Prediction of rain showers in the areas of big cities. Model and methodology

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We describe the increasing anthropogenic influence on the processes resulting in the formation of convective clouds and rain showers in the areas of big cities. The three-dimensional model of a convective cloud is proposed that allows for the microphysical processes of the drop generation and growth. The numerical scheme for the model realization, based on the splitting method, is developed for the use at the advection stage of the monotonic scheme. The methodology is proposed for predicting rain showers in the areas of big cities.

Introduction

The convective airflows depend both on thermal factors (buoyancy) and dynamic ones; the thermal instability of the atmosphere plays a key role in the development of convection. A local superheat of the air near the underlying surface can serve the starting stimulus for the development of convection, as well as the turbulence or macroscale (mesoscale) dynamic interaction of the flow with the underlying surface. It should be noted that cumulonimbus clouds are produced not by separate thermal springs but by quasi-ordered mesoscale air ascent.[1] Such an ascent occurs most often above the windward mountain slopes, in the gravitational wave crests, and above the areas where thermal characteristics and the roughness of the underlying surface change sharply (near the banks of rivers and sea cost, in the vicinity of urban areas). The contaminants emitted to air over big cities, in combination with the peculiarities of the underlying surface (considerable roughness, increased thermal conductivity, small albedo, etc.), and the availability of industrial heat sources have a noticeable effect on the micro- and mesoclimate regime in the city and its environments. Under the action of these factors in big cities considerable changes occur in the distributions of temperature and humidity of the air, wind velocity, radiation, visibility, the amount of precipitation, the conditions of formation of clouds and fogs.[2]

Thus, it becomes evident that it is necessary to take into account the peculiarities of the underlying surface, heat release, distribution of condensation centers in the regions of big cities in modeling cloud formation and in predicting the precipitation.

This paper describes the problem of predicting rain showers in big cities with the account of the above factors.

Set of equations of the model

As shown in Refs. 3–6 one can take, as the model of a convective cloud, the set of three-dimensional nonstationary equations of thermohydrodynamics in the nonhydrostatic approximation written with the conventional simplifications for the phenomena of mesometeorological scale (without the account of the Coriolis force) and the kinetic equations of coagulation (KEC) for the particle spectra in a three-phase medium.

Equations of motion, taking into account the advective and convective transfer, turbulence, forces of buoyancy and baric gradient, are written in the form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\partial \pi'}{\partial x} + \Delta' u, \qquad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial \pi'}{\partial y} + \Delta' v, \qquad (2)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} =$$

$$= -\frac{\partial \pi'}{\partial z}\,\Delta' u + g\left[\frac{\theta'}{\theta_0} + 0.61q' - q_L\right], \qquad (3)$$

where u, v, w are the x, y, and z components of the vector of the air motion velocity, respectively; θ' and q' are the deviations of the potential temperature and water vapor mass mixing ratio from those, $\theta_0(z)$ and $q_0(z)$, of the unperturbed atmosphere; q_L is the specific water content; the dimensionless pressure π is connected with the pressure p by the relationship $\pi = c_p\,\bar{\theta}(p/1000)^{R/c_p}$, where R is the universal gas constant, c_p is the specific heat of the air at constant pressure, $\bar{\theta}$ is the mean potential temperature; π' is the deviation of the dimensionless pressure from its background value;

$$\Delta' \equiv \frac{\partial}{\partial x}K\frac{\partial}{\partial x} + \frac{\partial}{\partial y}K\frac{\partial}{\partial y} + \frac{\partial}{\partial z}K\frac{\partial}{\partial z},$$

where K is the turbulence coefficient calculated based on the equation of turbulence energy balance.[7]

The continuity equation is written in the form

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=\sigma w, \qquad (4)$$

where the factor $\sigma = \mathrm{d}\ln\rho_0(z)/\mathrm{d}z$ allows for the change of air density $\rho_0(z)$ with height. At deep convection σ is nonzero. Otherwise it is assumed that $\sigma = 0$.[8]

The equation of energy conservation is written as applied to the potential temperature θ

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{L}{c_p}\frac{\theta}{T}M_c+\Delta'\theta, \qquad (5)$$

where M_c is the mass of vapor condensed (evaporated) per unit time; L is the specific heat of condensation; T is the thermodynamic temperature.

