Application of wavelet transform to analysis of interannual fluctuations of solar activity and surface air temperature in Tomsk

I.I. Ippolitov, M.V. Kabanov, and S.V. Loginov

Institute of Optical Monitoring,
Siberian Branch of the Russian Academy of Sciences, Tomsk

Received February 23, 2001

Latent periodicity in the interannual fluctuations of temperature was analyzed using, as an example, the data on annually mean temperature in Tomsk for the period since 1881 until 1998. Wavelet and Fourier transforms were used for the analysis. The power spectrum $|W_k(s)|^2$ of observations of the surface air temperature revealed periodic structures of the scales of 2–3, 7–10, 12–25, and 30–50 years. The smallest scale of 2–3 year was assigned to the quasibiannual cycle. Wavelet analysis of the Wolf number for the period of 1700–1998 and critical frequencies of the $F2$ ionospheric layer for the period of 1936–1998 was carried out. The correlation analysis showed the presence of significant correlation only between the 22-year cycle of the Wolf number and temperature fluctuations on the scales of 12–25 years. The quasi-periodic structure on the scale of 30–50 years was noticed; in the 20th century, this structure tends to a smaller scale. The time interval between 1940 and 1960, during which the solar activity increased markedly, was revealed. It was assumed that this increase could cause the change in the atmospheric circulation, which was completed by the late 70s. This change, in its turn, caused changes in the temperature conditions.

Introduction

Observed changes in the climate and ecology under the effect of both natural and anthropogenic factors have a complex spatiotemporal character. Therefore, revealing these changes implies not only continuous improvement of the monitoring instrumentation, but also a search for more efficient methods for analysis of the results of monitoring with respect to various climatic and ecological parameters.[1]

In analysis and interpretation of the data of climatic and ecological monitoring, the surface temperature of air is one of the key atmospheric parameters in view of several circumstances. First, under continental conditions, this parameter correlates very closely with the temperature of the Earth's surface, and therefore, in accordance with the Stephan–Boltzmann law, it characterizes the fluxes of longwave optical radiation, which is a decisive component of the radiative balance in the atmospheric surface layer. Second, this parameter is a very important climatic characteristic that generalizes the results of numerous atmospheric (through variable parameters of humidity, wind, etc.) and ecosystem (through the variable coefficient of emittance) processes. Third, long-term homogeneous series of instrumental observations of the temperature are the longest and most representative for statistical processing.

When studying long-term variability of the surface air temperature, the monthly mean temperature for a long observational period is commonly used. This excludes temperature fluctuations on the synoptic scale. Then, the components of the long-term and seasonal trends are separated from the initial series with the use of various statistical processing procedures.[2] The rest of the series describes temperature anomalies due to fluctuations of the amplitude and phase in the seasonal and interannual trends. Temperature changes are formed under the effect of natural and anthropogenic climate-forming factors, which are usually divided into internal and external relative to the climatic system.[3–6]

Spectral analysis based on the Fourier transform method or the method of maximum entropy is the most-used in study of latent periodicities in temperature series.[7] Along with all obvious merits of the Fourier transform method, it also has some demerits: analyzing functions (harmonics) of the method cover the entire time axis, therefore it provides the information on characteristic frequencies and intensities for the process under study, rather than on time localization and characteristic scales of latent periodicities. The demerits can be eliminated by using the wavelet transform[8] that projects a single-dimensional signal onto the time–frequency plane and allows tracing the temporal change of the signal spectral properties.

The series of the surface air temperature ($\bar{T}$) in Tomsk for the period from 1881 to 1998 are treated below using the wavelet transform and Fourier transform methods. The series of the annually mean Wolf number ($\bar{W}$) and annually mean critical frequencies ($\overline{fF2}$) of the $F2$ ionospheric layer are analyzed in parallel. Along with separation of general periodicities in the series, we

0235-6880/01/04 253-06 $02.00

determine the normalized correlation coefficients R_{XY}, which characterize the statistical relation between the corresponding observational series.

