PARAMETRIC GENERATION OF SQUEEZED QUANTUM STATES OF RADIATION AND THE POSSIBILITY OF THEIR APPLICATION TO OPTICAL COMMUNICATIONS

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Received November 29, 1989

An optical communication channel design is proposed to exploit the advantageous properties of light in its squeezed quantum states. A quantum theory of two-mode nondegenerate parametric generation of radiation in a laser cavity of arbitrary Q value is suggested. It is shown that the corresponding semiclassical approach is inadequate in this case.

INTRODUCTION

The task of generating and implementing various states of electromagnetic fields, and above all the "squeezed" states of light, has lately attracted much attention because it is related to solving a whole range of various fundamental and applied problems. This interest stems from the possibility of reducing the photodetection shot noise, and of achieving higher operational characteristics of various high-precision measuring and transceiving systems.[1–5]

The most vivid examples of obtaining such states of a light field are parametric generation or amplification in a resonance cavity.[2–7] Both degenerate and nondegenerate modes can be used for this purpose. In the first case a squeezed state of the signal wave is generated which, after mixing in a heterodyne, can reduce detection shot noise.[2,6] The same effect can be achieved in a nondegenerate mode through preliminary mixing of the signal with "blank" waves.[1,2,8] In addition the nondegenerate interaction is specific in that the signal and blank photons are generated simultaneously, so that they appear to be strongly intercorrelated.[9] This correlation is reflected in photocurrents when both the signal and the blank waves are independently detected. As a result the noise level of the differential photocurrent drops lower than the shot noise.[7]

Below we consider the possibility of employing these phenomena in optical communication lines. But first we present a consistent quantum theory of parametric interaction in a resonance cavity of arbitrary Q, used in experiments. The results we arrived at differ from those already known, derived from both the semiclassical and quantum approaches.[2,8,10–13] One should note that certain studies include no explicit statements on the applicability of the semiclassical approximation. For example, in Refs. 8, 10, and 11 the authors operate with the Bose-operators of photon generation and extinction, although it is easy to demonstrate that commutational relationships are not satisfied for them within the cavity. The common weak point of certain other studies, e.g., Refs. 2, 12, and 13, is the approximation of a high-Q cavity they assume, which reduces the scope of applicability for the results from these works.

I. THE QUANTUM MODEL OF NONLINEAR INTERACTION IN THE CAVITY

To describe the electromagnetic field in a nonlinear cavity we employ the following interaction model.

Let us the ring cavity shown in Fig. I. One of its mirrors (the exit one) has non-unit amplitude reflectance R and transmittance τ. In addition, we assume that there are no dissipative losses, i.e., $R^2 + \tau^2 = 1$.

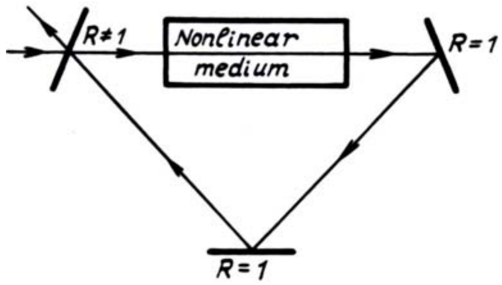

FIG. 1. Nonlinear cavity scheme.

Let us assume the signal and blank fields are given within the cavity and have certain average Intensities. The resonator can be either illuminated or not illuminated by an external coherent wave. We will be interested in quantum fluctuations of radiation from the average zero intensity in front of the light-splitting mirror. To describe them we employ a model combining the fluctuational component of the fields from the fluctuations of the initial vacuum waves, which enter the resonator through its exit mirror, and which have completed different numbers of circuits (see Fig. 2). If there is no cross-influence between these waves during their nonlinear interaction, the resulting field is represented simply by their superposition:

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$$a_j(t) = \sum_{m=1}^{\infty} a_{mj}(t), \quad a_j^+(t) = \sum_{m=1}^{\infty} a_{mj}^+(t), \tag{1}$$

where a and a^+ are the slowly changing photon generation and extinction operators in the Heisenberg representation, which describes the fluctuational component of the field; $j = 1, 2$ corresponds to the signal ($j = 1$) and blank ($j = 2$) modes. i.e., we assume parametric interaction to be frequency-degenerate but not degenerate in terms of the polarization state (the signal and blank waves have mutually orthogonal polarization planes); m is the number of complete circuits of the cavity, t is the time.

