A MODEL OF PLANETARY REFRACTION IN THE EARTH'S ATMOSPHERE

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Some model dependences of the planetary refractive angle and its derivative on the height of the perigee of the viewing ray are calculated employing the models of vertical air density profiles available from the literature. The formulas are given for calculating the coefficient of the light flux extinction due to refraction when viewing the extraterrestrial light sources through the atmosphere from satellite. The developed model may be employed to estimate the hight of the perigee of the viewing ray from the measurements of the atmospheric refraction angles, in particular, to solve the problems of autonomous satellite navigation.

INTRODUCTION

Planetary refraction in the Earth's atmosphere has an appreciable effect on the conditions of viewing the extraterrestrial sources of the electromagnetic radiation from satellite. The refraction angle of the optical ray which is tangent to the Earth's surface is about $2 \cdot 10^{-2}$ rad, i.e., is approximately equal to two angular diameters of the sun. In addition to the angular displacement of the images of the point sources (for example, the stars) atmospheric refraction results in the significant distortion of the images of the sources with the finite angular dimensions (sun and moon) due to the variation in the refractive angle vs the hight of the perigee of the sensing ray. The atmospheric refraction can also result in the significant change (on the average, to the decrease) of the radiant flux density because of its defocusing in the atmosphere. Finally, the dependence of the refractive index of air on the light wavelength results in the dispersion smearing of the images of the sources which, on the one hand, can be considered as noise in measuring the refractive angles and, on the other, being measured, it carries the information about the refractive angles. Foregoing discussion indicates the necessity of taking into consideration the atmospheric refraction in solving the numerous problems of radio and optical satellite observations using a limb.

Since the atmospheric refraction is determined by the refractive index distribution, i. e. , in fact, by the density of air, the relatively small variability of density results in higher stability of the refractive properties of the atmosphere in comparison with other optical properties which are associated with the extinction phenomenon. In this connection, the methods based on the measurements of the refraction angles for determining the height of the perigee of the ray and for solving the problems of autonomous satellite navigation are of definite interest.[1] In addition, the refraction reference of the viewing ray to the hight can be employed to interpret the data obtained from other optical measurements using a limb.

For solving the enumerated problems, an adequate model of the atmospheric refraction is required, i.e., some standard dependences of the atmospheric refractive angle and the coefficient of extinction of the radiantant flux density due to refraction on the hight of the perigee of the sensing ray and on the heights of a source and a receiver at different latitudes and in different seasons. The variability of the above–indicated dependences are required as well. Such a model was developed on the basis of calculations made during several years in the Institute of Atmospheric Physics of the Academy of Sciences of the USSR. The main results of these calculations were given in Ref. 2. The calculations were based on the standard model of the atmosphere[3] as well as on the data on the statistical structure of meteorological parameters.[4] Now a new model of the atmosphere is developed. The model is more accurate and detailed and incorporates the statistical characteristics of the variability of the meteorological parameters at the high altitudes.[5] In addition, the technique for evaluating the refraction effects were modified. All this made it possible to develop a new model of the atmospheric refraction whose main points (the technique for calculating and some results) are given in this paper.

PRINCIPAL RELATIONS

We shall consider the propagation of electromagnetic waves within the geometric optics approximation and assume the spherical symmetry of the refractive index distribution whose vertical profile intersects the point of the perigee the sensing ray. In the optical range the refractive index n is given (with an accuracy of 0.3%) by the formula[5]

$$n = 1 + C(\lambda)\rho = 1 + 2.227 \cdot 10^{-4}(1 + 7.526 \cdot 10^{-3}/\lambda^2)\rho , \quad (1)$$

where ρ is the density of air in kg/m^3 and λ is the wavelength, in μm.

The trajectory of ray may be described by the well–known Snell's law (see Fig. 1a)

$$rn(r) \sin\varphi = p = \text{const} , \quad (2)$$

where $\rho = r_0 n(r_0)$ is the impact parameter and r_0 is the radius of the perigee of the ray.

The atmospheric refraction angle ε of the ray is expressed in terms of the index of refraction as follows:

$$\varepsilon = -2r_0 n(r_0) \int_{r_0}^{\infty} \frac{dn/dr}{n(r)n[r^2(r) - r_0^2(r_0)]^{1/2}} \, dr \equiv \quad (3)$$

$$\equiv -2p \int\limits_{p}^{\infty} \frac{d\ln n/dx}{(x^2 - p^2)^{1/2}} dx \simeq -(2p)^{1/2} \int\limits_{p}^{\infty} \frac{dn/dx}{(x - p)^{1/2}} dx, \quad (3a)$$

where $x = nr(r)$ and Eq. (3a) is satisfied with the accuracy of 0.05%.