1. Method of wavelet transform of time series

The method is based on the wavelet transforms and procedures of analysis that allow us to reveal a fine structure of a time series. Wavelet transform of a single-dimensional signal consists in its resolution with respect to the basis made of a soliton-like function (wavelet), having certain properties, by means of scale changes s and shifts k. Every basis function characterizes both a certain spatial (temporal) frequency and its location in space (time). The continuous wavelet transform of a discrete series X_n with a constant space between neighboring measurements δt can be formally written as a convolution of the series with the wavelet function ψ (Ref. 9):

$$W_k(s) = \sum_{n}^{N-1} X_n\, \psi^* \left[\frac{(k-n)\,\delta t}{s}\right],$$

$$\text{or } W_k(s) = \sum_{n}^{N-1} \hat{X}_n\, \hat{\psi}^*\,(s\,\omega_n)\, e^{i\omega_n k}, \qquad (1)$$

where

$$\hat{\psi}\,(s\,\omega_n) = \left(\frac{2\pi\, s}{\delta t}\right)^{1/2} \hat{\psi}_0\,[s\,\omega_n]; \quad \int_{-\infty}^{\infty} |\hat{\psi}_0(\omega)|^2\, d\omega = 1;$$

the sign * denotes complex conjugation; the sign ˆ is for the Fourier transform; ψ_0 is the basis wavelet function; N is the number of points in the series.

When drawing the pattern of the energy density $W_k(s)$ for the wavelet transform of the time series X_n, we used, as a basis wavelet function, the Morlet wavelet that is well localized in the time and frequency spaces[8]:

$$\psi_0(t) = e^{is_0 t}\, e^{(-t^2/2)},$$

where the increase of s_0 leads to the increase of resolution in the frequency region at the cost of worsening of the spatial resolution.

The use of the wavelet transform, unlike the Fourier transform, provides 2D scanning of the studied one-dimensional signal, in which the frequency and shift are considered as independent coordinates. The wavelet transform can reveal the location of singularities of the studied function. Thus, for example, the coefficients of wavelet transform of a smooth function are small and increase sharply at a singular point, marking its location by the lines of local extrema. The inverse wavelet transform is written as

$$X_n = \frac{\delta j\ \delta t^{1/2}}{C_\delta\, \psi_0(0)} \sum_{j=0}^{J} \frac{\Re\,\{W_n(s_j)\}}{s_j^{1/2}},$$

where C_δ is the normalizing factor following from the δ-function reconstruction; δj is the step for the power 2 (it is assumed that the scale changes according to the power law).

The energy of the wavelet transform spectrum is connected with the variance σ^2 by a relation similar to the Parseval's theorem

$$\sigma^2 = \frac{\delta j\ \delta t^{1/2}}{C_\delta\, N} \sum_{n=0}^{N-1} \sum_{j=0}^{J} \frac{|W_n(s_j)|^2}{s_j},$$

and the confidence interval is defined as a probability that the actual wavelet spectrum at the given shift k and scale s is inside the interval with the estimated wavelet power spectrum[9]:

$$\frac{2}{\chi_2^2(p/2)}|W_k(s)|^2 \le |W_k^2(s)|^2 \le \frac{2}{\chi_2^2(1-p/2)}|W_k(s)|^2, \quad (2)$$

where $|W_k^2(s)|^2$ is the actual wavelet spectrum; p is the level of significance ($p = 0.05$ for the 95% confidence interval); $\chi_2^2(p/2)$ is the χ^2–distribution with ν degrees of freedom.

Averaging over one coordinate gives some interesting estimates. One of such estimates is a *time-averaged* power spectrum

$$\overline{W_n^2(s)} = \frac{1}{\Delta n} \sum_{k=n_1}^{n_2} |W_k(s)|^2,$$

where $\Delta n = n_2 - n_1 + 1$ is the number of points of averaging. The limiting case is averaging over the entire range n, and the obtained function called the "global" or "integral" wavelet spectrum is a smoothed Fourier spectrum:

$$\overline{W^2(s)} = \frac{1}{N} \sum_{k=0}^{N} |W_k(s)|^2. \qquad (3)$$

Since averaging is carried out over the entire range n, the number of degrees of freedom increases, and, consequently, the confidence interval decreases. Thus, the significance of peaks in the wavelet spectrum increases.

Another estimate is a *scale-averaged* spectrum, for which the weighted sum of the wavelet power spectrum from the scale s_1 to the scale s_2 is written as

$$\overline{W_k^2(s)} = \frac{\delta j\ \delta t}{C_\delta} \sum_{j=j_1}^{j_2} \frac{|W_k(s_j)|^2}{s_j}. \qquad (4)$$

The scale-averaged wavelet spectra allow determining the modulation of one time series by another series or modulation of one selected frequency/scale by another frequency/scale. This is possible both between different series and inside the same series. Some examples of analysis of the series of indices of El Niño Oscillation with the use of scale-averaged wavelet spectra are given in Refs. 8 and 9.

