Noise immunity of optical systems for information transfer through the turbulent atmosphere

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The noise immunity of optical systems for information transfer is studied as a function of the distribution law of laser radiation intensity fluctuations in the turbulent atmosphere.

The interest in atmospheric optical information transfer systems (AOITSs) has recently been revived.[1,2] In this connection, study of the noise influence on the reliability of such systems is of interest as well. AOITSs operate under conditions of additive (background) and multiplicative noise in the radiation propagation channel.[3] The atmospheric turbulence, in particular, introduces a multiplicative noise, which causes some undesired effects, including fluctuations of signal intensity (signal fading). As a result, the probability of errors in the transmitted information increases, and noise immunity of the system drops. The influence of the distribution laws of intensity fluctuations on the AOITS noise immunity has been studied in Ref. 4. However, some incorrect assumptions were made in this paper as far as they concerned the distribution laws themselves and the domain of their applicability. Therefore, it is worth considering the AOITS noise immunity based on the contemporary data on the distribution laws.

As a rule, AOITSs are digital systems using power reception of binary amplitude-modulated signals. In this method, unit elements of a signal differ by presence or absence ("passive pause") of a radiation pulse. A receiver in this case counts the number of photons in a time interval corresponding to binary pulsing and compares the results with the receiver threshold.

As known,[5] the statistics of received photons for a multimode laser in the presence of some background is well described by the Poisson law. For the false alarm probability we have[6]:

$$p_{\mathrm{f.a}} = 1 - \exp(-n_{\mathrm{b}}) \sum_{n=0}^{n_{\mathrm{thr}}} \frac{n_{\mathrm{b}}^{n}}{n!}, \qquad (1)$$

where n_{b} is the number of background photons.

The number of background photons n_{b} is usually small. Thus, the threshold ensuring low probability of false alarm $p_{\mathrm{f.a}}(\approx 10^{-4} - 10^{-8})$, usually does not exceed several units. At the same time, the probability to skip signal photons depends on a number of factors, among them there are not only the threshold value, but also the distribution function of the signal intensity fluctuations:

$$P_{\mathrm{sk}} = \sum_{n=0}^{n_{\mathrm{thr}}} p(n), \qquad (2)$$

where $p(n)$ is the probability of receiving n photons during the radiation pulse duration.

Taking into account that the fading correlation time in AOITS, that is, the time during which the correlation coefficient decreases down to zero level, is far longer than the signal length, we obtain

$$p_{\mathrm{sk}} = \sum_{n=0}^{n_{\mathrm{thr}}} \int_{0}^{\infty} p(n/n_0)\, p(n_0)\, \mathrm{d}n_0, \qquad (3)$$

where $p(n_0)$ is the probability density of the number n_0 of photons per radiation pulse.

To calculate the noise immunity of reception, one has to know the distribution law of the probability of laser radiation intensity fluctuations. It is found[7] that the universal dimensionless parameter, which determines the form of the distribution law of the intensity fluctuations, is the variance of logarithm of the intensity of a plane wave β_0^2. It can be calculated in the approximation of the method of smooth perturbations (MSP):

$$\beta_0^2 = 1.23\, C_n^2\, k^{7/6}\, L^{11/6}, \qquad (4)$$

where C_n^2 is the structure characteristics of the air refractive index, which determines the degree of turbulence; $k = 2\pi/\lambda$, λ is the wavelength; L is the path length.

”y now it is proved, both theoretically and experimentally,[7] that under conditions of the MSP applicability, when $\beta_0^2 \ll 1$ (the region of weak fluctuations), intensity fluctuations of the received radiation obey the lognormal distribution law

$$P(I) = \frac{1}{\sqrt{2\pi}\,\sigma\, I} \exp\left[-\frac{1}{2\sigma^2}(\ln I - \Lambda)^2\right], \qquad (5)$$

where $\sigma^2 = \ln(\sigma_I^2 + 1)$ is the variance of the mean level of intensity; $\sigma_I^2 = \langle (I - \langle I \rangle)^2 \rangle / \langle I \rangle^2$ is the relative intensity variance; $\Lambda = \ln \langle I \rangle / (\sigma_I^2 + 1)^{1/2}$ is the mean value of the intensity level; $\langle\ \rangle$ denote averaging.