The coefficient of variation of the radiant flux density due to refraction is determined by the ratio of the cros sections of the ray tubes $K = \delta s_0/\delta s$ (see Fig. 1b) and can be written on the basis of Eqs. (2) and (3) in the form

$$K = l_0(l_1 + l_2 - l_1 l_2 d\varepsilon/dp)^{-1}, \quad (4)$$

where $l_1 = r_1\cos\varphi_1$, $l_2 = r_2\cos\varphi_2$, $l_0 = (r_1^2 + r_2^2 - 2r_1 r_2\cos\vartheta)^{1/2}$. For the infinitely remote sources (for example, the stars $(l_0 = l_1 = \infty)$ we obtain instead of Eq. (4)

$$K = (1 - l_2 d\varepsilon/dp)^{-1}. \quad (5)$$

It should be noted that in Eqs. (4) and (5) the value $d\varepsilon/dp$ is determined only by the vertical profile of the refractive index and is independent of r_1, r_2, and ϑ and, therefore, can be represented in the form of the model and used for calculating the variation of the radiant flux density due to refraction from formulas (4) and (5) for different viewing geometry.
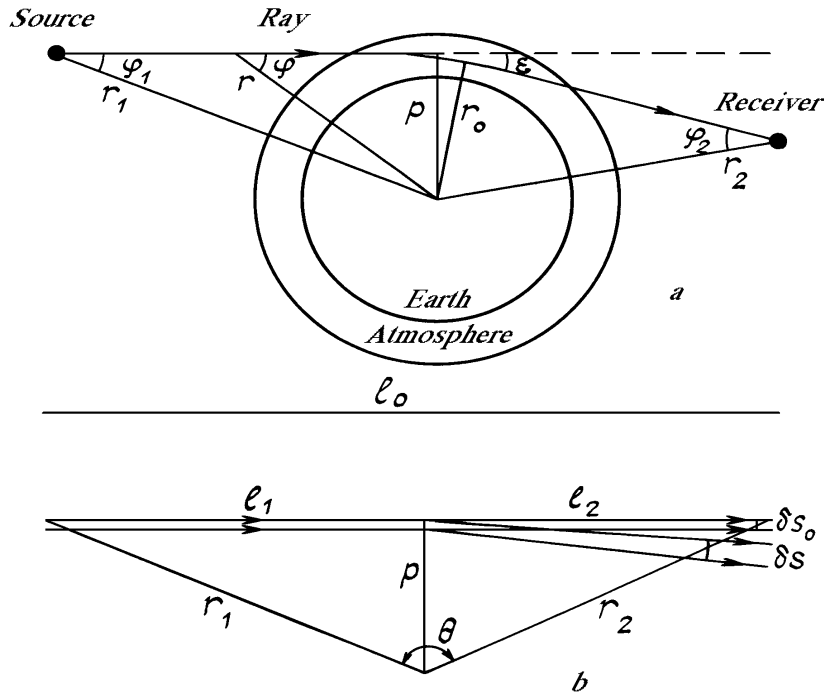

FIG. 1. The sensing ray geometry (a) and the variation of the cross section of the ray tube when viewing the extraterrestrial light sources through the atmosphere from satellite (b).

From formula (4) it follows that the coefficient K is invariant with respect to the transposition of the source and receiver of radiation (that corresponds to the well known reciprocity theorem. In addition, as it follows from the definition, the value K characterizes also the change of the visible vertical angular dimension of the small element of the image of the source with finite angular dimensions.

To obtain the main statistical characteristic of the variability of the refraction angles, namely, the standard deviation σ_ε, we can use relation (3a) which is linear in n, from which we obtain

$$\sigma_\varepsilon^2(p) = <\delta\varepsilon(p)^2> = 2p \int\limits_{p}^{\infty}\int \frac{\partial^2}{\partial x'\partial x''} <\delta n(x') \, \delta n(x'')> \times$$

$$\times (x' - p)^{-1/2}(x'' - p)^{-1/2}dx'dx'', \quad (6)$$

where $\delta n = n - <n>$, $\delta\varepsilon = \varepsilon - <\varepsilon>$ are the standard deviations from the their mean values. Thus, to calculate σ_ε, the correlation function of variations in the refractive

index is required. However, in practice the aerological measurements are made at the finite number of discrete hights and, as a rule, the correlation matrices of the density $<\delta\rho_i\delta\rho_j> = <\delta\rho(z_i)\rho\delta(z_j)>$, where i, $j = 1, ..., m$, can be experimentally obtained, from which it follows that $<\delta n_i\delta n_j> = C^2(\lambda)<\delta\rho_i\delta\rho_j>$. Therefore, in calculations according to Eq. (6), we must use the interpolation, and this means that the contribution of the high—frequency range in the spectrum of variations of the refractive index cannot be correctly taken into account. Evidently, the most reliable way to estimate σ_ε is the direct measurements of changes of the atmospheric refractive angles.