The equation of water mass conservation is analogous to Eq. (5) and is written for the mass mixing ratio of water vapor

$$\frac{\partial q}{\partial t}+u\frac{\partial q}{\partial x}+v\frac{\partial q}{\partial y}+w\frac{\partial q}{\partial z}=-M_c+\Delta' q. \qquad (6)$$

To describe the processes of occurrence, growth, and disappearance of cloud particles, the kinetic equations are included in the model. These equations describe the transformation of densities of size distribution of cloud drops and condensation nuclei (CN). The kinetic equation for the cloud drop size-distribution function $f(x, y, z, m, t)$ is written in the form

$$\frac{\partial f}{\partial t}+u\frac{\partial}{\partial x}(uf)+v\frac{\partial}{\partial y}(vf)+w\frac{\partial}{\partial z}[(w-v_m)f]=$$
$$=\left(\frac{\partial f}{\partial t}\right)_{\text{cond-evap}}+\left(\frac{\partial f}{\partial t}\right)_{\text{coag}}+\left(\frac{\partial f}{\partial t}\right)_{\text{decay}}+\left(\frac{\partial f}{\partial t}\right)_{\text{nucl}}+\Delta' f, \qquad (7)$$

where v_m is the fall velocity of a drop with the mass m. The terms in the right-hand side of Eq. (7) take into account the transformation of the function f due to the processes of condensation and evaporation, coagulation, cloud drop decay, the drop increase due to activation of condensation nuclei and turbulent transfer. The range of cloud drop radii (in the model it is 4 – 3250 μm) is divided into 30 logarithmic-equal classes and Eq. (7) is solved for each specific class separately.

The equation for the distribution density $n(x, y, z, r_n, t)$ of condensation nuclei is a little bit simpler than Eq. (7) and has the form

$$\frac{\partial n}{\partial t}+u\frac{\partial}{\partial x}(un)+v\frac{\partial}{\partial y}(vn)+w\frac{\partial}{\partial z}(wn)=$$
$$=-\left(\frac{\partial n}{\partial t}\right)_{\text{nucl}}+\Delta' n. \qquad (8)$$

The range of radii of condensation nuclei (0.0076–7.58 μm) is divided into 19 intervals, as is done in Ref. 9, and Eq. (8) is solved for each of them.

For the numerical solution of a given set of equations it is necessary to determine the boundary conditions, and for initiating the convection one should define the corresponding initial conditions.

Boundary and initial conditions

The boundary conditions of the type of open boundary are used for the fields of temperature, humidity, turbulence energy, and drop concentration at the side walls of the atmospheric volume considered. By this it is meant that at those boundary points, where the vector of air velocity is directed inside the volume modeled, the values of the above listed characteristics at the boundary are equal to their initial values. Otherwise the derivative of these characteristics along the normal to the corresponding boundary of the region is equal to zero. At the lower boundary, the condition of unflowing is set and at the upper boundary the condition is set on the free surface.

The initial distributions of these parameters are used as background distributions of temperature and humidity during the entire calculation. The initial conditions are set based on the data of temperature and wind profiling of the atmosphere. Thus obtained profiles of the components of wind velocity, temperature, and humidity are transformed to three-dimensional horizontally homogeneous fields of the corresponding characteristics. In this case the fields of temperature and pressure are adapted in such a manner that the hydrostatics ratio holds.