Within the region of weak intensity fluctuations, the following relations between σ_I^2 and β_0^2 are valid[8,9]:

for a plane wave

$$\sigma_I^2 \cong \beta_0^2, \qquad (6)$$

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for a spherical wave

$$\sigma_I^2 \cong 0.41\ \beta_0^2\ , \tag{7}$$

for a collimated beam under typical conditions of AOITS operation ($\Omega \gg 1$, where $\Omega = k\ a^2/L$, a is the transmitting antenna radius)

$$\sigma_I^2 \cong 0.84\ \beta_0^2\ . \tag{8}$$

At a further increase of β_0^2 along the path, strong intensity fluctuations arise, the above relations between σ_I^2 and β_0^2 break. Then fluctuations become saturated. It has been found experimentally[10] that the maximum level for the plane wave $\sigma_I^2 \cong 1.34 - 1.36$ corresponds to $\beta_0^2 \cong 4$. It is shown in Ref. 11 that for a spherical wave the value of the maximum level of σ_I^2 is higher than for a plane wave. The growth of β_0^2 above this level results in a smooth decrease of σ_I^2 in the region of highly saturated fluctuations.

The lognormal law has earlier been proposed[8] as the distribution law for the region of highly saturated fluctuations as well. However, recently it has been convincingly demonstrated[10] that for the values of β_0^2 from 36 to 324, the experimental data are well approximated by a K-distribution of the following form:

$$\langle I\rangle\ P(I) = \frac{2}{c\,(y)}\ y^{(y+1)/2}\ I^{(y-1)/2}\ K_{y-1}(2\sqrt{I\ y})\ , \tag{9}$$

where $y = 2/(\sigma_I^2 - 1)$; $y > 0$; $K_\nu(z)$ is the McDonald function (ν is the order; z is the argument of the function). ”esides, at $\sigma_I^2 \to 1$ and $y \to \infty$ the distribution transforms into the exponential one:

$$P(I) = \frac{1}{\langle I\rangle} \exp\left(-\frac{I}{\langle I\rangle}\right) . \tag{10}$$

In the region of $\beta_0^2 \gg 1$ the following relations between σ_I^2 and β_0^2 have been established:

for a plane wave

$$\sigma_I^2 \cong 1 + 0.86\ \left(\beta_0^2\right)^{-2/5}\ , \tag{11}$$

for a spherical wave

$$\sigma_I^2 \cong 1 + 2.8\ \left(\beta_0^2\right)^{-2/5}\ , \tag{12}$$

for a collimated beam, depending on the relation between Ω and β_0^2, the expressions will be identical to those for a plane or a spherical wave.

Upon substituting Eqs. (5), (9), and (10) into Eq. (3) and assuming $I = n_0$, $\langle I\rangle = n_{\rm av}$, where $n_{\rm av}$ is the average number of photons per radiation pulse, we have for the lognormal and K-distribution, respectively

$$P_{\rm sk}^{\ln} = \sum_{n=0}^{n_{\rm thr}} \int_0^\infty \frac{n_0^{n-1}}{\sqrt{2\pi}\ n!\ \sqrt{\ln\left(\sigma_I^2 + 1\right)}} \times$$

$$\times \exp\left\{ -\ n_0 - \frac{\ln\dfrac{n_0}{n_{\rm av}} + \left[\ln(\sigma_I^2 + 1)\right]^{1/2}}{2\ \ln(\sigma_I^2 + 1)} \right\} {\rm d}n_0\ , \tag{13}$$

$$P_{\rm sk}^{K} = \sum_{n=0}^{n_{\rm thr}} \int \frac{n_0^n \exp(-n_0)}{n!\ n_{\rm av}} \frac{2}{c\,(y)} \times$$

$$\times\ y^{(y+1)/2}\ (n_0)^{(y-1)/2}\ K_{y-1}(2\sqrt{n_0\ y})\ {\rm d}n_0\ . \tag{14}$$

For the exponential distribution, upon some transformations,[11] we have

$$P_{\rm sk}^{\rm ex} = \sum_{n=0}^{n_{\rm thr}} \frac{n_{\rm av}^n}{(n_{\rm av} + 1)^{n+1}}\ . \tag{15}$$

Following Ref. 4, the AOITS noise immunity was estimated using a power parameter – the average number of photons per radiation pulse that provides a given probability of the error. Therefore, the value of $n_{\rm thr}$ has been first determined by Eq. (1) with the Tables of the Poisson functions[12] assuming certain preset value of the false alarm probability. For the sake of convenience, the latter was set equal to the probability to skip a signal, which varied from 10^{-2} to 10^{-6}. The number of background photons varied from 1 to 10. Then for all distributions, within their applicability domains, we calculated the value of $n_{\rm av}$, which ensures the required probability of signal skipping at different values of σ_I^2 assuming the value β_0^2 to be set.

