MINIMIZATION OF THE ANGULAR CHARACTERISTICS OF PARTIALLY COHERENT OPTICAL RADIATION

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The possibility of achieving minimum angular divergence in the far zone of partially coherent radiation is studied for a nonlinear medium of the Kerr type. The condition for obtaining maximum intensity at the receiving point is formulated and it is shown that it is in principle possible to achieve angular divergence less than the diffraction limit in focusing media. The optimal values of the initial parameters of the radiation and the effect of its coherent properties in minimizing the angular divergence are determined.

The results presented make it possible to determine the range of values of the parameters for which, a fixed value of the angular divergence can be achieved.

The propagation of laser radiation through nonlinear media is accompanied by attenuation of the power density. Both methods for optimizing the parameters of the optical system and recently developed adaptive control methods are employed to minimize the nonlinear divergence. These questions are review of Refs. 1 and 2.

Programmed phase correction is most simply, but very efficiently, implemental for the case when the main nonlinear distortions of the phase of the radiation occur on the section of the propagation paths located near the radiating aperture.

These problems were studied in Refs. 1–5 for coherent radiation on vertical and scanning paths.

The angular characteristics of a beam in the far zone were studied in Ref. 6 for partially coherent radiation transmitted vertically through a thin layer of a nonlinear medium. The problems of minimizing the angular divergence in the far zone for a layer with nonlinearity of the Kerr type were studied in Ref. 7 in the aberration-free approximation.

We shall study the pulse-frequency mode when

$$\tau_p \ll \tau_{nl}, \; T \sim \tau_{nl},$$

where τ_p is pulse duration, τ_{nl} is nonlinear response time of the medium and T is the pulse repetition period. Then, it may be assumed that starting with the second pulse the propagation occurs in the refraction channel with the dielectric constant distribution formed after the passage of the preceding pulses. In the process a given pulse propagates without any self-action.

We pose the problem of determining the minimum angular divergence in the far diffraction zone after passage through a thin layer of nonlinear medium with thickness z_s (z is the longitudinal coordinate), in which the radiation interacts with the medium. We regard the angular divergence as having reached its maximum value when intensity at the point of reception becomes maximum. In Ref. 8 it was shown for partially coherent radiation, whose propagation is described by the equation of radiation transfer, that if the point of reception lies in the nonlinear zone, the optimum phase front in the starting plane will be the front that would arrive from a point source placed at the point of reception Then, placing the source in the far zone we get in the plane z_s a plane wave arriving from it. This wave having passed through the nonlinear layer, gives in the plane $z = 0$ the optimum phase front. We shall call this wave, which makes it possible to determine the optimal phase front, the reference wave (an example of a source of such a wave is a star).

We will prove this assertion in the aberration-free approximation, and the discussions below will be based on this approximation.

For nonlinearities of any type the dimensionless beam width $g(z)$ in aberration-free approximation satisfies the equation:

$$\frac{d^2 g}{dz^2} = f(z)g + \beta g^{-3}. \tag{1}$$

Here and below z is normalized to the refraction length

$$L_R = \frac{1}{2}|\varepsilon_2(z=0)|^{-1/2}; \qquad f(z) = \varepsilon_2(z)/|\varepsilon_2(z=0)|,$$

$\varepsilon_2(z) = \left.\dfrac{d^2\varepsilon(z, R)}{dR^2}\right|_{R=0}$, ε is the perturbation of the dielectric constant, whose functional form is determined by the type of nonlinearity $\beta = (L_R/L_D)$, where L_D is the diffraction length:

$$L_D = ka_0^2/\sqrt{1+(a_0/a_k)^2},$$

a_k is the coherence, radius and a_0 is the radius of the beam in plane $z = 0$.