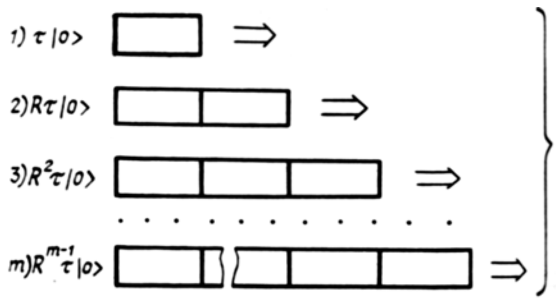

FIG. 2. Diagram of transformation of quantum fluctuations in the cavity. The vector $|0\rangle$ denotes the initial vacuum state of the field, and R and τ are the amplitude reflectance and transmittance of the entrance mirror. The rectangles represent the nonlinear medium.

The outlined approach is justified for quasi linear transformation of fluctuations in the medium, which is typical for parametric interaction in the inexhaustive pumping field

$$a_{m\,1,2}(t) = \tau R^{m-1} e^{-im\theta} [\mathrm{ch}(m\gamma)\, a_{0m\,1,2}(t - mT) + e^{-i\varphi}\, \mathrm{sh}(m\gamma)\, a^+_{0m\,2,1}(t - mT)]. \tag{2}$$

Here the operators a_{0mj} and a^+_{0mj} correspond to vacuum radiation entering the cavity and serving as "priming" for parametric amplification,[9] it then makes m circuits of the cavity; γ is the increment in parametric amplification per circuit; T is the time interval needed for one complete circuit of the cavity by the radiation; $\Theta = \omega T$ is the phase change during the same time interval at the carrier frequency $\omega = \omega_1 = \omega_2$ is the pumping phase.

To analyze the statistical characteristics of radiation formed in the cavity we need to know for commutative properties of the introduced operators. It is apparent that the operators $a_{mj}(t_1)$ and $a_{nj}(t_2)$ commutate for $m = n$, since they describe radiation which has suffered various numbers of reflections from the light-splitting mirror. It is also clear that

$$[a_{0mj}(t_1), a^+_{0mj'}(t_2)] = \delta_{jj'}, \tag{3}$$

for any t_1 and t_2. The situation with the operators $a_{0mj}(t_1)$ and $a^+_{0mj}(t_2)$ is somewhat more complex. When $m \neq n$

$$[a_{0mj}(t_1), a^+_{0mj'}(t_2)]_{m\neq n} = \begin{cases} \delta_{jj'}, & \text{for } |\Delta t| < T \\ 0 & \text{for } |\Delta t| \geq T, \end{cases} \tag{4}$$

where $\Delta t = t_2 - t_1$. These operators commutate for $|\Delta t| \geq T$, since at a separation interval of T or longer they describe Independent vacuum waves.

To obtain light in a squeezed state the signal and blank modes have to be mixed according to

$$a = (a_1 + a_2)/\sqrt{2}. \tag{5}$$

(see. e.g., Ref. 8); this operation can be realized by employing a light-splitting device or a polarization prism. Meanwhile, to lower the shot noise of photodetection this wave has to be further mixed with the heterodyne signal. Let us assume the carrier frequency of the latter coincide with the natural frequency ω of the signal and blank modes formed in the cavity. Then, as follows from the general description of the photodetection process (see, for example, Ref. 1), the correlation function of the photocurrent will be

$$G(\Delta t) = \eta^2 \langle I\rangle^2 + \eta \langle I\rangle \delta(\Delta t) + \eta^2 I_n (\tau^2/T) \{ [\langle a^+(t_1)\, a^+(t_2)\rangle + e^{-i2\varphi_n} \langle a^+(t_1)\, a(t_2)\rangle + \mathrm{c.c}]\, \Theta(t_2 - t_1) + (t_1 \leftrightarrow t_2). \tag{6}$$

Here $\langle I\rangle$ is the average intensity of the recorded radiation; I_h and φ_h are the intensity and phase of the heterodyne signal; η is the quantum efficiency of the photodetector; $\Theta(\Delta t) = 1$ for $\Delta t \geq 0$ and is equal to zero for $\Delta t < 0$, and the operators a and a^+ are determined in accordance with (5). In the derivation of (6) we disregarded the heterodyne natural quantum fluctuations; their effect can be practically completely neutralized by the choice of an appropriate mixing regime.[8] The expression (6) is valid for broadband detection and the shot noise for it is determined by the δ-correlated component.