TECHNIQUE FOR CALCULATIONS AND THE RESULTS

As the input data for calculating the standard dependences $\varepsilon(p)$ and $K(p)$, the standard vertical profiles of the air density were employed. These profiles corresponded to the different latitudes and seasons and were taken with a step of 2 km within the 0—90 km

altitude range[5] (below the height $z = r - r_E$ together with r as well as the height of the perigee of the viewing line $y = p - r_E$ will be used as the independent variables. Here r_E is the Earth's radius). The index of refraction was calculated from formula (1) at $\lambda = 0.8\ \mu m$. The profile $n(x) - 1$ specified at the nodes of the grid $x_i = r_i\, n(r_i)$ was interpolated between these nodes with the help of real cubic spline[6] for the altitudes $0 \leq z \leq 90$ km. This made it possible, on the one hand, to calculate integral (3a) analytically in calculating the dependence $\varepsilon(p)$ and on the other, to ensure sufficiently smooth profile $n(x)$ and, consequently, the profile $\varepsilon(p)$, taking into account the mathematical incorrectness of the problem of determining $\varepsilon(p)$ from formulas (3) and (3a), which include the differentiation operation. Since the problem of determining $d\varepsilon/dp$ is also mathematically incorrect, the function $\varepsilon(p)$ obtained at the nodes of the grid was interpolated again by the real cubic spline for the altitudes $0 \leq z \leq 50$ km, after that the derivative $d\varepsilon/dp$ was calculated analytically. The results of calculations for the winter and summer atmospheres at mid−latitudes (45°) are listed in Table I. The data presented in Table I make it possible to calculate the refractive index n, the atmospheric refractive angle ε, and the coefficient K of the extinction of the radiant flux density caused by refraction for the point source (the vertical component of the angular deformation of the element of the image of the extended source) from formulas (4) and (5) at $\lambda = 0.8\ \mu m$.

To calculate $\sigma_\varepsilon(p)$, for the initial data we used the vertical correlation matrices of the density obtained in Ref. 5 with a step of 2 km within the 0−60 km altitude ranges for the winter and summer atmospheres at several stations located at different latitudes. Since the realization ensemble consisted of numerous measurements carried out at the same point in the same season, we finally obtained the spread in values which was caused by the changes in the weather conditions. We calculated the vertical correlation matrices of the refractive index on the basis of these data. The matrices columns $\langle \delta n(x_i')\delta n(x_j'')\rangle$ were interpolated by real cubic splines and, hence, the array of the first derivative $\partial/\partial x_i' \langle \delta n(x_i')\,\delta n(x_j'')\rangle$ was calculated analytically. After that the obtained array was analogously interpolated over the rows by real cubic splines and in this way the array of the second derivative $\partial^2/\partial x_i' x_j'' \langle \delta n(x')\,\delta n(x'')\rangle$ was calculated. The second derivative specified in such a way in the form of the matrix was determined between the nodes as arithmetic mean of four nearest values and then the integral in Eq. (6) was calculated analytically. Table II gives the results σ_ε calculated from the data obtained in winter and summer at the station Wallops Island (38°N). It can be seen from Table II that in winter the weather variations of the atmospheric refraction angles were more significant than in summer and, in addition, they reach the local maximum at the 10−12 km altitude ranges (in tropopause).

TABLE I. Mid−latitudes, January.