Checking of the significance of the obtained values for the wavelet power spectrum has shown that the upper boundary of white noise at the 95% confidence probability is equal to 3. Red noise was modeled by the process of autoregression of the first kind $X_n = 0.7X_{n-1} + Z_n$, where $X_0 = 0$, Z_n is the white noise. The upper boundary of the red noise at 95% confidence probability at scales more than 100 years turned to be 19. This value is several times less than the values in the region of energy density peaks. On the scales less than 65 years, the noise component is determined by white noise. The boundaries of the confidence probability of the calculated energy density are, respectively, $0.33|W_k(s)|^2$ and $19.5|W_k(s)|^2$. Thus, the values of local maxima even for the scales of 8–16 years are significant.

2. Wavelet analysis of time series of temperature

Figure 1 shows the series of annually mean temperatures in Tomsk for the period from 1881 to 1998 (Fig. 1*a*) and the power spectrum $|W_k(s)|^2$ of the wavelet transform of this series (Fig. 1*b*). Shifts k are plotted along the abscissa, and the scale variable s is plotted along the ordinate. For the Morlet wavelet, the values of s are equal to the Fourier period. The peaks of the Wolf numbers for the given time interval are shown by arrows in Fig. 1*b*. Analysis of the power spectrum of wavelet transforms allows us to reveal quasiperiodic structures of several time scales: 2–3 years, 7–10, 12–25, and 30–50 years. In addition, there exists the periodicity of 120–150 years, which is not shown in Fig. 1*b*.

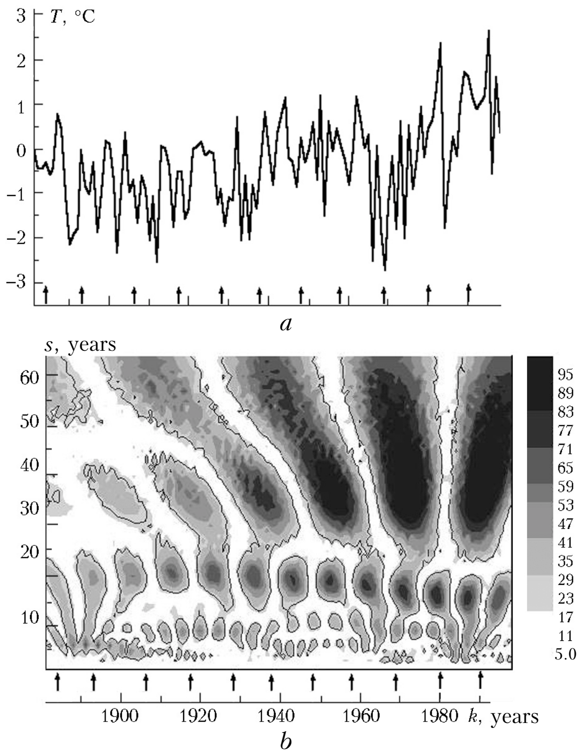

Fig. 1. Series of annually mean temperatures in Tomsk (*a*); power spectrum of wavelet transform (*b*). Arrows indicate the peaks of the Wolf number in the 11-year cycle.

The result of application of the fast Fourier transform to the initial temperature series show that peaks or groups of peaks in the Fourier spectrum correspond to the structures mentioned above. The obtained pattern of the power spectrum of the wavelet transforms is in a qualitative agreement with the results of Ref. 10, in which the wavelet analysis of the series of annually mean temperatures in Central England was conducted for the period from 1659 to 1996. In this series, the periodicities of the scales close to 7, 14, 23, 39, 65, and 100 years were found. Note also that the spectral analysis of the monthly mean temperatures in Central England established[11] the presence of rather strong spectral peaks corresponding to 2.1, 3.1, 5.1, 14.5, 23, and 76 years.