The optical suppression of shot noise is achieved for $\Theta = 2\Pi k$, $\varphi_h - \varphi = \Pi + 2\Pi k$, where k is an integer. We calculate the correlation function (6) for these conditions, using the relationships (1)–(5). One has to recall that in accordance with (4) the correlations $\langle a^+_{mj}(t_1) a^+_{nj}(t_2)\rangle$ and $\langle a^+_{mj}(t_1) a_{nj}(t_2)\rangle$ differ from zero only in case $\Delta t/T + m \leq n < \Delta t/T + m + 1$, i.e., when $n = m + (\Delta t/T)_0$, where $(\Delta t/T)_0$ is the value of $\Delta t/T$, which we consider positive, and which we round-off to the nearest lower integer.

As a result we have

$$G(\Delta t) = \eta^2 \langle I\rangle^2 + \eta \langle I\rangle \delta(\Delta t) + \eta^2 I_n \times$$

$$\times \frac{(1-R^2)(e^{-2\gamma}-1)}{T\,[1-(Re^{-\gamma})^2]} \cdot [1+(Re^{-\gamma})^{|\Delta t/T|_0}\Theta(|\Delta t|-T)]. \quad (7)$$

The value $|\Delta t / T|_0$ here is rounded-off to the nearest integer, lowest in modulus. The fact that

$$\sum_{m=1}^{\infty} R^{2(m-1)}(e^{-2m\gamma}-1) = (e^{-2\gamma}-1) / [1-(Re^{-\gamma})^2]. \quad (8)$$

is also taken into account.

Thus, the correlation function (7) is a stepwise dependence with its steps of a size T, and having a pedestal given by the values of the function for $0 < \Delta t < T$ and $|\Delta t| \to \infty$.

The results obtained are valid not only for the pre-threshold operation regime, but also when the threshold of parametric generation is exceeded. It is achieved when $R = e^{-\gamma}$. The convergence of the seried (8) depends only on satisfying the condition $Re^{-\gamma} < 1$, which is not violated at any finite Q of the cavity. Note that the known options of the semi-classical approach to the problem of parametric amplification in a cavity of finite Q[8,10,11] are only correct for the pre-threshold regime.

The spectrum of the correlation function (7) has the form

$$G(\Omega) = \eta \langle I\rangle - 2\eta^2 I_n\, Re^{-\gamma}\, \mathrm{sinc}(\Omega T) \times$$

$$\times (2\cos\Omega T - 1 - Re^{-\gamma})\cdot(1 - R^2)(1 - e^{-2\gamma}) /$$

$$/[1 - 2Re^{-\gamma}\cos\Omega T + (Re^{-\gamma})^2][1 - (Re^{-\gamma})^2], \quad (9)$$

where Ω is the detuning from the carrier frequency ω, and the terms produced by the constant component of the signal (they are filtered out in the receiving transmission line) are omitted. Finally, $\mathrm{sinc}x = \sin x / x$.

An optimal suppression of shot noises in the spectrum (9) is achieved at $\eta = 1$ and $\langle I\rangle = I_h$ (the latter is quite simply achieved by increasing the heterodyne power). In this case the Fano factor, defined as the ratio of the variance of the intensity fluctuations to the corresponding variance of the radiation in its coherent state, i.e., $F(\omega) = G(\omega)/\langle I\rangle$ is equal to

$$F(\Omega \neq 0) = 1 - 2Re^{-\gamma}\mathrm{sinc}\Omega T \times$$

$$\times \frac{(1 - R^2)(1 - e^{-2\gamma})(2\cos\Omega T - 1 - Re^{-\gamma})}{(1 - R^2e^{-2\gamma})(1 - 2Re^{-\gamma}\cos\Omega T + Re^2)^{-2\gamma}}. \quad (10)$$

Comparing the obtained results with the data from available studies[2,8,10–13] one should note the following principal difference between them. The authors of the studies mentioned above conclude that, apart from $\Omega = 0$, the ideal regime for squeezed states of radiation is achieved at generation threshold.