z, (km)	y, (km)	$N-1$	ε, (rad)	$d\varepsilon/dp$, (rad/km)
0	1.867	.2932E−03	.2172E−01	−.2883E−02
1	2.670	.2615E−03	.1938E−01	−.2791E−02
2	3.486	.2333E−03	.1723E−01	−.2557E−02
3	4.326	.2084E−03	.1523E−01	−.2149E−02
4	5.187	.1863E−03	.1352E−01	−.1762E−02
5	6.062	.1666E−03	.1217E−01	−.1503E−02
6	6.948	.1487E−03	.1097E−01	−.1304E−02
7	7.843	.1322E−03	.9801E−02	−.1119E−02
8	8.748	.1174E−03	.8811E−02	−.9701E−03
9	9.663	.1042E−03	.8057E−02	−.8815E−03
10	10.583	.9154E−04	.7307E−02	−.8708E−03
11	11.503	.7891E−04	.6408E−02	−.9375E−03
12	12.430	.6743E−04	.5488E−02	−.9590E−03
13	13.368	.5777E−04	.4668E−02	−.8271E−03
14	14.315	.4959E−04	.3975E−02	−.6589E−03
15	15.271	.4251E−04	.3404E−02	−.5437E−03
16	16.232	.3642E−04	.2921E−02	−.4642E−03
17	17.199	.3120E−04	.2502E−02	−.3997E−03
18	18.170	.2672E−04	.2141E−02	−.3433E−03
19	19.146	.2286E−04	.1832E−02	−.2931E−03
20	20.124	.1954E−04	.1568E−02	−.2504E−03
21	21.106	.1669E−04	.1339E−02	−.2153E−03
22	22.090	.1425E−04	.1142E−02	−.1846E−03
23	23.077	.1217E−04	.9741E−03	−.1562E−03
24	24.066	.1040E−04	.8322E−03	−.1314E−03
25	25.056	.8886E−04	.7128E−03	−.1110E−03
26	26.048	.7587E−04	.6110E−03	−.9517E−04
27	27.041	.6463E−05	.5227E−03	−.8318E−04
28	28.035	.5492E−05	.4452E−03	−.7313E−04
29	29.029	.4659E−05	.3769E−03	−.6360E−04
30	30.025	.3953E−05	.3181E−03	−.5420E−04
31	31.021	.3362E−05	.2689E−03	−.4514E−04
32	32.018	.2865E−05	.2279E−03	−.3736E−04
33	33.015	.2445E−05	.1939E−03	−.3128E−04
34	34.013	.2089E−05	.1652E−03	−.2651E−04
35	35.010	.1785E−05	.1406E−03	−.2265E−04
36	36.009	.1527E−05	.1197E−03	−.1935E−04
37	37.008	.1308E−05	.1019E−03	−.1641E−04
38	38.007	.1122E−05	.8683E−04	−.1383E−04
39	39.006	.9645E−06	.7413E−04	−.1163E−04
40	40.004	.8308E−06	.6343E−04	−.9818E−05
41	41.004	.7171E−06	.5442E−04	−.8368E−05
42	42.003	.6198E−06	.4670E−04	−.7166E−05
43	43.003	.5363E−06	.4001E−04	−.6125E−05
44	44.002	.4655E−06	.3434E−04	−.5189E−05
45	45.002	.4058E−06	.2962E−04	−.4342E−05
46	46.001	.3549E−06	.2568E−04	−.3611E−05
47	47.002	.3110E−06	.2236E−04	−.3023E−05
48	48.001	.2730E−06	.1951E−04	−.2602E−05
49	49.001	.2403E−06	.1705E−04	−.2341E−05
50	50.000	.2118E−06	.1490E−04	−.2161E−05

TABLE I (continued). Mid–latitudes, July.