The convection is initiated by the disturbance of the temperature from a heat source located on the Earth's surface. At the time $t = 0$ it is assumed that

$$\theta = \theta_0(z) + \theta'(x, y, z);\ q = q_0(z);\ f(r_i) = 0,\ i = 1(1)30;$$
$$n(r_j) = n_{j0}(z),\ j = 1(1)19.$$

Numerical scheme of model realization

The set of equations of the model of convective cloud (1)–(8), describing the change of its dynamic and microphysical characteristics in time, consists of 3 equations of motion, equations of heat and humidity balance, 30 kinetic equations for cloud drops, and 19 kinetic equations for the condensation centers. Besides, in order that the solution satisfies the continuity equation (2), it is necessary to solve, at each time step, the Poisson three-dimensional equation for the pressure perturbation. For solving these problems the spectroscopic splitting method developed by G.I. Marchuk[10] is widely used.

The splitting method can be realized by successively taking into account separate operators of the set of equations. In solving the model equations we used different numerical schemes. At the stage of advective transfer the solution technique mainly follows the scheme described in Ref. 6. The sole exception is that the Smolarkewicz scheme was used except for that of the type of the predictor-divergent corrector.[11] The use of this scheme was caused by the fact that in passing to the implicit scheme the fully positive functions can take negative values, while the additional use of the monotonic scheme enables us to avoid the above drawback.

At the next stage, the condensation process is taken into account. Equations, solved at this stage have the form

$$\frac{\partial\theta}{\partial t} = \frac{L}{c_p}\frac{\theta^{m+4/6}}{T^{m+4/6}} M_c ,$$

$$\frac{\partial q}{\partial t} = - M_c , \frac{\partial n}{\partial t} = -\left(\frac{\partial n}{\partial t}\right)_{\text{nucl}} , \quad (9)$$

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_{\text{cond-evap}} + \left(\frac{\partial f}{\partial t}\right)_{\text{nucl}}, \; t_m \le t \le t_m + \Delta t.$$

The set of equations includes Eq. (9) and the equation of condensation drop growth written as

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{b(T, p)\,(1 + \xi - B^*)}{r + a(T, p)/\alpha} , \quad (10)$$

where $a(T, p)$, $b(T, p)$ are the known functions of temperature and pressure[6]; B^* is the factor allowing for the effect of drop curvature and drop-soluble salts on the pressure of saturated vapor (equilibrium vapor pressure); ξ is the supersaturation per unit volume; r is the drop radius; α is the condensation coefficient.

This set is solved using the initial conditions

$$\theta\Big|_{t=t_m} = \theta^{m+4/6};\; q\Big|_{t=t_m} = q^{m+4/6},$$

$$f_i\Big|_{t=t_m} = f_i^{m+4/6};\; n_j\Big|_{t=t_m} = n_j^{m+4/6}, \quad (11)$$

where $\theta^{m+4/6}$, $q^{m+4/6}$, $f_i^{m+4/6}$, $n_j^{m+4/6}$ are the values of the appropriate parameters after the realization of splitting stages describing the advective transfer.

For stability, the condensation process is calculated using a smaller step $\Delta\tau$.

The successive calculations are performed at this stage:

1. We determine the initial values of the potential temperature, mass mixing ratio of water vapor as well as the functions n and f according to Eq. (11).

2. Using the values of the potential temperature, we calculate the absolute temperature T and the corresponding value of the saturating specific humidity of water vapor $q_s(T)$.

3. The supersaturation $\xi = q/q_s(T) - 1$ is determined.

4. We determine the growth, during the time $\Delta\tau$, of the radius of cloud drops and condensation nuclei activated at a given supersaturation parameter, ξ.

5. The mass of condensed (evaporated) humidity is calculated by the formula

$$M_c\,\Delta t = \rho_w \left[\int_{r_0^{\text{nucl}}}^{r_{\max}^{\text{nucl}}} 4\pi r^2\,\dot r\, n(r)\,\mathrm{d}r + \int_{r_0^{\text{c.d}}}^{r_{\max}^{\text{c.d}}} 4\pi r^2\,\dot r\, f(r)\,\mathrm{d}r\right]\Delta t.$$

6. We determine the variations in the temperature $\Delta T_{\text{cond}}(\Delta\tau)$ and mass mixing ratio of water vapor $\Delta q_{\text{cond}}(\Delta\tau)$ due to the condensation (evaporation) of the water vapor mass δM.