In our case, however, the Fano factor at generation threshold ($R = e^{-\gamma}$)

$$F(\Omega \to 0) = (1 - R^2)/(1 + R^2) \quad (11)$$

would only become minimal if $R \to 1$, i.e., for an infinite Q of the cavity. In every other practically achievable case ($R < 1$) complete squeezing is not achieved, so that a threshold regime becomes ideal ($R < e^{-\gamma}$). This statement is illustrated by the graphs of the minimum possible values of F and their respective optimal values of $e^{-2\gamma} = e_0^{-2\gamma}$ as a function of ΩT for various R^2 given in Fig. 3. It is seen from the figure that decreasing the cavity Q reduces the ultimately achievable squeezing and removes the optical amplification regime further from the generation threshold.

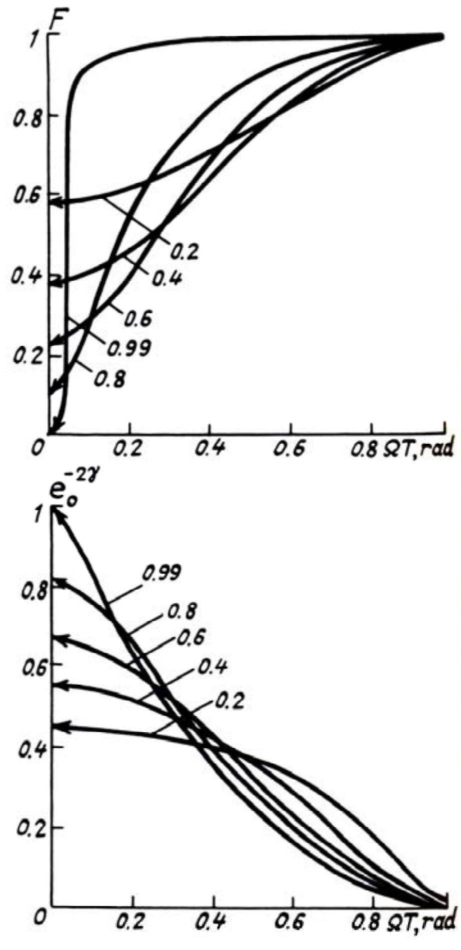

FIG. 3. Optimized graphs of the minimum possible Fano factors F and their respective optimal values of $e^{-2\gamma}$ as a function of ΩT. The numbers near the curves represent the intensity reflectances for the light-splitting mirror: R^2=0.2–0.99.

As noted in the introduction, to realize the advantages of quantum states of the light field generated during nondegenerate parametric interaction is also possible when the signal and blank modes are independently detected. Let us proceed to demonstrate that their mutual quantum fluctuations, which determine the noise level of the differential photocurrent, can be suppressed to a considerable extent. In the simplest case, when light is subjected to multiple passes through nonlinear medium, and two-mode parametric amplification of vacuum fluctuations occurs in a given classical pumping field, the interaction is described by the equations

$$a_{1,2} = e^{-i\theta_{1,2}}\left[a_{01,02}\operatorname{ch}\gamma + e^{-i\varphi}\, a^{+}_{02,01}\operatorname{sh}\gamma\right]. \qquad (12)$$

The notations here match to those in (2).

It is easy to see that the average differential photocurrent is zero for the case of independent detection of the signal and blank modes by receivers of unit quantum efficiency:

$$\langle I_{1-2}\rangle = \langle a_1^{+} a_1\rangle - \langle a_2^{+} a_2\rangle = 0. \qquad (13)$$

Zero variance of the fluctuations of this differential photocurrent, following from direct computations, is not as obvious

$$\langle I^2_{1-2}\rangle = \sum_{j=1}^{2}\langle a_j^{+} a_j\, a_j^{+} a_j\rangle - \langle a_1^{+} a_2^{+} a_1 a_2\rangle = 0. \qquad (14)$$

Thus, regardless of the efficiency of the parametric process, the photons in both modes turn out to be ideally correlated.