To reveal the pattern of high-frequency periodicity slightly manifesting itself in the series of monthly mean temperatures, we have made the wavelet transform of some anomalies in the annually mean temperatures for the period from 1875 to 2000 and then calculated the integral power spectrum using Eq. (3) (Fig. 2). One can see that the periodicities on the 2–3-year scale are formed exclusively by anomalies of the monthly mean temperatures, which also contribute to formation of the periodicities with the 7–10-year scale. Oscillations on the scale of 1.5–2 years are often combined by the term "quasibiannual cycle." This cycle is found in oscillations of zonal winds and temperatures and is most pronounced in the lower troposphere in tropics. The mechanism of the quasibiannual cycle is unclear now, although most often it is explained by trapping, at upper levels, of the upward energy of horizontal vortices of synoptic or somewhat larger scales and propagation of this energy through resonance mechanisms inherent in the structure of the atmosphere and ocean.[12]

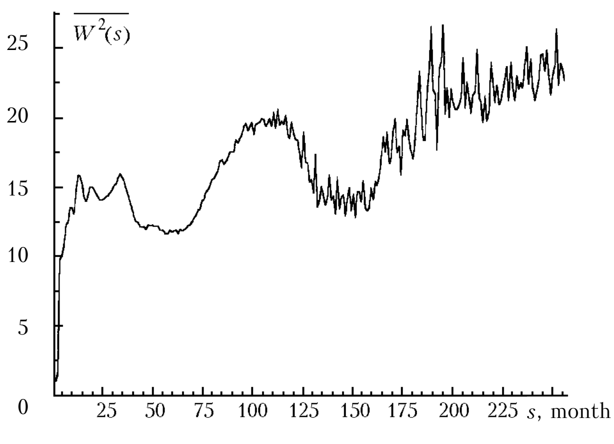

Fig. 2. Integral power spectrum of wavelet transform for some anomalies of monthly mean temperatures for the period of 1875–2000.

The scale of 7–10 years, as follows from Fig. 1*b*, has not a pronounced periodic character. Intense peaks corresponding to warming periods are noticed later in the 19th century, in the 1920–1940s, and after 1960. Numerous attempts were undertaken to relate the periodicity of this scale with the 11-year cycle of the solar activity.[13] Spectral analysis of long-term observation series has shown that the curve of the spectral power of the annually mean Wolf number has the peaks corresponding to 5.5, 8.1, 9.7, 11.2, 100, and 180 years. The most important of these peaks is the 11-year peak, but its relation with temperature fluctuations based on the data of different authors is contradictory. For different observation stations both positive and negative correlation with the coefficients varying from 0.27 to 0.58 was found.[13] Violation of the correlation, including alternation of the coefficient sign, is typical as well.

3. Wavelet analysis of time series of solar activity

The series of the annually mean Wolf number for the period from 1700 to 1998 and the distribution of the spectral power of the wavelet transform of the series are shown in Fig. 3 (positions of the 11-year peaks are shown by arrows). As is seen from the figure, intense peaks in 1750–1800, 1850–1900, and after 1950 are clearly pronounced. The Hale 22-year cycle is slightly expressed. The periodicity with the scale of 40–60 years is seen until the end of the 19th century; then it disappears and after 1956 transforms into the 30–40-year scale periodicity with simultaneous intensification of the activity in the 11-year cycle.

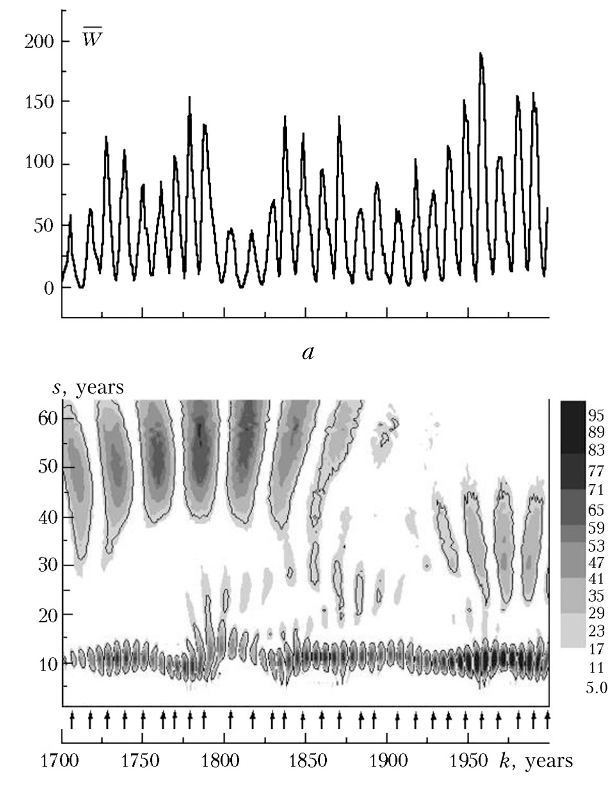

Fig. 3. Series of the annually mean Wolf number for the period of 1700–1998 (*a*) and distribution of spectral power of wavelet transform (*b*).