7. We calculate new values of T and q

$$T \to T + \Delta T_{\text{cond}}(\Delta\tau);\; q \to q + \Delta q_{\text{cond}}(\Delta\tau).$$

8. The operations 3–7 are repeated for the next moment in time during the entire dynamic step.

9. In the end of the dynamic stage, at $t = t_m + \Delta t$, new values of $n(r_j)$ and $f(r_i)$ are calculated by the interpolation method conserving the particle number and mass.[12] At the final stage the set of equations is solved describing the adaptation of the dynamic fields

$$u^{m+1} = u^{m+4/6} - \frac{\partial\pi'}{\partial x}\Delta t ,$$

$$v^{m+1} = v^{m+4/6} - \frac{\partial\pi'}{\partial y}\Delta t , \quad (12)$$

$$w^{m+1} = w^{m+4/6} - \frac{\partial\pi'}{\partial z}\Delta t +$$

$$+ g\,\Delta t \left\{\frac{\theta^{m+1} - \theta_0(z)}{\theta_0(z)} + 0.61\,[q^{m+1} - q_0(z)] - q_L^{m+1}\right\} .$$

Having differentiated Eqs. (6) and called for solving the continuity equation, we are led to the three-dimensional elliptic equation for the pressure perturbation:

$$\frac{\partial^2\pi'^{\,m+1}}{\partial x^2} + \frac{\partial^2\pi'^{\,m+1}}{\partial y^2} + \frac{\partial^2\pi'^{\,m+1}}{\partial z^2} -$$

$$- \sigma\frac{\partial^2\pi'^{\,m+1}}{\partial z} = F^{m+1}(x, y, z);$$

$$F^{m+1} = \frac{1}{\Delta t}\left[(\nabla\mathbf{U})^{m+4/6} - \sigma w^{m+4/6}\right] + g\left(\frac{\partial}{\partial z} - \sigma\right)\times$$

$$\times\left\{\frac{\theta^{m+1} - \theta_0(z)}{\theta_0(z)} + 0.61\,[q^{m+1} - q_0(z)] - q_L^{m+1}\right\} , \quad (13)$$

$$\left.\frac{\partial\pi'}{\delta x}\right|_{z=0,L_x} = \left.\frac{\partial\pi'}{\delta y}\right|_{z=0,L_y} = 0 ,$$

$$\left.\frac{\partial\pi'}{\delta z}\right|_{z=0,L_x} = g\left(\frac{\theta'}{\theta_0} + 0.61q' - q_L\right).$$

The technique of solving the Eq. (13) is similar to that used in Ref. 6, i.e., we used an implicit two-step method of the upper block relaxation.[13] In contrast to Ref. 6, taking into account the difference in horizontal and vertical scales of the running processes, we used different spatial steps. In the final-difference form Eq. (13) is written as

$$B^2 \pi'^{m+1}_{i+1,j,k} + B^2 \pi'^{m+1}_{i-1,j,k} + C^2 \pi'^{m+1}_{i,j+1,k} + C^2 \pi'^{m+1}_{i,j-1,k} + A^2 \pi'^{m+1}_{i,j,k+1} + A^2 (1 + \Delta z\sigma) \pi'^{m+1}_{i,j,k-1} = 2 (A^2 + B^2 + C^2 + \frac{1}{2} A^2 \Delta z\sigma) \pi'^{m+1}_{i,j,k} + ABCF'^{m+1}_{i,j,k}(x,y,z), \quad (14)$$

where i, j, k are the indices corresponding to the variables along the x, y, and z axes, respectively;

$$A = \Delta x \; \Delta y; \; B = \Delta y \; \Delta z; \; C = \Delta x \; \Delta z.$$

Equation (14) is solved by the iteration method of block relaxation[13] divided into two stages. At the first stage the proper relaxation is achieved along a certain fixed direction, and at the second stage we calculate the final values of $\pi'^{m+1}_{i,j,k}$ for a given iteration.