Let us now turn to parametric generation in the cavity. For the correlation function of the differential photocurrent when both photodetectors are ideal we have

$$G_{1-2}(\Delta t) = \eta\tau^2T^{-1}\Big\{\sum_{j=1}^{2}\{\langle a_j^{+}(t_1)\, a_j(t_2)\rangle\delta(\Delta t) + \{\eta\tau^2T^{-1}[\langle a_j^{+}(t_1)a_j(t_2)\rangle + \langle a_j^{+}(t_1)a_j^{+}(t_2)\rangle a_j(t_2)a_j(t_1)\rangle\,]\Theta(\Delta t) + (t_2\leftrightarrow t_1)\}\} - 2\eta\tau^2T^{-1}\times [\langle a_1^{+}(t_1)a_2^{+}(t_2)a_2(t_2)a_1(t_1)\rangle\times \Theta(\Delta t) + (1\leftrightarrow 2)\}\Big\}. \qquad (15)$$

The optimal suppression of mutual quantum fluctuations is also achieved at the resonance frequencies of the cavity, i.e., for $\Theta = 2\pi k$, where k is an integer (to be accounted for below).

Substitution of (1) and (2) into (15) and considering the commutational relationships (3), and (4) give

$$G_{1-2}(\Delta t) = 2\eta\tau^4T^{-1}\{\, S(0)\,\delta(\Delta t) + \{\eta\tau^2T^{-1}\times [S(\Delta t) + 2\tau^2 S(0)\, S(\Delta t) + \tau^2S^2(\Delta t) - 2\tau^2C(0)\times C(\Delta t) - \tau^2C^2(\Delta t)]\,\Theta(\Delta t - T) + (t_1\leftrightarrow t_2)\}\}, \qquad (16)$$

$$S(\Delta t) = R^{(\Delta t/T)_0}\sum_{m=1}^{\infty}R^{2(m-1)}\operatorname{sh}m\gamma\ \operatorname{sh}[m + (\Delta t_0/T)]\gamma,$$

$$C(\Delta t) = R^{(\Delta t/T)_0}\sum_{m=1}^{\infty}R^{2(m-1)}\operatorname{sh}m\gamma\ \operatorname{ch}[m + (\Delta t_0/T)]\gamma.$$

The dependence (16) is also a stepwise function with its steps being of width T. Direct computation of the sums makes it possible to simplify the expression (16) to:

$$G_{1-2}(\Delta t)/2\eta\,\langle I\rangle = \delta(\Delta t) - \eta T^{-1}R^{2|\Delta t/T|_0}(1 - R^2)\times (1 + R^2)^{-1}\Theta\,(|\Delta t| - T) \qquad (17)$$

where $\langle I\rangle = \tau^4T^{-1}S(0)$ is the average radiation intensity in the modes due to amplification of the vacuum fluctuations (we assume here that only they affect the cavity, i.e., it is not externally illuminated).

As for the spectrum of fluctuations of the differential photocurrent, we obtain

$$\frac{G_{1-2}(\Omega)}{2\eta\,\langle I\rangle} = 1 - 2\eta\,\frac{R^2(1 - R^2)}{1 + R^2}\times \frac{\cos\,(3\Omega T/2) - R^2\cos\,(\Omega T/2)}{1 - 2R^2\cos\,\Omega T + R^4}\times \operatorname{sinc}\,(\Omega T/2). \qquad (18)$$

Lack of any dependence on γ is what first attracts one's attention in the final relationships (17) and (18), with the latter determining only the average intensity in each channel; in other words, the situation appears to be similar to parametric "throughput" amplification. This fact, which has a completely obvious quantum explanation, finds no such explanation within the framework of the well-known semiclassical description (Ref. 2, p. 1520), where the dependence on γ enters the final results. Lack of the effect of the parameter γ on the degree of suppression of shot noises prompts one to conclude that a simpler way exists to such quantum states in which variance of the fluctuations in one of the field quadratures is decisively suppressed by the increment of the field (as compared to single-mode (degenerate) parametric generation or to the above technique of producing squeezed states by mixing the signal and blank modes). This consideration is supported by the latest experimental data from Ref. 5, p. 361: quantum noise could be suppressed in a cavity PLC (Parametric Light Generator) only by employing a scheme similar to that from Ref. 7, i.e., by independent detection of the signal and blank modes. Attempts to produce squeezed states in the degenerate regime failed.