We have made also the wavelet analysis of critical frequencies of the $F2$ ionospheric layer for the period of 1936–1998. These frequencies were observed at the ionospheric station of the Siberian Physical-Technical Institute in Tomsk[14] (Fig. 4). Arrows in Fig. 4*b* show the peaks of solar activity in the 11-year cycle.

It is seen from Fig. 4 that variations of the critical frequencies are pronounced in both the 11-year and 22-year cycles. Note, however, that the reaction of the ionosphere to 22-year variations of the solar activity (this reaction manifests itself in the critical frequencies of the $F2$ layer) has the nonstationary character. It is also seen from Fig. 4*b* that the peaks in the 22-year scale shift with respect to the peaks of the Wolf number in the 11-year cycle. This behavior is likely caused by the fact that the 11-year and 22-year cycles of variation of the hard, ionizing component of solar radiation are of different nature.

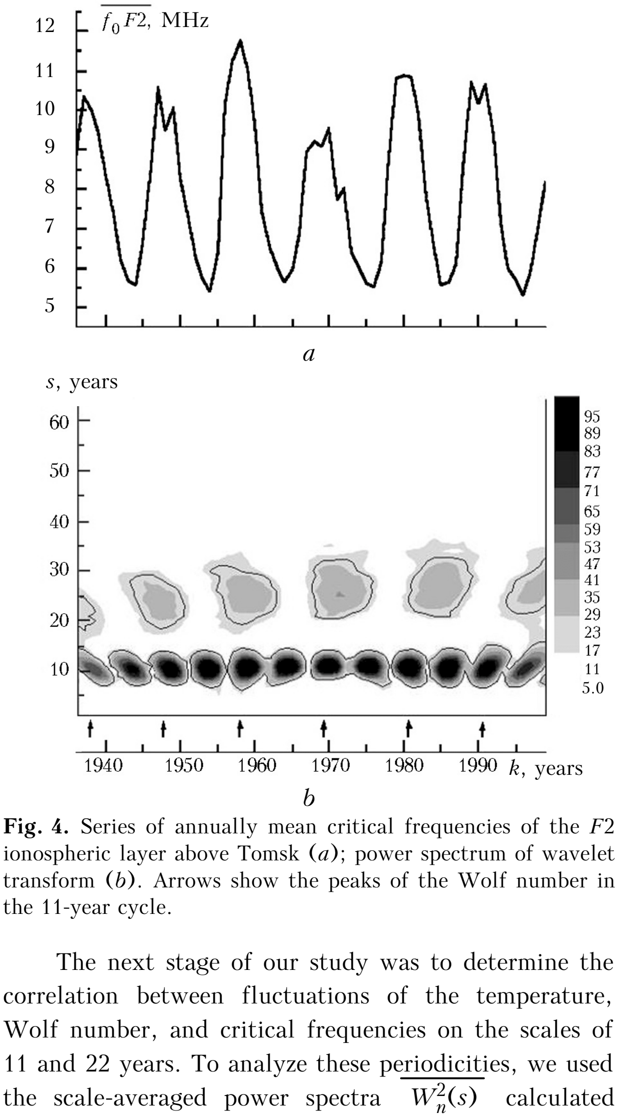

Fig. 4. Series of annually mean critical frequencies of the $F2$ ionospheric layer above Tomsk (*a*); power spectrum of wavelet transform (*b*). Arrows show the peaks of the Wolf number in the 11-year cycle.

The next stage of our study was to determine the correlation between fluctuations of the temperature, Wolf number, and critical frequencies on the scales of 11 and 22 years. To analyze these periodicities, we used the scale-averaged power spectra $\overline{W_n^2(s)}$ calculated according to Eq. (4). Analysis of the calculated power spectra $|W_k(s)|^2$ (see Figs. 1, 3, and 4) and time-averaged global spectra (see Fig. 2) drawn for each series allowed us to determine the regions of the scales needed for calculation of $\overline{W_n^2(s)}$. Thus, for example, for the 11-year periodicity, we separated the range of 6–15 years for the series of the Wolf number and critical frequencies $\overline{f_0 F2}$, as well as the range of 4–12 years for the temperature series. The 22-year periodicity is enclosed in the range of 18–35 years for the series of the Wolf number and critical frequencies $\overline{f_0 F2}$ and in the range of 13–24 years for the series of temperature data. In addition, two following time intervals were distinguished.