The volume simulated is a parallelepiped with the number of nodes 32400 (30×30×36 nodes of regular grid with a computing volume element 1×1×0.4 km^3).

Prediction technique

The rain shower prediction technique, based on the proposed model, is as follows. First, we assign the physical and mathematical constants necessary for solving the equations of the model. Then we perform the procedure of assigning the initial and boundary conditions. In this procedure:

1. Setting the positions of nodes in space on a regular grid within the volume simulated.

2. The values of the functions are set at the boundaries of the volume.

3. Based on the predicted results on the fields of wind, temperature, and air humidity with the use of the model of processes of synoptic scale, the vertical distributions of the above parameters are determined at the moments to which the developed prediction refers.

4. These data agree with each other, as it is stated above.

5. The values are found of the superheat near the underlying surface and the vertical distribution of the concentration of condensation nuclei of different size in the region of a big city.

6. The numerical realization of the model is performed by successively performing the following procedures:

– solution of equations determining the predictor where the equations of transfer and turbulent exchange are solved separately for each spatial coordinate;

– correction of the solution, in which the values of the functions at the temporal step $t_m + 4/6 \, \Delta t$ are refined;

– taking account of the condensation process, in which the amount of condensed water is determined as well as the variation of the size distribution of the condensation nuclei and cloud droplets.

– adaptation of dynamic fields where the three-dimensional elliptic equation is solved for the pressure disturbance that provides satisfying the solutions of the continuity equation.

These procedures are performed during one step in time; next the entire procedure of integration over the time is performed cyclically up to the moment in time equal to the directive time of prediction plus one hour. If it has been found from the simulation that rain showers are formed, then the forecast of this phenomenon is given for the above-mentioned time.

The results of numerical experiments using the above technique, performed by the authors to date, point to the technique practicability and high quality of the forecasts. Thus, as an experiment, we analyzed the case with the intense precipitation in Moscow on April 12, 1998. The weather on that day was conditioned by the passage of cold weather front. During the period from 6:00 a.m. to 18:00 p.m. of GMT in Moscow 21 mm precipitation (snow and snow and rain) was recorded at 0–1.8°C temperature.

It can be assumed that the precipitation on that day was steady. Then, using the A.F. Dyubyuk method of forecasting the steady rain based on the determination of individual variation of water vapor mass portion in the saturated air passing both vertically and horizontally, we could forecast rain at the intensity of 5.8 mm/12 h. In this case, the forecast error would be about 77%.

Taking into account the convection effects, based on the hypothesis of conditional instability of the second type, the predicted amount of precipitation during the considered period was 8.4 mm. Thus, the account of the forced convection makes it possible to decrease the forecast error for this case by 17%.

When using the model of atmospheric front, proposed by S.A. Soldatenko, where the forced convection and frontal effects are considered, the following results are obtained. In this case, the temperature contrast in the front area is about 8°C, and the velocity gap of geostrophic wind is about 10 m/s. At relative humidity of warm air within the limits from 90 to 95% the amount of precipitation can reach 12.2 mm during 12 hours. The forecast error (as compared with the standard methods) is reduced by 45%.

When using the proposed technique of the rain showers forecast, which takes into account the effect of heat source in the area of big city, the prognostic estimate of the precipitation amount during the considered period (only if taking into account the convective effects) is 11.8 mm. This estimate is comparable with the estimate obtained with the use of the model of atmospheric front that illustrates the practicability of the developed technique.

Statistically significant estimates of the quality of the proposed technique are planned to be obtained during further investigations.

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