Some calculated results are shown in Fig. 1.

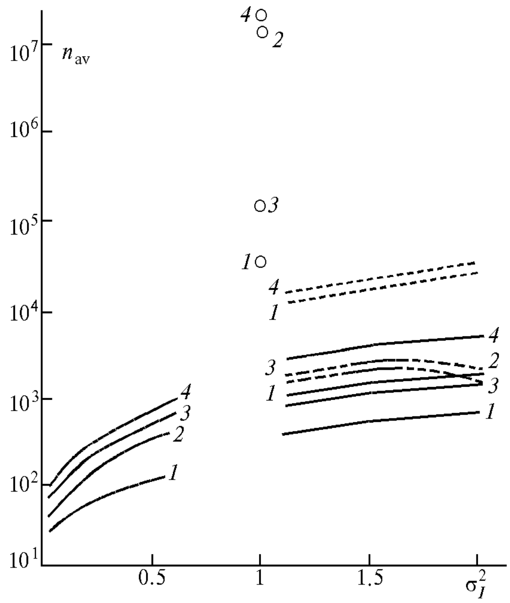

Fig. 1. Dependence of $n_{\rm av}$ on σ_I^2: lognormal distribution (solid curves), K-distribution (dashed curves); exponential distribution (circles); $n_{\rm b} = 1$, ..., $P_{\rm sk} = 10^{-4}$, ..., $n_{\rm thr} = 6$ (curve *1*); $n_{\rm b} = 1$, ..., $P_{\rm sk} = 10^{-6}$, ..., $n_{\rm thr} = 9$ (curve *2*); $n_{\rm b} = 10$, ..., $P_{\rm sk} = 10^{-4}$, ..., $n_{\rm thr} = 24$ (curve *3*); $n_{\rm b} = 10$, ..., $P_{\rm sk} = 10^{-6}$, ..., $n_{\rm thr} = 28$ (curve *4*).

From analysis of the data obtained we can conclude the following:

1. With increasing background illumination, as we might expect, the average number of photons in the pulse needed to provide the required value of $p_{\rm sk}$ increases.

2. In the presence of signal fading obeying the K-distribution along the path with $\beta_0^2 \gg 1$, the number of photons per radiation pulse necessary for reliable AOITS operation is larger than in the presence of lognormal fading, the values of n_b and p_{sk} being the same.

3. The effect of saturation is well pronounced for K-distribution, what is clearly seen in Fig. 2, while the lognormal distribution does not give this effect.

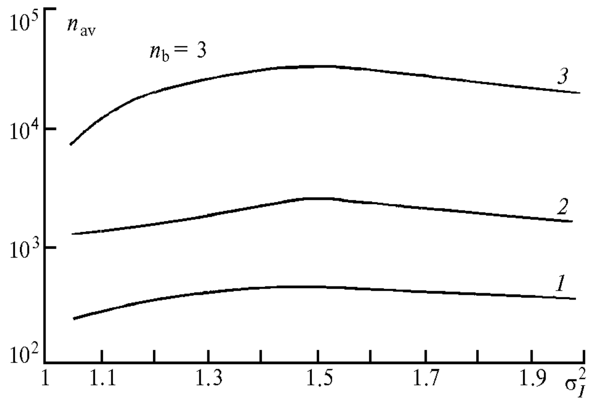

Fig. 2. The dependence of n_{av} on σ_I^2 for K-distribution of optical wave intensity fluctuations: $P_{sk} = 10^{-3}$ (curve *1*); $P_{sk} = 10^{-4}$ (curve *2*); $P_{sk} = 10^{-6}$ *(curve 3)*.

4. The poorest conditions for signal reception occur, when signal fading along the path are distributed by the exponential law.

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