_c k_1 (1 - k_2 H_0); \tag{7}$$

$g_c = 9.80665$ m/sec^2 is the acceleration of gravity at sea level and latitude $\varphi = 45°$; T_0^v is the virtual temperature at the observation point and is related with the measured temperature T_0 by the relation[5]:

$$T_0^v = T_0(1 + 0.378\, e_0/P_0), \tag{8}$$

where e_0 is the partial vapor of water pressure.

Comparing the values of the ΔS_e with the exact values ΔS calculated from exact formulas using real profiles of the refractive index shows that the formula

(1) has an error δS. The formula (4) also contains a systematic error. The magnitude and behavior of this error is illustrated in Table 1, which gives the values of δS (mm) for two extreme states of the atmosphere corresponding to temperatures of –60°C (numerator) and +60°C (denominator) at the ground. The calculations showed that δS also depends on the wavelength.

TABLE 1.

The values of $\delta S = \Delta S - \Delta S_e$ (mm) for ground temperatures of –60°C (numerator) and +60°C (denominator). λ = 0.6943 μm.

ALTITUDE km	ZENITH ANGLE, deg						
	45	70	80	85	88	89	90
1	$\frac{-0.2}{0}$	$\frac{-0.3}{0}$	$\frac{0.2}{0.1}$	$\frac{5.5}{0.7}$	$\frac{18.5}{6.4}$	$\frac{*}{-117}$	$\frac{*}{*}$
5	$\frac{-3.0}{-0.3}$	$\frac{-6.9}{-1.2}$	$\frac{-17.9}{-5.2}$	$\frac{-64.2}{-29.0}$	$\frac{-282}{-170}$	$\frac{380}{-201}$	$\frac{29013}{1466}$
10	$\frac{-4.5}{-0.3}$	$\frac{-14.2}{-3.8}$	$\frac{-61.8}{-28.5}$	$\frac{-327}{-176}$	$\frac{-1628}{-880}$	$\frac{-1149}{-1267}$	$\frac{31773}{368}$
25	$\frac{-6.4}{-1.0}$	$\frac{-30.0}{-16.6}$	$\frac{-168}{-126}$	$\frac{-915}{-693}$	$\frac{-4024}{-2705}$	$\frac{-4283}{-3704}$	$\frac{32322}{-2107}$
100	$\frac{-6.9}{-1.3}$	$\frac{-35.5}{-21.7}$	$\frac{-200}{-156}$	$\frac{-1075}{-833}$	$\frac{-4658}{-3139}$	$\frac{-5128}{-4249}$	$\frac{35376}{-2461}$

Note: The asterisk * corresponds to the regions where the formula (1) is inapplicable.

Our investigations of the dependence of δS on H, ξ and the meteorological conditions enabled representing δS in terms of an empirical function which is the correction to ΔS_e

$$\delta S = \delta S_{100}\{1-\exp[-0.0027(H - H_0)^2]\}. \quad (9)$$

In the formula (9) δS_{100} is the value of δS at an altitude H = 100 km, calculated from the formula

$$\delta S_{100}=\exp\{(\alpha_1+\beta_1 T_0^{\nu}+\gamma_1 P_0)+(\alpha_2+\beta_2 T_0^{\nu}+\gamma_2 P_0)\times$$
$$\times\ \mathrm{tg}[\xi-(\alpha_3+\beta_3 T_0^{\nu}+\gamma_3 P_0)]\} \quad (10)$$

The coefficients α_3, β_3, and γ_3, which are found from the solution of the system of nonlinear equations (10), are as follows:

α_1 = 0.65329 β_1 = -0.009955 γ_1 = 0.001514
α_2 = 0.67008 β_2 = 0.007787 γ_2 = -0.000195
α_3 = 5.34512 β_3 = 0.035466 γ_3 = -0.000745

In using the formulas (9) and (10) it should be kept in mind that the numerical values of the coefficients α_3, β_3, and γ_3 correspond to ξ expressed in degrees and H and H_0 expressed in km; the correction ΔS will then be given in mm. It should also be kept in mind that the coefficients α, β, and γ were obtained for λ = 0.6943 μm. To calculate ΔS for other wavelengths δS must be multiplied by the coefficient K_λ given by

$$k_\lambda = N_0^g(\lambda) \,/\, N_0^g(0.6943), \quad (11)$$

where $N^g = (n^g-1)\times10^6$ is the group refractivity at the required wavelength and λ = 0.6943 μm. The value of N^g is calculated using Owens' formulas[6].