z,(km)	y, (km)	$N-1$	ε, (rad)	$d\varepsilon/dp$, (rad/km)
0	1.722	.2704E−03	.1876E−01	−.2181E−02
1	2.557	.2440E−03	.1692E−01	−.2045E−02
2	3.403	.2203E−03	.1528E−01	−.1860E−02
3	4.268	.1993E−03	.1381E−01	−.1612E−02
4	5.147	.1801E−03	.1250E−01	−.1385E−02
5	6.034	.1623E−03	.1133E−01	−.1231E−02
6	6.930	.1459E−03	.1027E−01	−.1123E−02
7	7.835	.1311E−03	.9309E−02	−.1032E−02
8	8.749	.1175E−03	.8417E−02	−.9355E−03
9	9.669	.1049E−03	.7590E−02	−.8262E−03
10	10.596	.9350E−04	.6862E−02	−.7219E−03
11	11.530	.8317E−04	.6246E−02	−.6414E−03
12	12.468	.7347E−04	.5675E−02	−.6065E−03
13	13.409	.6415E−04	.5092E−02	−.6256E−03
14	14.353	.5538E−04	.4484E−02	−.6507E−03
15	15.302	.4740E−04	.3866E−02	−.6277E−03
16	16.257	.4040E−04	.3294E−02	−.5613E−03
17	17.219	.3343E−04	.2802E−02	−.4765E−03
18	18.187	.2934E−04	.2381E−02	−.3988E−03
19	19.159	.2497E−04	.2021E−02	−.3391E−03
20	20.135	.2125E−04	.1714E−02	−.2896E−03
21	21.115	.1811E−04	.1454E−02	−.2440E−03
22	22.098	.1545E−04	.1234E−02	−.2040E−03
23	23.084	.1319E−04	.1050E−03	−.1708E−03
24	24.071	.1128E−04	.8954E−03	−.1434E−03
25	25.061	.9652E−04	.7650E−03	−.1210E−03
26	26.052	.8262E−04	.6544E−03	−.1029E−04
27	27.045	.7069E−05	.5597E−03	−.8850E−04
28	28.038	.6046E−05	.4780E−03	−.7640E−04
29	29.033	.5170E−05	.4072E−03	−.6568E−04
30	30.028	.4424E−05	.3467E−03	−.5595E−04
31	31.024	.3793E−05	.2956E−03	−.4716E−04
32	32.021	.3258E−05	.2525E−03	−.3966E−04
33	33.018	.2802E−05	.2161E−03	−.3353E−04
34	34.015	.2413E−05	.1852E−03	−.2848E−04
35	35.013	.2082E−05	.1590E−03	−.2422E−04
36	36.011	.1798E−05	.1366E−03	−.2061E−04
37	37.010	.1555E−05	.1176E−03	−.1757E−04
38	38.008	.1347E−05	.10143E−04	−.1501E−04
39	39.007	.1168E−06	.8750E−04	−.1284E−04
40	40.006	.1015E−06	.7561E−04	−.1100E−05
41	41.005	.8824E−06	.6542E−04	−.9436E−05
42	42.004	.7683E−06	.5648E−04	−.8089E−05
43	43.004	.6698E−06	.4917E−04	−.6962E−05
44	44.003	.5846E−06	.4268E−04	−.6041E−05
45	45.002	.5107E−06	.3704E−04	−.5299E−05
46	46.002	.4469E−06	.3213E−04	−.4584E−05
47	47.002	.3921E−06	.2788E−04	−.3838E−05
48	48.001	.3450E−06	.2430E−04	−.3213E−05
49	49.001	.3048E−06	.2135E−04	−.2784E−05
50	50.001	.2697E−06	.1886E−04	−.2467E−05

TABLE II.

z (km)	σ_ε (rad)	σ_ε (rad)
0	.79E−02	.50E−02
2	.68E−02	.23E−02
4	.55E−02	.12E−02
6	.33E−02	.80E−03
8	.21E−02	.71E−03
10	.20E−02	.86E−03
12	.29E−02	.81E−03
14	.22E−02	.85E−03
16	.14E−02	.81E−03
18	.11E−02	.50E−03
20	.61E−03	.25E−03
22	.52E−03	.15E−03
24	.30E−03	.10E−03
26	.16E−03	.81E−04
28	.12E−03	.81E−04
30	.95E−04	.64E−04
32	.73E−04	.50E−04
34	.64E−04	.37E−04
36	.49E−04	.26E−04
38	.42E−04	.22E−04
40	.41E−04	.22E−04

CONCLUSION

Because of limitations on the length of the article published in the journal the present paper gives only some basic results of calculations for solving a number of applicated problems associated with viewing the extrpaterrestrial light sources through the Earth's atmosphere from satellite. More complete data on the model of atmospheric refraction at different latitudes and in different seasons including the data which take into account the effects of dispersion are available from the Institute of Atmospheric Physics of the Academy of Sciences of the USSR.

REFERENCES

1. R.L. White, S. W. Thurman, and F. A. Barnes, Navigation **32**, No. 4, 317333 (Winter 19851986). .
2. G.M. Grechko, A.S. Gurvich, et al., *Optical Phenomena in Space Environment during Sunset and Sunrise*, Trudy S. I. Vavilov State Optical Institute, Leningrad (1989), Vol. 71, No. 205, Ch. 2.
3. U.S. Standard Atmosphere Supplements 1966/U.S. Air Force, Washington D. C. 20402, 289 pp.
4. *Reference Book on the Vertical Correlation of the Meteorological Parameters in the Upper Atmosphere above the USSR* (Gidrometeoizdat, Moscow, 1980), 397 pp.
5. *Handbook of Geophysics and the Space Environment*, Air Force Geophysics Laboratory, U. S. Air Force (1985), Ch. 14.
6. T.A. Shup, *Computation of the Engineering Problems* [Russian translation] (Mir, Moscow, 1982), 235 pp.