The solution of Eq. (1)[9] with the initial conditions $g(z = 0) = 1$, $g'(z = 0) = 1/F \equiv \alpha$, where F is the focal length, can be written as

$$g^2=v_1^2+\beta v_2^2,$$

where v_1 and v_2 are solutions of the equation

$$\frac{d^2v}{dz^2}=f(z)v \qquad (2)$$

with the initial conditions $v_1(z = 0) = 1$, $\frac{dv_1(z=0)}{dz} = \alpha$; $v_2(z = 0) = 0$; $dv_2(z = 0)dz = 1$.
After passing through the layer z_s the beam acquires the angular divergence

$$\gamma^2=\left[\frac{dg_s}{dz}\right]^2+\beta g_s^{-2}=g_s^{-2}\left[\left[v_{1s}\frac{dv_{1s}}{dz}+\beta v_{2s}\frac{dv_{2s}}{dz}\right]^2+\beta\right], \qquad (3)$$

Here the index s indicate that the function is evaluated at $z = z_s$. Then $g_s(z)$ has the form

$$g_s=(v_{0s}+\alpha v_{2s})^2+\beta v_{2s}^2.$$

Due to the nonlinearity of Eq. (2) the following linear combination was used her for v_1

$$v_1=v_0+\alpha v_2, \qquad (4)$$

where v_0 satisfies (2) with the initial conditions

$$v_0(z=0)=1, \quad dv_0(z=0)/dz=0.$$

We shall find the focusing for which the angular divergence is minimum. For this we shall find that values of a for which $d\gamma^2/d\alpha$ vanishes. We solve the equation

$$\frac{d\gamma^2}{d\alpha}=\frac{d\gamma^2}{dv'_{1s}}\frac{dv'_{1s}}{d\alpha}=0, \text{ where } v'_1=\frac{dv_1}{dz}.$$

The condition that the Wronskian for Eq. (2) determinant is constant implies that $v_1(z) = 1/v'_2(z)$. It is easy to show that

$$\frac{dv_1}{d\alpha}\Big/\frac{dv'_1}{d\alpha}=\frac{v_2(z)}{v'_2(z)}.$$

Using these conditions we find that the minimum angular divergence is obtained with $\alpha = \alpha_{opt}$ for which

$$v'_{1s} = 0 \qquad (5)$$

Then from Eq. (4) we obtain the value of α_{opt} corresponding to optimal focusing of the starting limited beam described by Eq. (1):

$$\alpha_{opt} = -v'_{0s}/v'_{2s}$$

In Eq. (6), however, v'_{1s} is the derivative with respect to the z in the plane $z = z_s$ of the function v_1 which is the solution of Eq. (2) with the initial conditions

$$v_1(z = 0) = 1,\ v'_1(z = 0) = \alpha = \alpha_{opt}.$$

Since Eq. (2) is obtained from Eq. (1) in the limit $\beta \to 0$, v_1 describes the propagation of a beam with radius a_0 ∞.

Thus the wave propagating from a point source in the far zone arrives at $z = z_s$ as an infinite plane wave and reaching the starting plane it has a phase front with the slope $\alpha = \alpha_{opt}$. Such a wave can therefore be used as the "reference" wave. It is interesting that α_{opt} is independent of β and, consequently, of the radius of initial beam and the coherence radius.

Substituting Eq. (5) into Eq. (3) it is easy to obtain the minimum angular divergence for the optimal focused beam in the far zone:

$$\gamma_{min}^2=\beta v'^2_{2s}. \qquad (6)$$

We note that for the case under study the angular divergence $\gamma^2 = \beta$ corresponds to the diffraction divergence, i.e., the divergence of a beam propagating in a linear medium. As one can see from Eq. (2). for a defocusing nonlinear layer ($\varepsilon_2(z) > 0$) $v'_{2s} > 1$. Then even with optimal focusing in the starting plane the divergence in the far zone is larger than the diffraction divergence, i.e., γ^2min $> \beta$. On the other hand, for a focusing medium ($\varepsilon_2(z) < 0$) $v'_{2s} < 1$ and therefore the minimum angular divergence can be less than the diffraction divergence, i.e., γ^2min $< \beta$.

It is obvious from Eq. (6) that the more coherent the radiation (i.e., the less β) the smaller the angular divergence is.