At low altitudes (H < 5 m) δS is small and random, so that it can be neglected in calculating ΔS. The value of ΔS is finally calculated by the proposed method using the formula

$$\Delta S = \begin{cases} \Delta S_e & H<5\ \mathrm{km} \\ \Delta S_e + k_\lambda \delta S & H\ge5\ \mathrm{km} \end{cases} \quad (12)$$

with ΔS_e is calculated from the formula (1). The formula (12) is valid at all altitudes and zenith angles with the exception of the case where the condition $R_0 n_0 \sin\xi/(R_0 + H_e) > 1$ holds. This condition corresponds to altitudes $H < R_0(n_0\sin\xi - 1)$, i.e., to the case of horizontal and slightly inclined paths.

The foregoing method for eliminating the systematic error ΔS in the formula (1) was also employed to improve the accuracy of the formula (4). The behavior and magnitude of the systematic error δS^* of the formula (4) differs somewhat from those of δS. For $\xi \le 87$ and $H \ge$ 8km δS^* is described well by formulas analogous to (9) and (10):

$$\delta S^* = \delta S^*_{100}\{-\exp[-0.0027\,(H-H_0)^2]\} \quad (13)$$

δS^*_{100} is calculated from the formula (10) using the following values of the coefficients α, β, and γ (λ = 0.6943 μm):

α_1 = 0.97747 β_1 = -0.011137 γ_1 = 0.001316
α_2 = 0.38951 β_2 = 0.008695 γ_2 = -0.000210
α_3 = 4.87994 β_3 = 0.035608 γ_3 = -0.000048

For $\lambda \ne$ 0.6943 μm the value of δS^* must also be multiplied by the coefficient K_λ given by the formula (11). For H < 8 km $\delta S^* \approx 0$ and the final formula for calculating ΔS (taking into account now the extension of the ray trajectory owing to refraction) will have the form

$$\Delta S = \begin{cases} \Delta S_e^* & H<8km \\ \Delta S_e^* + k_\lambda \delta S & H\ge8km \end{cases} \quad (14)$$

where ΔS_e^* is calculated from the formula (4). For $\xi > 87°$ the formula (14) should not be used owing to its large error.

In calculating ΔS from the formulas (12) or (14) for altitudes $H \le$ 60 km the air pressure $P(H)$ in Eq. (6) must be determined. The error in determining $P(H)$ can be easily determined starting from the formulas (1) or (4) and the admissible computational error $\sigma_{\Delta S}$. For example, differentiating, the formula (4) with respect to P gives the following formula for the error $\sigma_{\Delta S}$ owing to the error σ_p in the determination of the pressure:

$$\sigma_{\Delta S} = \frac{n_0^g - 1}{\left[1 - [A_1/(R_0+H_e)]^2\right]^{1/2}} \frac{R_c T_0^v}{g_0} \frac{c_p}{P_0} \qquad (15)$$

Equation (13) was derived under the assumption that $H_e = H_e^0$. This assumption has virtually no effect on the value of $\sigma_{\Delta S}$. The formula (4) also gives virtually the same values of $\sigma_{\Delta S}$.

Using (15) it is easy to find the error in determining the pressure at an altitude H, if $\sigma_{\Delta S}$ is given. Setting, for example, $\sigma_{\Delta S} = 1$ cm with $\xi = 85°$ we obtain $\sigma_p = 0.7$ mbar. To achieve the same accuracy at $\xi = 88°$ the error in the pressure must not exceed 0.2 mbar.