The time interval of 1936–1998. In this interval, three series of the parameters of our interest were determined: the series of $\overline{f_0 F2}$, the series of the Wolf number, and the series of annually mean temperature.

The time interval of 1881–1998. In this interval, only two series were determined: the series of the Wolf number and the series of annually mean temperature.

For the separated series, the scale-averaged power spectra $\overline{W_n^2(s)}$ were normalized and subjected to correlation analysis. The obtained results, namely, the normalized correlation coefficients and 90% confidence boundaries are given in the Table.

Table. Comparison of normalized correlation coefficients: I is the series of critical frequencies of the $F2$ layer, $\bar{W}$ is the series of the Wolf number, $\bar{T}$ is the series of annually mean temperature in Tomsk

Interval	Cycle	$R_{I\bar{W}}$	$R_{\bar{W}\bar{T}}$	$R_{I\bar{T}}$
1936–1998	11-year	0.940 ± ± 0.024	0.184 ± ± 0.202	0.195 ± ± 0.201
	22-year	0.686 ± ± 0.110	−0.152 ± ± 0.204	−0.086 ± ± 0.207
1881–1998	11-year	–	0.055 ± ± 0.153	–
	22-year	–	0.327 ± ± 0.137	–

The normalized correlation coefficient is the largest, as expected, for the pair $I - \bar{W}$, i.e., Wolf number – critical frequency $\overline{f_0 F2}$. The smallest values of about ±0.2 are for the pairs $\bar{W} - \bar{T}$ and $I - \bar{T}$, i.e., the pairs with participation of the temperature series $\bar{T}$. Only for the 22-year periodicity in the time interval of 1881–1998 the correlation coefficient takes the value of 0.327. To evaluate the significance of the obtained estimates, we determined the confidence boundaries of single correlations of white noise.[15] Since the standard deviation of the sample estimate of a single value of the correlation coefficient is equal to $1/\sqrt{N}$, the 90% confidence boundaries for the single correlation R_{XY} are roughly equal to ±0.209 and ±0.153 for the samples with the lengths of 64 and 117, respectively. Therefore, starting from the confidence level of 90%, we can conclude that the values of the correlation coefficient $R_{I\bar{W}}$ for the 11-year and 22-year scales, as well as the value of $R_{\bar{W}\bar{T}}$ for the 22-year scale are significant.

As to the scales of 30–50 years, it follows from Fig. 1*b* that a quasiperiodic structure is observed in the considered time series, and the intensity of the spectral peaks increases during the 20th century with simultaneous scale transformation of the periodic structure from large scales to smaller ones. The nature of this periodicity is now unclear and likely connected with a change in atmospheric circulation.

It should be noted that the periodicities of the scales more than 60 years were studied thoroughly in Ref. 16 using, as an example, long-term observations of the air temperature in Europe. In Ref. 16, no convincing proofs were found for the periodicity of the scale of 60–80 years, although noticed in various observations, because oscillations of the air temperature on the scales turned out to be nonsynchronous in different seasons.

Thus, we can assert that the power spectrum of wavelet transform of the series of the surface air temperature in Tomsk shown in Fig. 1*b* demonstrates a presence of the pronounced 22-year periodicity corresponding to periodicity in variations of the solar activity. The value of the correlation coefficient for the 22-year cycle (0.327) indicates that this correlation is statistically significant. The obtained results do not demonstrate a statistically significant correlation between temperature oscillations and the 11-year cycle of solar activity.

Joint analysis of the power spectra of the series of temperature, Wolf number, and critical frequencies of the $F2$ ionospheric layer suggests that some event happened between the 1940s and 1950s, which entailed deep changes in the dynamics of global atmospheric circulation and was reflected in the dynamics of temperature. The event might be the monotonous increase in the intensity of solar radiation between 1940 and 1960 discovered by Abbot.[13] During the increase, the solar constant grew by 0.25% and the Wolf number increased from 25 to 175. Additional heating of the atmosphere and the surface due to the increase of the solar constant could cause changes in the atmospheric circulation, whose indices were considered in Ref. 16. The authors of the reference, based on the analysis of the 105-year series of atmospheric pressure, have shown that since 1950 an approximately linear growth of the pressure at the center of the winter Siberian anticyclone was observed (from 1025 to 1035 mbar by 1980). It was also shown that the 1950 falls on the extreme point on the curves of pressure differences, and latitude and longitude differences of the atmospheric centers of action in the Northern Hemisphere.