TABLE 1.

z_s	β=0.001	β=0.05	β=1
0	0.001 / 0.001	0.050 / 0.050	1.0 / 1.0
0.2	0.042 / 0.001	0.093 / 0.052	1.081 / 1.041
0.5	0.273 / 0.001	0.335 / 0.064	1.543 / 1.272
1.0	1.384 / 0.002	1.500 / 0.119	3.762 / 2.381
1.5	4.539 / 0.006	4.811 / 0.277	10.068 / 5.534

Table 1 gives the results of calculations of the γ^2 angular divergence γ^2 of α^2 collimated (numerator) and an optimally focused (denominator) beams as a function of the thickness of the nonlinear layer for different values of β. The calculations are performed for the case $f(z) = 1$ for $0 \le z \le z_s$ and $f(z) = 0$ for $z > z_s$.

Analysis of these results shows that in posing the problem on obtaining minimum angular divergence, we obtain as the thickness of the nonlinear layer is increased, for an optimally focused beam an increased gain as compared with a collimated beam, and this gain is all the higher the small the value of β. However the obtained level γ_{min} remains higher than the diffraction divergence and their difference increases as the thickness of the layer z_s is increased. The increase is approximately the same for all the values of β and for $z_s = 1.5\gamma_{min}$ is 5 to 6 times greater than the diffraction divergence.

In the case of continuous radiation with $\tau_{cor} \geq \tau_{nl}$ where τ_{cor} is the correction time of the phase front at the radiation source we have the possibility of controlling the dielectric constant distribution formed up to a fixed moment in time with the reference wave and by the adjusting of mirrors to produce a phase front that is optimal at each moment in time.

In the case $\tau_{cor} \gg \tau_{nl}$ phase correction based on the use of an infinite plane (at $z = z_s$) front as the reference wave will not be optimal since the change in the corrected focus will lag the changes in the distribution of dielectric constant of the medium.

In this case the situation is similar to that discussed above, and, therefore the same results obtained and reasoning employed for the pulse-frequency radiation mode remain valid.

We shall examine the possibility of obtaining optimal focusing, as done in Ref. 7, for the example of nonlinearity of the Kerr type.

It is well known that for nonlinearity of the Kerr type the solution of Eq. (1) with the initial conditions $g(z = 0) = 1$ and $dg(z = 0)/dz = -\alpha$ has the form $g_s^2 = (1 + z_s\alpha)^2 + z_s^2(1 + \beta)$. The degree of optimal focusing can be found from the equation $d\gamma^2/d\alpha = 0$. However it is difficult to obtain an exact analytic solution. Numerical investigations of $\gamma = \gamma(z, \alpha, \beta)$ showed that, for example, for $z_s = 0.2$ and $\beta = 0.01$ and 0.05 the function has two minima; for $z_s = 0.5$ and the same values of β there is only one minimum. In addition in the case of two minima the deeper minima corresponds to focusing with a longer focal length.

Figure 1 shows the dependence of the angular divergence on the thickness z of the nonlinear layer for optimally focused (solid curve) and collimated (dashed curve) beams and the diffraction divergence limit (dotted curve). As one can see from the figure, if the thickness of the layer doesn't exceed one- half the refraction length, optimal focusing permits obtaining divergence close to the diffraction limit. For thicknesses greater than the refraction length the gain for those values of β is less than 10% of the collimated case. Here it is also possible to find the thickness for which the gain for an optimally focused beam will be several times to fractions of a percent of the gain for as collimated beam, and it is thus possible to determine how effective focusing is for each specific case.