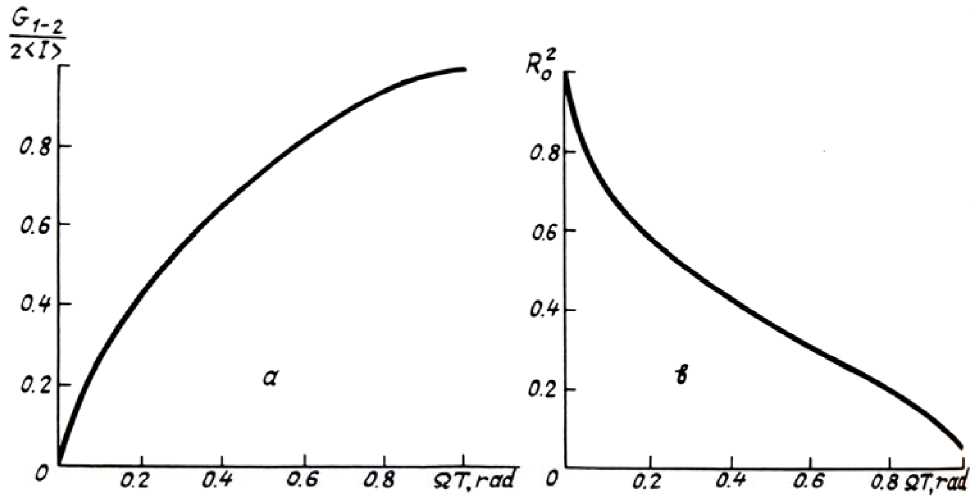

FIG. 4 Graphs of the minimum possible values of $G_{1-2}(\Omega)\,/\,2\langle I\rangle$ (a) and the respective optimal R_0^2 (b) as a function of ΩT for $\eta = 1$.

The important difference of the outlined quantum treatment from the semiclassical approach is also the fundamental impossibility of achieving ideal suppression of differential photocurrent noise at any given finite Q of the cavity ($|R| < 1$). Semiclassical theory, however, predicts such a possibility for the case of $\Omega = 0$.

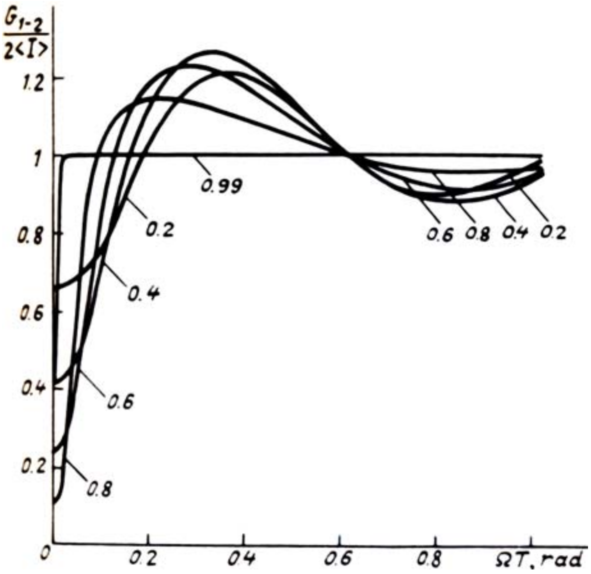

FIG. 5. Normalized spectra of differential photocurrent noises for various $R^2 = 0.2$–0.99 at $\eta = 1$. The value of one corresponds to the noise level during detection of two coherent modes.

Note the following fact, important for the experiment. In accordance with (18) there exists a certain optimal cavity Q, which minimized $G_{1-2}(\Omega)$ for each frequency Ω. The dependences of the limiting values of $G_{1-2}(\Omega)\,/\,\eta\langle I\rangle$ and of their respective R_0^2 on ΩT are presented in Fig. 4. Examples of the differential photocurrent noise spectra, computed for various Q'values, are given in Fig. 5.