It is difficult to achieve this accuracy in determining $P(H)$ in the entire atmosphere, even by direct measurements. For this reason the simplest method for determining $P(H)$ is to use the average, multilayer pressure profiles for a specific region and season. The variance of the pressure for the average profiles is .quite significant: the value of σ_p equals on the average 3–5 mbar for the troposphere and about lmbar in the tropopause. Therefore the error of the formulas (12) and (14) $\sigma_{\Delta S}$, obtained using the average seasonal pressure profiles (for the example of the Balkhash site), is given in Table 2.

TABLE 2.

The magnitude of the additional error (mm) in determining ΔS using the average, seasonal pressure profiles.

H km	ZENITH ANGLE, deg										
	30	60	70	75	80	82	85	86	87	88	89
1-5	11	19	27	38	53	66	102	122	152	194	192
5-10	10	18	26	36	52	64	98	116	141	173	192
15	4.3	7.5	11	14	21	26	39	46	55	66	74
20	2.7	4.7	6.8	9.0	13	16	24	29	34	41	46
25	1.9	3.3	4.8	6.3	9.2	11	17	20	24	29	32
30	1.1	1.9	2.7	3.6	5.3	6.5	10	12	14	16	18
50	0.5	0.9	1.4	1.8	2.6	3.2	4.0	5.8	7.2	8.0	8.8
60	0	0	0	0	0	0.1	0.1	0.2	0.3	0.4	0.4

TABLE 3.

Root-mean-square error (in mm) in determining ΔS using the formulas (12) and (14).

ALTITUDE RANGE km	ZENITH ANGLE, deg								
	60	70	75	80	82	85	86	87	88
	FORMULA (12)								
$H<5$	0.6	0.9	1.5	2.6	4.9	16	20	50	153
$5 \le H<100$	0.6	0.9	1.6	4.1	8.9	33	70	136	471
$H \ge 100$	0.3	0.6	1.0	3.0	5.6	18	30	50	392
	FORMULA (14)								
$H<8$	0.5	0.7	1.1	1.9	5.8	26	38	107	$\sim 8\cdot 10^2$
$8 \le H<100$	0.5	0.6	1.6	3.90	10.9	59	70	138	~1000
$H \ge 100$	0.3	0.4	1.3	2.8	4.8	17	28	58	$\sim 9\cdot 10^2$

It is obvious that the maximum values of $\sigma_{\Delta S}$ occur in the troposphere; at altitudes $H > 15$ km $\sigma_{\Delta S}$ drops rapidly. For $H \ge 60$ km the effect of the pressure is already negligibly small.

On the whole the error in determining ΔS from the formulas (12) and (14) was determined by comparing the values obtained with the exact values of ΔS. For comparison 43 profiles of meteorological variables for ground temperatures ranging from –60 to +60°C, ground pressures ranging from 500 to 1100 mbar, and values of e_0 ranging from 0 to 50 mbar for wavelengths in the range 0.4 ... 10 μm were employed. The rms error, obtained in this manner, in determining ΔS is given in Table 3.

The additional error owing to errors in the method employed to determine the pressure $P(H)$ is not taken into account in Table 3. Its value, naturally will be different depending on the method employed to determine $P(H)$ and can be easily calculated using the formula (13), Table 2 gives a general idea of the magnitude and behavior of this error.

Comparing the accuracies of the formulas (12) and (14) shows that for zenith angles $\xi \le 86°$ the more convenient formula (12) can be used to calculate ΔS with the same accuracy, with formula (14) should be used only $\xi > 86°$. For $\xi \le 89°$ both formulas give a significant error (up to 10m) and they cannot be employed to calculate ΔS. For zenith angles $\xi \le 87°$ the accuracy of the formulas (12) and (14) is comparable to that of the more complicated methods[1], but they are simpler and more convenient for practical applications than the latter methods. The measured parameters of the atmosphere are the values of the meteorological parameters at the point of observation only. The accuracy of the formulas can be improved, if the profile $P(H)$ or the temperature profile can be measured in the region of the observations.

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