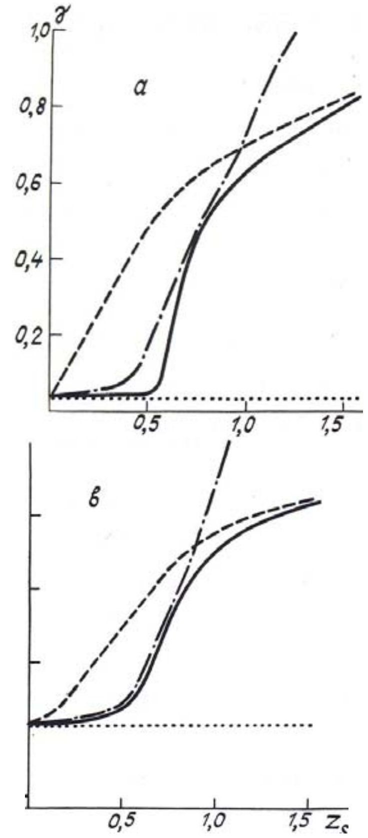

FIG. 1. The angular divergence γ versus the normalized values of the . thickness z_s of the factor of the nonlinear layer with $\beta = 0,001$ (a) and $\beta = 0.05$ (b). The solid line corresponds to the optimally focused beam, the dashed line corresponds to a collimated beam, the dot-dashed line was calculated using the algorithm, and the dotted line shows the diffraction limit.

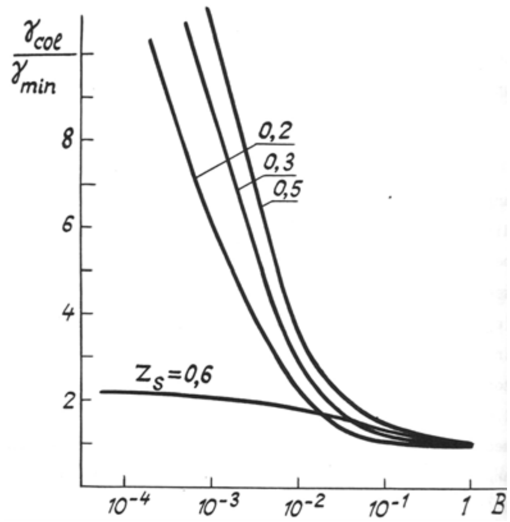

FIG. 2. $\gamma_{col}/\gamma_{min}$ vs β for different normalized values of thickness Z_s of the nonlinear layer.

The dependence of the angular divergence on β is shown in Fig. 2. On can see that for β of the order of 0.05 and less and for the thicknesses of the nonlinear presented in the figure the gain (relative to the angular divergence for a collimated beam γ_{col}) amount to a factor of two and more. It is interesting that the greatest gain is observed for z_s of the order of one half the refraction length for any value of β and equals a factor of 30 for $\beta = 10^{-1}$. Thus as β is increased the phase correction becomes less effective.

The dot-dashed curve in Fig. 1 shows the numerical calculations of the angular divergence of a beam which passes through a nonlinear layer and whose phase front is corrected using the algorithm based on the concept of a plane reference wave and representing one possible variant of programmed phase correction. According to the algorithm a beam whose radius equals that of the initial beam ($g = 1$) and which has a plane phase front is given in the plane $z = z_s$. Next the propagation of this beam into the starting plane is calculated and the arriving distribution of the phase front in this plane is taken as the starting distribution for a beam transmitted into the far zone through the nonlinear layer. It is obvious that for $z_s \le 0.8$ the angular divergence for an optimally focused beam closely matches that of a beam whose focusing is determined by the algorithm. For larger values of z_s the algorithm no longer works and for $z_s \approx 1$ focusing of a beam following the algorithm gives a divergence that exceeds that of a collimated beam. As follows from the figures, however, phase correction is very effective precisely for those values of z_s for which the algorithm works.

Thus, for the case $\tau_{cor} \ge \tau_{nl}$ we find that phase correction permits increasing substantially (by tens of times) the efficiency of energy transfer as compared with a collimated beam when the thickness of the nonlinear layerz $z_s \le 0.5$ and $\beta \le 0.0005$. In the process divergence close to the diffraction limit is reached. As the coherent characteristics of the beam are degraded (i.e., as β increases α) phase correction gives a smaller gain. For $z_s > 1$ even optimal focusing gives a gain of several percent. It is obvious that in this case nonoptimal algorithms (for example, with programmed correction) can degrade the characteristics of the radiation relative to those of a collimated beam.

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