When considering the large-scale elements of atmospheric circulation (cyclones, anticyclones) as turbulent formations, the changes in the dynamics (spectrum) of scales and temperature must be coordinated with each other, namely, the growth of the temperature must correspond to transformation of the turbulent spectrum toward smaller scales, what is clearly seen in Fig. 1*b*.

Conclusion

When discussing the obtained results, we ignored the greenhouse effect as possible cause of climatic changes. Thus, it was shown that the possibilities to explain interannual fluctuations of the climatic system by a combination of external and internal factors are not exhausted. These factors, in the opinion of some authors (see the brief review in Ref. 17) can cause the "Brownian" character of the climate dynamics change, and the low-frequency components of the dynamics can explain the long-term trend in the temperature change that is observed in the last 100–150 years. Solid proofs that the climatic system is nearly intransitive, at least on regional scales, can play therewith an important role. The proofs would help to establish the fact that the atmospheric circulation oscillates between several different quasi-equilibrium states, and variations of the solar constant play the role of a driving force causing transitions between these states and related changes of climatic characteristics. In our opinion, thus we could reveal the differences between the purely stochastic behavior and noise-induced transitions in the climatic system, as in a nonlinear dynamic system,[18] as well as estimate adequately the influence of the greenhouse effect.

References

1. M.V. Kabanov, *Regional Monitoring of the Atmosphere*. Part 1. *Scientific and Methodical Principles* (SB RAS Publishing House, Tomsk, 1997), 210 pp.

2. G.O Zadde., S.G. Kataev, and A.I. Kuskov, in: *Regional Monitoring of the Atmosphere*. Part 4. *Natural Climatic Changes* (Rasko, Tomsk, 2000), pp. 11–56.

3. M.I. Budyko, Yu.A. Izrael, M.S. Mackraken, and A.D. Hekt, eds., *Future Climate Changes* (Gidrometeoizdat, Leningrad, 1991), 137 pp.

4. C.F. Mass and D.A. Portmann, J. Clim. **2**, 566–593 (1989).

5. S.S. Khmelevtsov, ed., *Volcanoes. Stratospheric Aerosol and Earth's Climate* (Gidrometeoizdat, Leningrad, 1986), 256 pp.

6. C. Frohlich, in: *Long and Short-Term Variability of Climate* (Springer-Verlag, 1988), Part 4, pp. 6–17.

7. R.G. Currie, J. Geophys. Res. **79**, 5627–5632 (1974).

8. N.M. Astaf'eva, Usp. Fiz. Nauk **166**, No. 11, 1145–1170 (1996).

9. C. Torrence and G.P. Compo, Bulletin of the American Meteorological Society **79**, No. 1, 61–78 (1998).

10. S. Balinnas, P. Frick, D. Sokoloff, and W. Soon, Geophys. Res. Lett. **24**, No. 11, 1351–1354 (1997).

11. B.I. Mason, Quart. J. R. Meteorol. Soc., No. 102, 473–486 (1976).

12. *Physical Principles of Theory of Climate and Its Simulation. Proceedings of the International Scientific Environmental Conference*, *Stockholm 1974* (Gidrometeoizdat, Leningrad, 1977), 277 pp.

13. J.R. Herman and R.A. Goldberg, *Sun, Weather, and Climate*, NASA Special Publication SP–426, NASA (Washington DC, 1978).

14. Yu.E. Tarashchuk, in: *Abstracts of Reports at Siberian Meeting on Climatic Monitoring,* Tomsk (1995), pp. 4–7.

15. G.M. Jenkins and D. G. Watts, *Spectral Analysis and Its Applications* (Holden-Day, San Francisco, 1968), 523 pp.

16. N.M. Dotsenko, D.M. Sonechkin, and M.V. Tabalova, Meteorol. Gidrol., No. 7, 33–41 (2000).

17. N.V. Vakulenko, A.S. Monin, and Yu.A. Shitkov, Dokl. Ros. Akd. Nauk **371**, No. 6, 802–805 (2000).

18. V.S. Anishchenko, A.B. Neiman, F. Moss, and L. Shimanskii-Gaier, Usp. Fiz. Nauk **169**, No. 1, 7–38 (1999).