2. A SPECIFIC SCHEME (A COMMUNICATION LINE)

We now turn to a specific example of a communication line employing the advantages of quantum states of light. It is depicted schematically in Fig. 6.

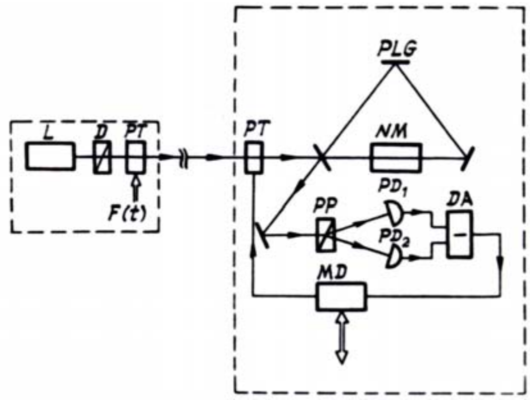

FIG. 6. Communication line scheme: L – laser; P – polarizer; PT – phase transformer, PLG – cavity parametric generator of light including nonlinear medium NM; PP – polarization prism; PD – photodetector; DA – differential amplifier; MD – matching device.

The transmitter consists of a source of coherent plane-polarized radiation and a phase transformer, PT, which follows the control signal $F(t)$, carrying the transmitted information, to rotate the polarization plane appropriately. In the receiver a squeezed quantum state the of the light field is produced in a cavity parametric generator. The polarization planes of its signal and blank modes are mutually orthogonal. The interaction in terms of the generated frequencies is degenerate. The emitted modes are mixed by a polarization prism PP in accordance with the relationship (5) and are sent to two photodetectors PD1 and PD2. The polarization planes of the prism are crossed with respect to those of the signal and blank waves at an angle of $\Pi/4$. Thus, in the absence of a signal the differential photocurrent remains zero. A signal, on the other hand, works as a heterodyne and drives the whole device out of this balance; it is restored by the feedback system, which produces the needed change in polarization by means of the detector phototransformer PT. The signal, driving the feedback system to control and restore the balance in the device, characterizes the value of $F(t)$. Thus the "operating point" of the system is its zero differential photocurrent, and, by mixing the received signal at the exit mirror of the cavity with radiation generated in it, its noises, in accordance with (10), can be suppressed, in the low-frequency spectral domain to below the level of the shot noise.

Photodetection shot noise does not play a limiting role in the considered scheme any more, and does not determine the threshold intensity of the received signal. Consequently, to transmit the same amount of information one would need fewer photons. The gain is then determined by the achieved depth of squeezing and by the degree to which phase fluctuations of the transmitted signal (in the classical sense) are suppressed with respect to the pumping phase, since the signal in that case plays the role of the heterodyne. It is also possible to track the signal phase in the detector via a feedback line. The minimal level of noise in the differential photocurrent serves as a criterion of optimum reaction to phase deviations.

Available experimental data[5–7] on producing quantum states indicate the possibility of halving the number of photons needed for confident reception of signals as an advantage of such systems. However, the untapped possibilities for improving this technology and choosing its optimal regimes give grounds for expecting even more spectacular results.

CONCLUSION

A quantum solution is suggested to the problem of light field evolution in a nonlinear ring cavity of arbitrary Q, in which nondegenerate parametric interaction takes place. The results obtained differ considerably from the already available conclusions and predictions stemming from a semiclassical description. The reason for this disagreement consists of the divergence between classical and quantum-mechanical descriptions of reflection from the semi-transparent exit mirror. While the classical wave splits in a regular way at such a mirror, a stream of quanta divides in a probabilistic manner. The lower the mirror reflectance, the stronger this feature is revealed in the resulting reflected wave. This explains the increasing quantitative differences between the semiclassical and quantum-mechanical descriptions at lower cavity Q's.

Furthermore, the scheme of an optical communication channel has been suggested, which exploits the advantages of the squeezed state of the light field. The limiting threshold intensity of the received signal can then be reduced because of the suppression of the shot detection noise. A more economic regime of information transmission is achieved because fewer photons have to be transmitted.

We are grateful to A.S. Chirkin for his helpful discussions.

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